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SCIENTIFIC CONTRIBUTIONS
Recent Developments in Particle Physics (*).

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When I received my B. S. degree in 1932, only two of the fundamental particles of physics were known. Every bit of matter in the universe was thought to consist solely of protons and electrons. But in that same year, the number of particles was suddenly doubled. In two beautiful experiments, Chadwick [1] showed that the neutron existed, and Anderson [2] photographed the first unmistakable positron track. In the years since 1932, the list of known particles has increased rapidly, but not steadily. The growth has instead been concentrated into a series of spurts of activity.

Following the traditions of this occasion, my task this afternoon is to describe the latest of these periods of discovery, and to tell you of the development of the tools and techniques that made it possible. Most of us who become experimental physicists do so for two reasons; we love the tools of physics because to us they have intrinsic beauty, and we dream of finding new secrets of nature as important and as exciting as those uncovered by our scientific heroes. But we walk a narrow path with pitfalls on either side. If we spend all our time developing equipment, we risk the appellation of «plumber», and if we merely use the tools developed by others, we risk the censure of our peers for being parasitic. For these reasons, my colleagues and I are grateful to the Royal Swedish Academy of Science for citing both aspects of our work at the Lawrence Radiation Laboratory at the University of California—the observations of a new group of particles and the creation of the means for making those observations.

As a personal opinion, I would suggest that modern particle physics started in the last days of World War II, when a group of young Italians, Conversi, Pancini, and Piccioni, who were hiding from the German occupying forces, initiated a remarkable experiment. In 1946, they showed [3] that the «mesotron», which had been discovered in 1937 by Neddermeyer and Anderson [4] and by Street and Stevenson [5], was not the particle predicted by

Yukawa [6] as the mediator of nuclear forces, but was instead almost completely unreactive in a nuclear sense. Most nuclear physicists had spent the war years in military-related activities, secure in the belief that the Yukawa mesons was available for study as soon as hostilities ceased. But they were wrong.

The physics community had to endure less than a year of this nightmarish state; Powell and his collaborators [7] discovered in 1947 a singly charged particle (now known as the pion) that fulfilled the Yukawa prediction, and that decayed into the « mesotron », now known as the muon. Sanity was restored to particle physics, and the pion was found to be copiously produced in Ernest Lawrence’s 184-inch cyclotron, by Gardner and Lattes [8] in 1948. The cosmic ray studies of Powell’s group were made possible by the elegant nuclear emulsion technique they developed in collaboration with the Ilford laboratories under the direction of C. Waller.

In 1950, the pion family was filled out with its neutral component by three independent experiments. In Berkeley, at the 184-inch cyclotron, Moyer, York, et al. [9] measured a Doppler-shifted γ-ray spectrum that could only be explained as arising from the decay of a neutral pion, and Steinberger, Panofsky and Steller [10] made the case for this particle even more convincing by a beatiful experiment using McMillan’s new 300 MeV synchrotron. And independently at Bristol, Ekspong, Hopper, and King [11] observed the two-γ-ray decay of the π° in nuclear emulsion, and showed that its lifetime was less than 5 \cdot 10^{-14} \text{s}.

In 1952 Anderson, Fermi, and their collaborators [12] at Chicago started their classic experiments on the pion-nucleon interaction at what we would now call low energy. They used the external pion beams from the Chicago synchrocyclotron as a source of particles, and discovered what was for a long time called the pion-nucleon resonance. The isotopic spin formalism, which had been discussed for years by theorists since its enunciation in 1936 by Cassen and Condon [13], suddenly struck a responsive chord in the experimental physics community. They were impressed by the way Brueckner [14] showed that « I-spin » invariance could explain certain ratios of reaction cross-sections, if the resonance, which had been predicted many years earlier by Pauli and Dancoff [15] were in the \frac{2}{3} isotopic spin state, and had an angular momentum of \frac{3}{2}.

By any test we can now apply, the « 3,3 resonance » of Anderson, Fermi et al. was the first of the « new particles » to be discovered. But since the rules for determining what constitutes a discovery in physics have never been codified—as they have been in patent law—it is probably fair to say that it was not customary, in the days when the properties of the 3,3 resonance were of paramount importance to the high energy physics community, to
regard that resonance as a «particle». Neutron spectroscopists study hundreds of resonances in neutron-nucleus system which they do not regard as separate entities, even though their lives are billions of times as long. I don’t believe that an early and general recognition that the 3,3 resonance should be listed in the «table of particles» would in any way have speeded up the development of high energy physics.

Although the study of the production and the interaction of pions had passed in a decisive way from the cosmic ray groups to the accelerator laboratories in the late 1940’s, the cosmic-ray-oriented physicists soon found two new families of «strange particles»—the K mesons and the hyperons. The existence of the strange particles has had an enormous impact on the work done by our group at Berkeley. It is ironic that the parameters of the Bevatron were fixed and the decision to build that accelerator had been made before a single physicist in Berkeley really believed in the existence of strange particles. But as we look back on the evidence, it is obvious that the observations were well made, and the conclusions were properly drawn. Even if we had accepted the existence—an more pertinently the importance—of these particles, we would not have known what energy the Bevatron needed to produce strange particles; the associated production mechanism of Pais [16] and its experimental proof by Fowler, Shutt et al. [17] were still in the future. So the fact that, with a few notable exceptions, the Bevatron has made its greatest contributions to physics in the field of strange particles must be attributed to a very fortunate set of accidents.

The Bevatron’s proton energy of 6.3 GeV was chosen so that it would be able to produce antiprotons, if such particles could be produced. Since, in the interest of keeping the «list of particles» tractable, we no longer count antiparticles nor individual members of I-spin multiplets, it is becoming fashionable to regard the discovery of the antiproton as an «obvious exercise for the student». (If we were to apply the «new rules» to the classical work of Chadwick and Anderson, we would conclude that they hadn’t done anything either—the neutron is simply another I-spin state of the proton, and Anderson’s positron is simply the obvious anti-electron!) In support of the non-obvious nature of the Segrè group’s discovery of the antiproton [18] I need only recall that one of the most distinguished high energy physicists I know, who didn’t believe that antiprotons could be produced, was obliged to settle a 500-dollar bet with a colleague who held the now universally accepted belief that all particles can exist in an antistate.

I have just discussed in a very brief way the discovery of some particles that have been of importance in our bubble chamber studies, and I will continue the discussion throughout my lecture. This account should not be taken to be authoritative—there is no authority in this area—but simply
as a narrative to indicate the impact that certain experimental work had on my own thinking and on that of my colleagues.

I will now return to the story of the very important strange particles. In contrast to the discovery of the pion, which was accepted immediately by almost everyone— one apparent exception will be related later in this talk— the discovery and the eventual acceptance of the existence of the strange particles stretched out over a period of a few years. Heavy, unstable particles were first seen in 1947, by Rochester and Butler [19], who photographed and properly interpreted the first two « V particles » in a cosmic-ray-triggered cloud chamber. One of the V's was charged, and was probably a K meson. The other was neutral, and was probably a K⁰. For having made these observations, Rochester and Butler are generally credited with the discovery of strange particles. There was a disturbing period of two years in which Rochester and Butler operated their chamber and no more V particles were found. But in 1950 Anderson, Leighton et al. [20] took a cloud chamber to a mountain top and showed that it was possible to observe approximately one V particle per day under such conditions. They reported, « To interpret these photographs, one must come to the same remarkable conclusion as that drawn by Rochester and Butler on the basis of these two photographs, viz., that these two types of events represent, respectively, the spontaneous decay of neutral and charged unstable particles of a new type ».

Butler and his collaborators then took their chamber to the Pic-du-Midi and confirmed the high event rate seen by the CalTech group on White Mountain. In 1952 they reported the first cascade decay [21]—now known as the Ξ⁻ hyperon.

While the cloud chamber physicists were slowly making progress in understanding the strange particles, a parallel effort was under way in the nuclear emulsion-oriented laboratories. Although the first K meson was undoubtedly observed in Leprince-Ringuet's cloud chamber [22] in 1944, Bethe [23] cast sufficient doubt on its authenticity that it had no influence on the physics community and on the work that followed. The first overpowering evidence for a K meson appeared in nuclear emulsion, in an experiment by Brown and most of the Bristol group [24], in 1949. This so-calledτ⁺ meson decayed at rest into three coplanar pions. The measured ranges of the three pions gave a very accurate mass value for the τ meson of 493.6 MeV. Again there was a disturbing period of more than a year and a half before another τ meson showed up.

In 1951, the year after the τ meson and the V particles were finally seen again, O'Ceallaigh [25] observed the first of his kappa mesons in nuclear emulsion. Each such event involved the decay at rest of a heavy meson into a muon with a different energy. We now know these particles as K⁺ mesons
decaying into \( \mu^+ + \pi^0 + \nu \), so the explanation of the broad muon energy spectrum is now obvious. But it took some time to understand this in the early 1950's, when these particles appeared one by one in different laboratories. In 1953, Menon and O'Ceallagh [26] found the first \( \mathrm{K}_{\pi 2} \) or \( \theta \) meson, with a decay into \( \pi^+ + \pi^0 \). The identification of the \( \theta \) and \( \tau \) mesons as different decay modes of the same \( \mathrm{K} \) mesons is one of the great stories of particle physics, and it will be mentioned later in this lecture.

The identification of the neutral \( \Lambda \) emerged from the combined efforts of the cosmic ray cloud chamber groups, so I will not attempt to assign credit for its discovery. But it does seem clear that Thompson et al. [27] were the first to establish the decay scheme of what we now know as the \( \mathrm{K}_0^\Lambda \) meson: \( \mathrm{K}_0^\Lambda \rightarrow \pi^+ + \pi^- \). The first example of a charged \( \Sigma \) hyperon was seen in emulsion by the Genoa and Milan group [28], in 1953. And after that, the study of strange particles passed, to a large extent, from the cosmic ray groups to the accelerator laboratories.

So by the time the Bevatron first operated, in 1954, a number of different strange particles had been identified; several charged particles and a neutral one all with masses in the neighborhood of 500 MeV, and three kinds of particles heavier than the proton. In order of increasing mass, these were the neutral \( \Lambda \), the two charged \( \Sigma \)'s (plus and minus), and the negative cascade \( (\Xi^-) \), which decayed into a \( \Lambda \) and a negative pion.

The strange particles all had lifetimes shorter than any known particles except the neutral pion. The hyperons all had lifetimes of approximately \( 10^{-10} \) s, or less than 1% of the charged pion lifetime. When I say that they were called strange particles because their observed lifetimes presented such a puzzle for theoretical physicists to explain, I can imagine the lay members in this audience saying to themselves, «Yes, I cannot see how anything could come apart so fast.» But the strangeness of the strange particles is not that they decay so rapidly, but that they last almost a million million times longer than they should—physicists could not explain why they did not come apart in about \( 10^{-21} \) s.

I will not go into the details of the dilemma, but we can note that a similar problem faced to physics community when the muon was found to be so inert, nuclearly. The suggestion by Marshak and Bethe [29] that it was the daughter of a strongly interacting particle was published almost simultaneously with the independent experimental demonstration by Powell et al. mentioned earlier. Although invoking a similar mechanism to bring order into the strange-particle arena was tempting, Pais [16] made his suggestion that strange particles were produced «strongly» in pairs, but decayed «weakly» when separated from each other.

Gell-Mann [30] (and independently Nishijima [31] then made the first
of this several major contributions to particle physics by correctly guessing the rules that govern the production and decay of all the strange particles. I use the word «guessing» with the same sense of awe I feel when I say that Champollion guessed the meanings of the hieroglyphs on the Rosetta Stone. Gell-Mann had first to assume that the $K$ meson was not an $I$-spin triplet, as it certainly appeared to be, but an $I$-spin doublet plus is antiparticles, and he had further to assume the existence of the neutral $\Sigma$ and to the neutral $\Xi$. And finally, when he assigned appropriate values of his new quantum number, strangeness, to each family, his rules explained the one observed production reaction and predicted a score of others. And of course it explained all the known decays, and predicted another. My research group eventually confirmed all of Gell-Mann's and Nishijima's early predictions, many of them for the first time, and we continue to be impressed by their simple elegance.

This was the state of the art in particle physics in 1954, when William Brobeck turned his brainchild, the Bevatron, over to his Radiation Laboratory associates to use as a source of high energy protons. I has been using the Berkeley proton linear accelerator in some studies of short-lived radioactive species, and I was pleased at the chance to switch to a field that appeared to be more interesting. My first Bevatron experiment was done in collaboration with Sula Goldhaber [32]; it gave the first real measurement of the $\tau$ meson lifetime. My next experiment was done with three talented young post-doctoral fellows, Frank S. Crawford jr., Myron L. Good and M. Lynn Stevenson. An early puzzle in K-meson physics was that two of the particles (the $\theta$ and $\tau$) had similar, but poorly determined lifetimes and masses. That story has been told in this auditorium by Lee [33] and Yang [34] so I will not repeat it now. But I do like to think that our demonstration [35], simultaneously with and independently from one by Fitch and Motley [36], that the two lifetimes were not measurably different, plus similar small limits on possible mass differences set by von Friesen et al. [37] and by Birge et al. [38], nudged Lee and Yang a bit toward their revolutionary conclusion.

Our experiences with what was then a very complicated array of scintillation counters led me and my colleagues to despair of making meaningful measurements of what we perceived to be the basic reactions of strange particle physics:

\[
\pi^- + p \rightarrow \Lambda + K^0 \quad \downarrow \quad p + \pi^- \rightarrow \pi^- + \pi^+ 
\]

the production reaction is indicated by the horizontal arrows, the subsequent decays by the vertical arrows. Figure 1 shows a typical example of this reac-
Fig. 1. $\pi^- + p \rightarrow K^0 + \Lambda$.

...tion, as we saw it later in the 10 in. bubble chamber. We concluded, correctly I believe, that none of the then known techniques was well suited to study this reaction. Counters appeared hopelessly inadequate to the task, and the spark chamber had not yet been invented. The Brookhaven diffusion cloud chamber group [17] had photographed only a few events like shown in Fig. 1, in a period of two years. It seemed to us that a track-recording technique was called for, but each of the three known track devices had drawbacks that ruled it out as a serious contender for the role we envisaged. Nuclear emulsion, which had been so spectacularly successful in the hands of Powell’s group, depended on the contiguous nature of the successive tracks at a production or decay vertex. The presence of neutral and therefore non-ionizing particles between related charged particles, plus lack of even a rudimentary time resolution, made nuclear emulsion techniques virtually unusable in this new field. The two known types of cloud chambers appeared to have equally insurmountable difficulties. The older Wilson expansion chamber had two difficulties that rendered it unsuitable for the job: if used at atmospheric pressure, its cycling period was measured in minutes, and if one increased its pressure to compensate for the long mean free path of nuclear interactions, its cycling period increased at least as fast as the pressure was increased. Therefore the number of observed reactions per day started at...
an almost impossibly low value, and dropped as «corrective action» was taken. The diffusion cloud chamber was plagued by «background problems», and had an additional disadvantage—its sensitive volume was confined in the vertical direction to a height of only a few centimeters. What we conclude from all this was simply that particle physicists needed a track-recording device with solid or liquid density (to increase the rate of production of nuclear events by a factor of 100), with uniform sensitivity (to avoid the problems of the sensitive layer in the diffusion chamber), and with fast cycling time (to avoid the Wilson chamber problems). And of course, any cycling detector would permit the association of charged tracks joined by neutral tracks, which was denied to the user of nuclear emulsion.

In late April of 1953 I paid my annual visit to Washington, to attend the meeting of the American Physical Society. At lunch of the first day, I found myself seated at a large table in the garden of the Shoreham Hotel. All the seats but one were occupied by old friends from World War II days, and we reminisced about our experiences at the MIT radar laboratory and at Los Alamos. A young chap who had not experienced those exciting days was seated at my left, and we were soon talking of our interests in physics. He expressed concern that no one would hear his 10 min contributed paper, because it was scheduled as the final paper of the Saturday afternoon session, and therefore the last talk to be presented at the meeting. In those days of slow airplanes, there were even fewer people in the audience for the last paper of the meeting than there are now—if that is possible. I admitted that I would not be there, and asked him to tell me what he would be reporting. And that is how I heard first hand from Donald Glaser how he had invented the bubble chamber, and to what state he had brought its development. And of course he has since described those achievements from this platform [39]. He showed me photographs of bubble tracks in a small glass bulb, about 1 cm in diameter and 2 cm long, filled with diethyl ether. He stressed the need for absolute cleanliness of the glass bulb, and said that he could maintain the ether in a superheated state for an average of many seconds before spontaneous boiling took place. I was greatly impressed by his work, and it immediately occurred to me that this could be the «big idea» I felt was needed in particle physics.

That night in my hotel room I discussed what I had learned with my colleague from Berkeley, Frank Crawford. I told Frank that I hoped we could get started on the development of a liquid hydrogen chamber, much larger than anything Don Glaser was thinking about, as soon as I returned to Berkeley. He volunteered to stop off in Michigan on the way back to Berkeley, which he did, and learned everything he could about Glaser's technique.

I returned to Berkeley on Sunday, May 1, and on the next day Lynn
Stevenson started to keep a new notebook on bubble chambers. The other day, when he saw me writing this talk, he showed me that old notebook with its first entry dated May 2, 1953, with Van der Waal’s equation on the first page, and the isotherms hydrogen traced by hand onto the second page. Frank Crawford came home a few days later, and he and Lynn moved into the «student shop» in the synchrotron building, to build their first bubble chamber. They were fortunate in enlisting the help of John Wood who was an accelerator technician at the synchrotron. The three of them put their first efforts into a duplication of Glaser’s work with hydrocarbons. When they have demonstrated radiation sensitivity in ether, they built a glass chamber in a Dewar flask to try first with liquid nitrogen and then with liquid hydrogen.

I remember that on several occasions I telephoned to the late Earl Long at the University of Chicago, for advice on cryogenic problems. Dr. Long gave active support to the liquid hydrogen bubble chamber that was being built at that time by Roger Hildebrand and Darragh Nagle at the Fermi Institute in Chicago. In August of 1953 Hildebrand and Nagle [40] showed that superheated hydrogen boiled faster in the presence of a gamma-ray source than it did when the source was removed. This is a necessary (though not sufficient) condition for successful operation of a liquid hydrogen bubble chamber, and the Chicago work was therefore an important step in the development of such chambers. The important unanswered question concerned the bubble density—was it sufficient to see tracks of «minimum ionizing» particles, or did liquid hydrogen—as my colleagues had just shown that liquid nitrogen did—produce bubbles but no visible tracks?

John Wood saw the first tracks in a 1.5 in.-diameter liquid hydrogen bubble chamber in February of 1954 [41]. The Chicago group could certainly have done so earlier, by rebuilding their apparatus, but they switched their efforts to hydrocarbon chambers, and were rewarded by being the first physicists to publish experimental results obtained by bubble chamber techniques. Figure 2 is a photograph of Wood’s first tracks.

At the Lawrence Radiation Laboratory, we have long had a tradition of close cooperation between physicists and technicians. The resulting atmosphere, which contributed so markedly to the rapid development of the liquid hydrogen bubble chamber, led to an unusual phenomenon: none of the scientific papers on the development of bubble chamber techniques in my research group were signed by experimenters who were trained as physicists or who had had previous cryogenic experience. The papers all had authors who were listed on the Laboratory records as technicians, but of course the physicists concerned knew what was going on, and offered many suggestions. Nonetheless, our technical associates carried the main responsibility, and published their findings in the scientific literature. I believe this is a healthy
change from practices that were common a generation ago; we all remember papers signed by a single physicist that ended with a paragraph saying, « I wish to thank Mr. __________, who built the apparatus and took much of the data ».

And speaking of acknowledgments, John Wood’s first publication, in addition to thanking Crawford, Stevenson, and me for our advice and help, said, « I am indebted to A. J. Schwemin for help with the electronic circuits ». « Pete » Schwemin, the most versatile technician I have ever known, became so excited by his initial contact with John Wood’s 1.5 in.-diameter all-glass chamber that he immediately started the construction of the first metal bubble chamber with glass windows. All earlier chambers had been made completely of smooth glass, without joints, to prevent accidental boiling at sharp points; such boiling of course destroyed the superheat and made the chamber insensitive to radiation. Both Glaser and Hildebrand stressed the long times their liquids could be held in the superheated condition; Hildebrand and Nagle averaged 22 s and observed one superheat period of 70 s. John Wood reported [41], « We were discouraged by our inability to attain the long times of superheat, until the track photographs showed that it was not important in the successful operation of a large bubble chamber ». I have always felt that second to Glaser’s discovery of tracks this was the key
observation in the whole development of bubble chamber technique. As long as one « expanded the chamber » rapidly, bubbles forming on the wall didn't destroy the superheated condition of the main volume of the liquid, and it remained sensitive as a track-recording medium.

Pete Schwemin, with the help of Douglas Parmentier [42], built the 2.5 in.-diameter hydrogen chamber in record time, as the world's first « dirty chamber ». I have never liked that expression, but it was used for a while to distinguish chambers with windows gasketed to metal bodies from all-glass chambers. Because of it « dirtiness », the 2.5 in. chamber boiled at its walls, but still showed good tracks throughout its volume. Now that « clean » chambers are of historical interest only, we can be pleased that the modern chambers need no longer be stigmatized by the adjective « dirty ».

Lynn Stevenson's notebook shows a diagram of John Wood's chamber dated January 25, 1954, with Polaroid pictures of tracks in hydrogen. A month later he recorded details of Schwemin's 2.5 in. chamber, and drew a complete diagram dated March 5. (That was the day after the Physical Review received Wood's letter announcing the first observation of tracks.) On April 29, Schwemin and Parmentier photographed their first tracks; these are shown in Fig. 3. (Things were happening so fast at this time that the 2.5 in. system was never photographed as a whole before it ended up on the scrap pile.)

In August, Schwemin and Parmentier separately built two different 4 in.-diameter chambers. Both were originally expanded by internal bellows, and Parmentier's 4 in. chamber gave tracks on October 6. Schwemin's chamber produced tracks three weeks later, and survived as the 4 in. chamber. (See Fig. 4.) The bellows systems in both chambers failed, but it turned out to be easier to convert Schwemin's chamber to the vapor expansion system that was used in all our subsequent chambers until 1962. (In that year, the 25 in. chamber introduced the « Ω bellow » that is now standard for large chambers.)

Figure 5 shows all our chambers displayed together a few weeks ago, at the request of Swedish Television. As you can see, we all look pretty pleased to see so many of our « old friends » side by side for the first time.

Figure 6 shows an early picture of multiple meson production in the 4 in. chamber. This chamber was soon equipped with a pulsed magnetic field, and in that configuration it was the first bubble chamber of any kind to show magnetically curved tracks. It was then set aside by our group as we pushed on to larger chambers. But it ended its career as a useful research tool at the Berkeley electron synchrotron, after almost two million photographs of 300 MeV bremsstrahlung passing through it had been taken and analyzed by Bob Kenney et al. [43].

In the year 1954, as I have just recounted, various members of my research
Fig. 3. – Tracks in \(2\frac{1}{2}\) in. chamber: \(a\) neutrons; \(b\) gamma rays.
group had been responsible for the successful operation of four separate liquid hydrogen bubble chambers, increasing in diameter from 1.5 to 4 in. By the end of that eventful year, it was clear that it would take a more concerted engineering-type approach to the problem if we were to progress to the larger chambers we felt were essential to the solution of high-energy physics problems. I therefore enlisted the assistance of three close associates, J. Donald Gow, Robert Watt and Richard Blumberg. Don Gow and Bob Watt had
taken over full responsibility for the development and operation of the 32 MeV linear accelerator that had occupied all my attention from its inception late in 1945 until it first operated in late 1947. Neither of them had any experience with cryogenic techniques, but they learned rapidly, and were soon leaders in the new technology of hydrogen bubble chambers. Dick Blumberg had been trained as a mechanical engineer, and he had designed the equipment used by Crawford, Stevenson and me in our experiments, then in progress, on the Compton scattering of $\gamma$-rays by protons [44].

Wilson Powell had built two large magnets to accommodate his Wilson Cloud Chambers, pictures from which adorned the walls of every cyclotron laboratory in the world. He very generously placed one of these magnets at our disposal, and Dick Blumberg immediately started the mechanical design of the 10 in. chamber—the largest size we felt could be accommodated in the well of Powell’s magnet. Blumberg’s drafting table was in the middle of the single room that contained the desks of all the members of my research group. Not many engineers will tolerate such working conditions, but Blumberg was able to do so and he produced a design that was quickly built in the main machine shop. All earlier chambers had been built by the experimenters themselves. The design of the 10 in. chamber turned out to be a much larger job than we had foreseen. By the time it was completed, eleven members of the Laboratory’s Mechanical Engineering Department had worked on it, including Rod Byrns and John Mark. The electrical engineering aspects of all our large chambers were formidable, and we are indebted to Jim Shand for his leadership in this work for many years.
Great difficulty was experienced with the first operation of the 10 in. chamber; too much hydrogen was vaporized at each «expansion». Pete Schwemin quickly diagnosed the trouble and built a fast-acting valve that permitted the chamber to be pulsed every 6 s, to match the Bevatron’s cycling time.

It would be appropriate to interrupt this description of the bubble chamber development program to describe the important observations made possible by the operation of the 10 in. chamber early in 1956, but instead, I will preserve the continuity by describing the further development of the hardware. In December of 1954, shortly after the 4 in. chamber had been operated in the cyclotron building for the first time, it became evident to me that the 10 in. chamber we had just started to design would not be nearly large enough to tell us what we wanted to know about the strange particles. The tracks of these objects had been photographed at Brookhaven [17], and we knew they were produced copiously by the Bevatron.
The size of the « big chamber » was set by several different criteria, and fortunately all of them could be satisfied by one design. (Too often, a designer of new equipment finds that one essential criterion can be met only if the object is very large, while an equally important criterion demands that it be very small.) All « dirty chambers » so far built throughout the world had been cylindrical in shape, and were characterized by their diameter measurement. By studying the relativistic kinematics of strange particles produced by Bevatron beams, and more particularly by studying the decay of these particles, I convinced myself that the big chamber should be rectangular, with a length of at least 30 in. This length was next increased to 50 in. in order that there would be adequate amounts of hydrogen upstream from the required decay region, in which production reactions could take place. Later the length was charged to 72 in., when it was realized that the depth of the chamber could properly be less than its width and that the change could be made without altering the volume. The production region corresponded to about 10% of a typical pion-proton mean free path, and the size of the decay region was set by the relativistic time-dilated decay lengths of the strange particles, plus the requirement that there be a sufficient track length available in which to measure magnetic curvature in a « practical magnetic field » of 15000 G. In summary, then, the width and depth of the chamber came rather simply from an examination of the shape of the ellipses that characterize relativistic transformations at Bevatron energies, plus the fact that the magnetic field spreads the particles across the width but not along the depth of the chamber.

The result of this straightforward analysis was a rather frightening set of numbers: The chamber length was 72 in.; its width was 20 in., and its depth was 15 in. It had to be pervaded by a magnetic field of 15000 G, so its magnet would weigh at least 100 tons and would require 2 or 3 MW to energize it. It would require a window 75 in. long by 23 in. wide and 5 in. thick to withstand the (deuterium) operating pressure of 8 atm, exerting a force of 100 tons on the glass. No one had any experience with such large volumes of liquid hydrogen; the hydrogen-oxygen rocket engines that now power the upper stages of the Saturn boosters were still gleams in the eyes of their designers—these were pre-Sputnik days. The safety aspects of the big chamber were particularly worrisome. Low temperature laboratories had a reputation for being dangerous places in which to work, and they did not deal with such large quantities of liquid hydrogen, and what supplies they did use were kept at atmospheric pressure.

For some time, the glass window problem seemed insurmountable—no one had ever cast and polished such a large piece of optical glass. Fortunately for the eventual success of the project, I was able to persuade myself that the chamber body could be constructed of a transparent plastic cylinder with
metallic end plates. This notion was later demolished by my engineering colleagues, but it played an important role in keeping the project alive in my own mind until I was convinced that the glass window could be built. As an indication of the cryogenic «state of the art» at the time we worried about the big window, I can recall the following anecdote. One day, while looking through a list of titles of talks at a recent cryogenic conference, I spotted one that read, «Large glass window for viewing liquid hydrogen». Eagerly I turned to the paper—but it described a metallic Dewar vessel equipped with a glass window 1 in. in diameter!

Don Gow was now devoting all his to hydrogen bubble chambers, and in January of 1955 we interested Paul Hernandez in taking a good hard engineering look at the problems involved in building and housing the 72 in. bubble chamber. We were also extremely fortunate in being able to interest the cryogenic engineers at the Boulder, Colorado, branch of the National Bureau of Standards in the project. Dudley Chelton, Bascomb Birmingham and Doug Mann spent a great deal of time with us, first educating us in large-scale liquid hydrogen techniques, and later cooperating with us in the design and initial operation of the big chamber.

In April of 1955, after several months of discussion of the large chamber, I wrote a document entitled The Bubble Chamber Program at UCRL. This paper showed in some detail why it was important to build the large chamber, and outlined a whole new way of doing high-energy physics with such a device. It stressed the need for semiautomatic measuring devices (which had not previously been proposed), and described how electronic computers would reconstruct tracks in space, compute momenta, and solve problems in relativistic mechanics. All these techniques are now part of the «standard bubble chamber method», but in April of 1955 no one had yet applied them. Of all the papers I have written in my life, none gives me so much satisfaction on rereading as does this unpublished prospectus.

After Paul Hernandez and Don Gow has estimated that the big chamber, including its building and power supplies, would cost about 2.5 million dollars, it was clear that a special AEC appropriation was required; we could no longer build our chambers out of ordinary laboratory operating money. In fact, the document I have just described was written as a sort of proposal to the AEC for financial support—but without mentioning money! I asked Ernest Lawrence if he would help me in requesting extra funds from the AEC. He read the document, and agreed with the points I had made. He then asked me to remind him of the size of the world’s largest hydrogen chamber. When I replied that it was 4 in. in diameter, he said the though I was making too large an extrapolation in one step, to 72 in. I told him that the 10 in. chamber was on the drawing board, and if we could make it work,
the operation of the 72 in. chamber was assured. (And if we could not make it work, we could refund most of the 2.5 million.) This was not obvious until I explained the hydraulic aspects of the expansion system of the 72 in. chamber; it was arranged so that the 20 in. wide, 72 in. long chamber could be considered to be a large collection of essentially independently expanded 10 in. square chambers. He was not convinced of the wisdom of the program, but in a characteristic gesture, he said, «I don't believe in your big chamber, but I do believe in you, and I'll help you to obtain the money». I therefore accompanied him on his next trip to Washington, and we talked in one day to three of the five Commissioners: Lewis Strauss, Willard Libby (who later spoke from this podium), and the late John von Neumann, the greatest mathematical physicist then living. That evening, at a cocktail party at Johnny von Neumann's home, I was told that the Commission had voted that afternoon to give the laboratory the 2.5 million dollars we had requested. All we had to do now was build the thing and make it work!

Design work had of course been under way for some time, but it was now rapidly accelerated. Don Gow assumed a new role that is not common in physics laboratories, but is well known in military organizations; he became my «chief of staff». In this position, he coordinated the efforts of the physicists and engineers; he had full responsibility for the careful spending of our precious 2.5 million dollars, and he undertook to become an expert second to none in all the technical phases of the operation, from low temperature thermodynamics to safety engineering. His success in this difficult task can be recognized most easily in the success of the whole program, culminating in the fact that I am speaking here this afternoon. I am sorry that Don Gow can not be here today; he died several years ago, but I am reminded of him every day—my three-year-old son is named Donald in his memory.

The engineering team under Paul Hernandez's direction proceeded rapidly with the design, and in the process solved a number of difficult problems in ways that have become standard «in the industry». A typical problem involved the very considerable differential expansion between the stainless steel chamber and the glass window. This could be lived with in the 10 in. chamber, but not in the 72 in. Jack Franck's «inflatable gasket» allowed the glass to be seated against the chamber body only after both had been cooled to liquid hydrogen temperature.

Just before leaving for Stockholm, I attended a ceremony at which Paul Hernandez was presented with a trophy honoring him as a «Master Designer» for his achievements in the engineering of the 72 in. chamber. I had the pleasure of telling in more detail than I can today of his many contributions to the success of our program. One of his associates recalled a special service that he rendered not only to our group but to all those who followed us in
building liquid hydrogen bubble chambers. Hernandez and his associates wrote a series of *Engineering Notes*, on matters of interest to designers of hydrogen bubble chambers, that soon filled a series of notebooks that spanned 3 ft of shelf space. Copies of theses were sent to all interested parties on both sides of the Atlantic, and I am sure that they resulted in a cumulative savings to all bubble chamber builders of several million dollars; had not all this information been readily available, the test programs and calculations of our engineering group would have required duplication at many laboratories, at a large expense of money and time. Our program moved so rapidly that there was never time to put the Engineering Notes into finished form for publication in the regular literature. For this reason, one can now read review articles on bubble chamber technology, and be quite unaware of the part that our Laboratory played in its development. There are no references to papers by members of our group, since those papers were never written—the data that would have been in them had been made available to everyone who needed them at a much earlier date.

And just to show that I was also deeply involved in the chamber design, I might recount how I purposely «designed myself into a corner» because I thought the result were important, and I thought I could invent a way out of a severe difficulty, if given the time. All previous chambers had had two windows, with «straight through» illumination. Such a configuration reduces the attainable magnetic field, because the existence of a rear pole piece would interfere with the light-projection system. I made the decision that the 72 in. chamber would have only a top window, thereby permitting the magnetic field to be increased by a lower pole piece and at the same time saving the cost of the extra glass window, and also providing added safety by eliminating the possibility that liquid hydrogen could spill through a broken lower window. The only difficulty was that for more than a year, as the design was firmed up and the parts were fabricated, none of us could invent a way both to illuminate and to photograph the bubbles through the same window. Duane Norgren, who has been responsible for the design of all our bubble chamber cameras, discussed the matter with me at least once a week in that critical year, and we tried dozens of schemes that did not quite do the job. But as a result of our many failures, we finally came to understand all the problems, and we eventually hit on the retrodirecting system known as coat hangers. This solution came none too soon; if it had been delayed by a month or more, the initial operation of the 72 in. chamber would have been correspondingly delayed. We took many other calculated risks in designing the system; if we had postponed the fabrication of the major hardware until we had solved all the problems on paper, the project might still not be completed. Engineers are conservative people by nature; it is the ultimate disgrace to have a boiler
explode or a bridge collapse. We were therefore fortunate to have Paul Hernandez as our chief engineer; he would seriously consider anything his physics colleagues might suggest, no matter how outlandish it might seem at first sight. He would firmly reject it if it could not be made safe, but before rejecting any idea for lack of safety he would use all the ingenuity he possessed to make it safe.

We felt that we needed to build a test chamber to gain experience with a single-window system, and to learn to operate with a hydrogen refrigerator; our earlier chambers had all used liquid hydrogen as a coolant. We therefore built and operated the 15 in. chamber in the Powell magnet, in place of the 10 in. chamber that had served us so well.

The 72 in. chamber operated for the first time on March 24, 1959, very nearly four years from the time it was first seriously proposed. Figure 7 shows it at about that time. The «start-up team» consisted of Don Gow, Paul Hernandez and Bob Watt, all of whom had played key roles in the initial operation of the 15 in. chamber. Bob Watt and Glenn Eckman have been responsible for the operation of all our chambers from the earliest days of the 10 in. chamber, and the success of the whole program has most often rested in their hands. They have maintained an absolutely safe operating record in the face of very severe hazard, and they have supplied their colleagues in the physics community with approximately ten million high-quality stereo photographs. And most recently, they have shown that they can design chambers as well as they have operated them. The 72 in. chamber was recently enlarged to an 82 in. size, incorporating to a large extent the design concepts of Watt and Eckman.

Although I have not done justice to the contributions of many close friends and associates who shared in our bubble chamber development program, I must now turn to another important phase of our activities—the data-analysis program. Soon after my 1955 prospectus was finished, Hugh Bradner undertook to implement the semiautomatic measuring machine proposal. He first made an exhaustive study of commercially available measuring machines, encoding techniques, etc., and then, with Jack Franck, designed the first «Frackenstein». This rather revolutionary device has been widely copied, to such an extent that objects of its kind are now called «conventional» measuring machines (Fig. 8). Our first Frackenstein was operating reliably in 1957, and in the summer of 1958 a duplicate was installed in the U.S. exhibit at the «Atoms for Peace» exposition in Geneva. It excited a great deal of interest in the high-energy physics community, and a number of groups set out to make similar machines based on its design. Almost everyone thought at first that our provision for automatic track following was a needless waste of money, but over the years, that feature has also come to be «conventional».
Jack Franck then went on to design the Mark II Franckenstein, to measure 72 in. bubble chamber film. He had the first one ready to operate just in time to match the rapid turn-on of the big chamber, and he eventually built three more of the Mark II's. Other members of our group then designed and perfected the faster and less expensive SMP system, which added significantly to our «measuring power». The moving forces in this development were Pete Schwemin, Bob Hulsizer, Peter Davey, Ron Ross and Bill Humphrey [45].
Our final and most rewarding effort to improve our measuring ability was fulfilled several years ago, when our first Spiral Reader became operational. This single machine has now measured more than one and a half million high energy interactions, and has, together with its almost identical twin, measured one and a quarter million events in the last year. The SAAB Company here in Sweden is now building and selling Spiral Readers to European laboratories.

The Spiral Reader had a rather checkered career, and it was on several occasions believed by most workers in the field to have been abandoned by our group. The basic concept of the spiral scan was supplied by Bruce McCormick, in 1956. Our attempts to reduce his ideas to practice resulted in failure, and shortly after that, McCormick moved to Illinois, where he has since been engaged in computer development. As the cost of transistorized circuits dropped rapidly in the next years, we tried a second time to implement the Spiral Reader concept, using digital techniques to replace the analog devices of the earlier machine. The second device showed promise, but its «hard-wired logic» made it too inflexible, and the unreliability of its electronic components kept it in repair most of the time. The mechanical and optical components of the second Spiral Reader were excellent, and we hated to drop the whole project simply because the circuitry did not come up to the
same standard. In 1963, Jack Lloyd suggested that we use one of the new breed of small high-speed, inexpensive computers to supply the logic and the control circuits for the Spiral Reader. He then demonstrated great qualities of leadership by delivering to our research group a machine that has performed even better than he had promised it would. In addition to his development of the hardware, he initiated POOH, the Spiral Reader filtering program, which was brought to a high degree of perfection by Jim Burkhard. The smooth and rapid transition of the Spiral Reader from a developmental stage into a useful operational tool was largely the result of several years of hard work on the part of Gerry Lynch and Frank Solmitz. Figure 9, from a talk I gave two and a half years ago [46], shows how the measuring power of our group has increased over the years, with only a modest increase in personnel.
According to a simple extrapolation of the exponential curve we had been on from 1957 through 1966, we would expect to be measuring 1.5 million events per year some time in 1969. But we have already reached that rate and we will soon be leveling off about there because we have stopped our development work in this area.

The third key ingredient of our development program has been the continually increasing sophistication in our utilization of computers, as they have increased in computational speed and memory capacity. While I can speak from a direct involvement in the development of bubble chambers and measuring machines, and in the physics done with those tools, my relationship to our computer programming efforts is largely that of an amazed spectator. We were most fortunate that in 1956 Frank Solmitz elected to join our group. Although the rest of the group thought of themselves as experimental physicists, Solmitz had been trained as a theorist, and had shown great aptitude in the development of statistical methods of evaluating experimental data. When he saw that our first Franckenstein was about to operate, and no computer programs were ready to handle the data it would generate, he immediately set out to remedy the situation. He wrote HYDRO, our first system program for use on the IBM 650 computer. In the succeeding twelve years he has continued to carry the heavy responsibility for all our programming efforts. A major breakthrough in the analysis of bubble chamber events was made in the years 1957 through 1959. In this period, Solmitz and Art Rosenfeld, together with Horace Taft from Yale University and Jim Snyder from Illinois, wrote the first «fitting routine», GUTS, which was the core of our first «kinematics program, KICK». To explain what KICK did, it is easiest to describe what physicists had to do before it was written. HYDRO and its successor, PANG, listed for each vertex the momentum and space angles of the tracks entering or leaving that vertex, together with the calculated errors in these measurements. A physicist would plot the angular coordinates on a stereographic projection of a unit sphere known as a Wolff-plot. If he was dealing with a three-track vertex—and that was all we could handle in those days—he would move the points on the sphere, within their errors, if possible, to make them coplanar. And of course he would simultaneously change the momentum values, within their errors, to insure that the momentum vector triangle closed, and energy was conserved. Since momentum is a vector quantity, the various conditions could be simultaneously satisfied only after the angles and the absolute values of the momenta had been changed a number of times in an iterative procedure. The end result was a more reliable set of momenta and angles, constrained to fit the conservation laws of energy and momentum. In a typical case, an experienced physicist could solve only a few Wolff-plot problems in a day. (Lynn Stevenson had written a specific program, COPLAN,
that solved a particular problem of interest to him that was later handled by
the more versatile GUTS.)

GUTS was being written at a time when one highly respected visitor to the
groups saw the large pile of PANG printout that had gone unanalysed because
so many of our group members were writing GUTS—a program that was
planned to do the job automatically. Our visitor was very upset at what he
told me was a « foolish deployment of our forces ». He said, « If you would
only get all those people way from their program writing, and put them to
work on Wolff-plots, we’d have the answer to some really important physics
in a month or two ». I said I was sure we would end up with a lot more
physics in the next years if my colleagues continued to write GUTS and
KICK. I am sure that those who wrote these pioneering « fitting and kinematics programs » were subjected to similar pressure. Everyone in the high-
energy physics community has long been indebted to these farsighted men
because they knew that what they were doing was right. KICK was soon
developed so that it gave an overall fit to several interconnected vertices,
with various hypothetical identities of the several tracks assumed in a series
of attempts at a fit. The relationship between energy and momentum depends
on mass, so a highly constrained fit can be obtained only if the particle
responsible for each track is properly identified. If the degree of constraint
is not so high, more than one « hypothesis » (set of track identifications) may
give a fit, and the physicist must use his judgment in making the identification.

As another example in this all-too-brief sketch of the computational aspects
of our work, I will mention an important program, initiated by Art Rosenfeld
and Ron Ross, that has removed much of the remaining drudgery from the
bubble chamber physicists’ life. SUMX is a program that can easily be in-
structed to search quickly through large volumes of « kinematics program
output », printing out summaries and tabulations of interesting data. (Like
all our pioneering programs, SUMX was replaced by an improved and more
versatile program—in this case, KIOWA. But I will continue to talk as
though SUMX were still used.) A typical SUMX printout will be a com-
puterprinted document 3 in.-thick, with hundreds of histograms, scatter
plots, etc.

Hundreds of histograms are similarly printed showing numbers of events
with effective masses for many different combinations of particles, with
various « cuts » on momentum transfer, etc. What all this amounts to is
simply that a physicist is no longer rewarded for his ability in deciding what
histograms he should tediously plot and then examine. He simply tells the
computer to plot all histograms of any possible significance, and then flips
the pages to see which ones have interesting features.

One of my few real interactions with our programming effort came when
I suggested to Gerry Lynch the need for a program he wrote that is known as GAME. In my work as a nuclear physicist before World War II, I had often been skeptical of the significance of the «bumps» in histograms, to which importance was attached by their authors. I developed my own criteria for judging statistical significance, by plotting simulated histograms, assuming the curves to be smooth; I drew several samples of «Monte Carlo distributions», using a table of random numbers as the generator of the samples. I usually found that my skepticism was well founded because the «faked» histograms showed as much structure as the published ones. There are of course many statistical tests designed to help one evaluate the reality of bumps in histograms, but in my experience nothing is more convincing than an examination of a set of simulated histograms from an assumed smooth distribution.

GAME made it possible, with the aid of a few control cards, to generate a hundred histograms similar to those produced in any particular experiment. All would contain the same number of events as the real experiment, and would be based on a smooth curve through the experimental data. The standard procedure is to ask a group of physicists to leaf through the 100 histograms—with the experimental histogram somewhere in the pile—and vote on the apparent significance of the statistical fluctuations that appear. The first time this was tried, the experimenter—who had felt confident that his bump was significant—did not know that his own histogram was in the pile, and did not pick it out as convincing; he picked out two of the computer-generated histograms as looking significant, and pronounced all other—including his own—as of no significance! In view of this example, one can appreciate how many retractions of discovery claims have been avoided in our group by the liberal use of the GAME program.

As a final example from our program library, I will mention FAKE, which, like SUMX, has been widely used by bubble chamber groups all over the world. FAKE, written by Gerry Lynch, generates simulated measurements of bubble chamber events to provide a method of testing the analysis programs to determine how frequently they arrive at an incorrect answer.

Now that I have brought you up to date on our parallel developments of hardware and software (computer programs), I can tell you what rewards we have reaped, as physicists, from their use. The work we did with the 4 in. chamber at the 184 in. cyclotron and at the Bevatron cannot be dignified by the designation «experiments», but it did show examples of π-μ-e decay and neutral strange-particle decay. The experiences we had in scanning the 4 in. film merely whetted our appetite for the exciting physics we felt sure would be manifest in the 10 in. chamber, when it came into operation in Wilson Powell’s big magnet.
Robert Tripp joined the group in 1955, and as his first contribution to our program he designed a «separated beam» of negative K mesons that would stop in the 10 in. chamber. We had two different reasons for starting our bubble chamber physics program with observations of the behavior of K\(^-\) mesons stopping in hydrogen. The first reason involved physics: The behavior of stopping \(\pi^-\) mesons in hydrogen had been shown by Panofsky and his co-workers [47] to be a most fruitful source of fundamental knowledge concerning particle physics. The second reason was of an engineering nature: Only one Bevatron «straight section» was available for use by physicists, and it was in constant use. In order not to interfere with other users, we decided to set the 10 in. chamber close to a curved section of the Bevatron, and use secondary particles, from an internal target, that penetrated the wall of the vacuum chamber and passed between neighboring iron blocks in the return yoke of the Bevatron magnet. This physical arrangement gave us negative particles (K\(^-\) and \(\pi^-\) mesons) of a well-defined low momentum. By introducing an absorber into the beam, we brought the K\(^-\) mesons almost to rest, but allowed the lighter \(\pi^-\) mesons to retain a major fraction of their original momentum. The Powell magnet provided a second bending that brought the K\(^-\) mesons into the chamber, but kept the \(\pi^-\) mesons out. That was the theory of this first separated beam for bubble chamber use. But in practice, the chamber was filled with tracks of pions and muons, and we ended up with only one stopped K\(^-\) per roll of 400 stereo pairs. It is now common for experimenters to stop one million K\(^-\) mesons in hydrogen, in a single experimental run, but the 137 K\(^-\) mesons we stopped in 1956 [48] gave us a remarkable preview of what has now been learned in the much longer exposures. We measured the relative branching of K\(^-\)+p into

\[ \Sigma^- + \pi^+; \Sigma^+ + \pi^-; \Sigma^0 + \pi^0; \Lambda + \pi^0. \]

And in the process, we made a good measurement of the \(\Sigma^0\) mass. We plotted the first decay curves for the \(\Sigma^+\) and \(\Sigma^-\) hyperons, and we observed for the first time the interactions of \(\Sigma^-\) hyperons and protons at rest. We felt amply rewarded for our years of developmental work on bubble chambers by the very interesting observations we were now privileged to make.

We had a most exciting experience at this time, that was the result of two circumstances that no longer obtain in bubble chamber physics. In the first place, we did all our own scanning of the photographic film. Such tasks are now carried out by professional scanner, who are carefully trained to recognize and record «interesting events». We had no professional scanners at the time because we would not have known how to train them before this first film became available. And even if they had been trained, we would not have let
them look at the film—we found it so completely absorbing that there was always someone standing behind a person using one of our few film viewers, ready to take over when the first person's eyes tired. The second circumstance that made possible the accidental discovery I am about to describe was the very poor quality of our separated K⁻ beam—by modern standards. Most of the tracks we observed were made by negative pions or muons, but we also saw many positively charged particles—protons, pions and muons.

At first we kept no records of any events except those involving strange particles; we would look quickly at each frame in turn, and shift to the next one if no «interesting event» showed up. In doing this scanning, we saw many examples of \( \pi^+ - \mu^+ - e^+ \) decays, usually from a pion at rest, and we soon learned about how long to expect the \( \mu^+ \) track to be—about 1 cm. I did my scanning on a stereo viewer, so I probably had a better feeling for the length of a \( \mu^+ \) track in space than did my colleagues, who looked at two projections of the stereo views, sequentially. Don Gow, Hugh Bradner and I often scanned at the same time, and we showed each other whatever interesting events came into view. Each of us showed the others examples of what we thought was an unusual decay scheme: \( \pi^- \rightarrow \mu^- \rightarrow e^- \). The decay of a \( \mu^- \) at rest into an \( e^- \), in hydrogen, was expected from the early observations by Conversi \textit{et al.} [3], but Paošsky \textit{et al.} [47] had shown that a \( \pi^- \) meson could not decay at rest in hydrogen. Our first explanation for our observations was simply that the pion had decayed just before stopping. But we gradually became convinced that this explanation really did not fit the facts. There were too many muons tracks of about the same length, and none that were appreciably longer or shorter, as the decay-in-flight hypothesis would predict. We now began to keep records of these «anomalous decays», as we still called them, and we found occasional examples in which the muon was horizontal in the chamber, so its length could be measured. (We had as yet no way of reconstructing tracks in space from two stereo views.) By comparing the measured length of the negative muon track with that of its more normal positive counterpart, we estimated that the negative muons had an energy of 5.4 MeV, rather than the well-known positive muon energy (from positive pion decay at rest) of 4.1 MeV. This confirmed our earlier suspicion that the long primary negative track could not be that of a pion, but it left us just as much in the dark as to the nature of the primary.

After these observations had been made, I gave a seminar describing what we had observed, and suggesting that the primary might be a previously unknown weakly interacting particle, heavier than the pion, that decayed into a muon and a neutral particle, either neutrino or photon. We had just made the surprising observation, shown in Fig. 10, that there was often a gap, measured in millimeters, between the end of the primary and the beginning of
the secondary. This finding suggested diffusion by a rather long-lived negative particle that orbited around and neutralized one of the protons in the liquid hydrogen. We had missed many tracks with these «gaps» because no one has seen such a thing before; we simply ignored such track configurations by subconsciously assuming that they were unassociated events in a badly cluttered bubble chamber.

One evening, one of the members of our research team, Harold Ticho from our Los Angeles campus, was dining with Jack Crawford, a Berkeley astrophysicist he had known when they were students together. They discussed our observations at some length, and Crawford suggested the possibility that a fusion reaction might somehow be responsible for the phenomenon. They cal-
culated the energy released in several such reactions, and found that it agreed with experiment if a stopped muon were to be binding together a proton and a deuteron into an HD $\mu^-$-molecular ion. In such a «molecule» the proton and deuteron would be brought into such close proximity for such a long time that they would fuse into $^3\text{He}$, and could deliver their fusion energy to the muon by the process of internal conversion. However, they could not think of any mechanism that would make the reaction happen so often—the fraction of deuterons in liquid hydrogen is only 1 in 5000. They had, however, correctly identified the reaction, but a key ingredient in the theoretical explanation was still missing.

The next day, when we had all accepted the idea that stopped muons were catalyzing the fusion of protons and deuterons, our whole group paid a visit to Edward Teller, at his home. After a short period of introduction to the observations and to the proposed fusion reaction, he explained the high probability of the reaction as follows: the stopped muon radiated its way into the lowest Bohr orbit around a proton. The resulting muonic hydrogen atom, $p\mu^-$, then had many of the properties of a neutron, and could diffuse freely through the liquid hydrogen. When it came close to the deuteron in an HD molecule, the muon would transfer to the deuteron, because the ground state of the $\mu^-d$ atom is lower than that of the $\mu^-p$ atom, in consequence of «reduced mass» effect. The new «heavy neutron» $d\mu^-$ might then recoil some distance as a result of the exchange reaction, thus explaining the «gap». The final stage of capture of a proton into a $pd\mu^-$ molecular ion was also energetically favorable, so a proton and deuteron could now be confined close enough together by the heavy negative muon to fuse into a $^3\text{He}$ nucleus plus the energy given to the internally converted muon.

We had a short but exhilarating experience when we thought we had solved all of the fuel problems of mankind for the rest of time. A few hasty calculations indicated that in liquid HD a single negative muon would catalyze enough fusion reactions before it decayed to supply the energy to operate an accelerator to produce more muons, with energy left over after making the liquid HD from sea water. While everyone else had been trying to solve this problem by heating hydrogen plasmas to millions of degrees, we had apparently stumbled on the solution, involving very low temperatures instead. But soon, more realistic estimates showed that we were off the mark by several orders of magnitude—a «near miss» in this kind of physics!

Just before we published our results [49], we learned that the «$\mu$-catalysis» reaction had been proposed in 1947 by Frank [50] as an alternative explanation of what Powell et al. had assumed (correctly) to be the decay of $\pi^+$ to $\mu^+$. Frank suggested that it might be the reaction we had just seen in liquid hydrogen, starting with a $\mu^-$, rather than with a $\pi^+$. Zel’dovitch [51] had
extended the ideas of Frank concerning this reaction, but because their papers were not known to anyone in Berkeley, we had a great deal of personal pleasure that we otherwise would have missed.

I will conclude this episode by noting that we immediately increased the deuterium concentration in our liquid hydrogen and observed the expected increase in fusion reactions, and saw two examples of successive catalyses by a single muon (Fig. 11). We also observed the catalysis of $D + D \rightarrow ^3H + ^1H$ in pure liquid deuterium.

Fig. 11. - Double muon catalysis.
A few months after we had announced our $\mu$-catalysis results, the world of particle physics was shaken by the discovery that parity was not conserved in beta decay. Madame Wu and her collaborators [52], acting on a suggestion by Lee and Yang [53], showed that the $\beta$ rays from the decay of oriented $^{60}$Co nuclei were emitted preferentially in a direction opposite to that of the spin. Lee and Yang suggested that parity nonconservation might also manifest itself in the weak decay of the $\Lambda$ hyperon into a proton plus a negative pion. Crawford et al. had moved the 10 in. chamber into a negative pion beam, and were analysing a large sample of $\Lambda$'s from associated production events. They looked for an «up-down asymmetry» in the emission of pions from $\Lambda$'s, relative to the «normal to the production plane», as suggested by Lee and Yang. As a result, they had the pleasure of being the first to observe parity nonconservation in the decay of hyperons [54].

Fig. 12. $-K^{-}$ beam in 72 in. bubble chamber. a) No spectrometers on; b) one spectrometer on; c) two spectrometers on.
In the winter of 1958, the 15 in. chamber had completed its engineering test run as a prototype for the 72 in. chamber, and was operating for the first time as a physics instrument. Harold Ticho, Bud Good and Philippe Eberhard [55] had designed and built the first separated beam of $K^-$ mesons with a momentum of more than 1 GeV/c. Figure 12 shows the appearance of a bubble chamber when such a beam is passed through it, and when one or both of the electrostatic separators are turned off. The ingenuity which has been brought to bear on the problem of beam separation, largely by Ticho and Murray, is difficult to imagine, and its importance to the success of our program cannot be overestimated [55]. Joe Murray has recently joined the Stanford Linear Accelerator Center, where he has in a short period of time built a very successful radiofrequency-separated $K$ beam and a back-scattered laser beam.

The first problem we attacked with the 15 in. chamber was that of the $\Xi^0$. Gell-Mann had predicted that the $\Xi^-$ was one member of an $I$-spin doublet, with strangeness minus 2. The predicted partner of the $\Xi^-$ would be a neutral hyperon that decayed into a $\Lambda$ and a $\pi^0$—both neutral particles that would, like the $\Xi^0$, leave no track in the bubble chamber. A few years earlier, as an after-dinner speaker at a physics conference, Victor Weisskopf had «brought down the house» by exhibiting an absolutely blank cloud chamber photograph, and saying that it represented proof of the decay of a new neutral particle into two other neutral particles! And now we were seriously planning to do what had been considered patently ridiculous only a few years earlier.

According to the Gell-Mann and Nishijima strangeness rules, the $\Xi^0$ should be seen in the reaction

$$K^- + p \rightarrow \Xi^0 \rightarrow \Sigma^+ + K^0$$

$$\Lambda + \pi^0 \rightarrow \pi^- + \pi^+$$

$$\pi^- + p$$

In the one example of this reaction that we observed, Fig. 13, the charged pions from the decay of the neutral $K^0$ yielded a measurement of the energy and direction of the unobserved $K^0$. Through the conservation laws of energy and momentum (plus a measurement of the momentum of the interacting $K^-$ track) we could calculate the mass of the coproduced $\Xi^0$ hyperon plus its velocity and direction of motion. Similarly, measurements of the $\pi^-$ and proton gave the energy and direction of motion of the unobserved $\Lambda$, and proved that it did not come directly from the point at which the $K^-$ meson interacted with the proton. The calculated flight path of the $\Lambda$ intersected the calculated flight path of the $\Xi^0$, and the angle of intersection of the two
unobserved but calculated tracks gave a confirming measurement of the mass of the $\Xi^0$ hyperon, and proved that it decayed into a $\Lambda$ plus a $\pi^-$. This single hard-won event was a sort of tour de force that demonstrated clearly the power of the liquid hydrogen bubble chamber plus its associated data-analysis techniques.

Although only one $\Xi^0$ was observed in the short time the 15 in. chamber was in the separated $K^-$ beam, large numbers of events showing strange-particle production were available for study. The Franckenstein's were kept busy around the clock measuring these events, and those of us who had helped to build and maintain the beam now concentrated our attention on the analysis of these reactions. The most copious of the simple «topologies» was $K^- p \rightarrow$ two charged prongs plus a neutral V-particle. According to the
strangeness rules, this topology could represent either

\[ K^- + p \rightarrow \Lambda + \pi^+ + \pi^- \]
\[ \downarrow \]
\[ \pi^- + p \]

or

\[ K^- \rightarrow p \rightarrow K^0 + p + \pi^- \]
\[ \downarrow \]
\[ \pi^- + \pi^+ \]

The kinematics program, KICK, was now available to distinguish between these two reactions, and to eliminate those examples of the same topology in which an unobserved \( \pi^0 \) was produced at the first vertex. SUMX had not yet been written, so the labor of plotting histograms was assumed by the two very able graduate students who has been associated with the \( K^- \) beam and its exposure to the 15 in. chamber since its planning stages: Stanley Wojcicki and Bill Graziano. They first concentrated their attention on the energies of the charged pions from the production vertex in the first of the two reactions listed above. Since there were three particles produced at the vertex—a charged pion of each sign plus a \( \Lambda \)—one expected to find the energies of each of the three particles distributed in a smooth and calculable way from a minimum value to a maximum value. The calculated curve is known in particle physics at the «phase-space distribution». The decay of a \( \tau \) meson into three charged pions was a well-known «three-particle reaction» in which the dictates of phase space were rather precisely followed.

But when Wojcicki and Graziano finished transcribing their data from KICK printout into histograms, they found that phase-space distributions were poor approximations to what they observed. Figure 14 shows the distribution of energy of both positive and negative mesons, together with the corresponding «Dalitz plot», which Richard Dalitz [56] had originated to elucidate the «\( \tau^0 \) puzzle», which had in turn led to Lee and Yang's parity-nonconservation hypothesis.

The peaked departure from a phase-space distribution had been observed only once before in particle physics, where it had distinguished the reaction \( p + p \rightarrow \pi^+ + d \) from the «three-body reaction» \( p + p \rightarrow \pi^+ + p + n \). (Although no new particles were discovered in these reactions, they did contribute to our knowledge of the spin of the pion [57]). But such a peaking had been observed in the earliest days of experimentation in the artificial disintegration of nuclei, and its explanation was known from that time. Oliphant and Rutherford [58] observed the reaction \( p + ^{11}B \rightarrow ^3He \). This is a three-body reaction, and the energies of the \( \alpha \) particles had a phase-space-like distribution except for the fact that there was a sharp spike in the energy distribution at the highest
\( \alpha \)-particle energy. This was quickly and properly attributed [58] to the reaction

\[
p + ^{11}\text{B} \rightarrow ^{8}\text{Be} + ^{4}\text{He}
\]

\[
\downarrow
\]

\(^{4}\text{He} + ^{4}\text{He}\)

In other words, some of the reaction proceeded via a two-body reaction, in which one \( \alpha \) particle recoiled with unique energy against a quasistable \(^8\text{Be}\) nucleus. But the \(^8\text{Be}\) nucleus was itself unstable, coming apart in \(10^{-16}\) s into two \( \alpha \) particles of low relative energy. The proof of the fleeting existence of \(^8\text{Be}\) was the peak in the high-energy \( \alpha \)-particle distribution, showing that initially only two particles, \(^8\text{Be}\) and \(^4\text{He}\), participated in the reaction.

The peaks seen in Fig. 14 were thus a proof that the \( \pi^\pm \) recoiled against a combination of \( \Lambda + \pi^\mp \) that had a unique mass, broadened by the effects of the uncertainty principle. The mass of the \( \Lambda \pi \) combination was easily calculable.
as 1382 MeV, and the $I$-spin of the system was obviously 1, since the $I$-spin of the $\Lambda$ is 0, and the $I$-spin of the $\pi$ is 1. This was then the discovery of the first «strange resonance», the $\Sigma_f^+(1385)$. Although the famous Fermi 3, 3 resonance had been known for years, and although other resonances in the $\pi^{-}$ nucleon system had since shown up in total cross-section experiments at

![Graph showing mass of K\pi system](image)

Fig. 15. – Discovery of the K*(890).
Brookhaven and Berkeley, CalTech and Cornell [59] the impact of the $Y_1^*$ resonance on the thinking of particle physicists was quite different—the $Y_1^*$ really acted like a new particle, and not simply as a resonance in a cross-section.

![Diagram of $Y_1^*$ and $Y_0^*$ resonances with mass and number distributions.]

**Fig. 16.** Discovery of the $Y_0^*(1405)$.

We announced the $Y_1^*$ at the 1960 Rochester High Energy Physics Conference [60], and the hunt for more short-lived particles began in earnest. The same team from our bubble chamber group that had found the $Y_1^*(1385)$
now found two other strange resonances before the end of 1960—the K*\(^{(890)}\) [61], and the \(Y_{\sigma}^{0}(1405)\) [62].

Although the authors of these three papers have for years been referred to as «Alston et al.», I think that on this occasion it is proper that the full list be named explicitly. In addition to Margaret Alston (now Margaret Garnjost) and Luis W. Alvarez, and still in alphabetical order, the authors are: Philippe Eberhard, Myron L. Good, William Graziano, Harold K. Ticho, and Stanley G. Wojcicki.

Figures 15 and 16 show the histograms from the papers announcing these two new particles; the K* was the first example of a «boson resonance» found by any technique. Instead of plotting these histograms against the energy of one particle, we introduced the now universally accepted technique of plotting them against the effective mass of the composite system: \(\Sigma + \pi\) for the \(Y_{\sigma}^{0}(1405)\) and \(K + \pi\) for the K*\(^{(890)}\). Figure 17 shows the present state of the art relative to the K*\(^{(890)}\); there is essentially no phase-space background in this histogram, and the width of the resonance is clearly measurable to give the lifetime of the resonant state via the uncertainty principle.

![Histogram of K* (890)](image)

**Fig. 17.** Present day K*\(^{(890)}\).

These three earliest examples of strange-particle resonances all had lifetimes of the order of \(10^{-23}\) s, so the particle all decayed before they could traverse more than a few nuclear radii. No one had foreseen that the bubble chamber could be used to investigate particles with such short lives; our chambers
had been designed to investigate the strange particles with lifetimes of $10^{-10}$ s—$10^{13}$ times as long.

In the summer of 1959, the 72 in. chamber was used in its first planned physics experiment. Lynn Stevenson and Philippe Eberhard designed and constructed a separated beam of about 1.6 GeV/c antiprotons, and a quick scan of the pictures showed the now famous first example of antilambda production, via the reaction

\[
\bar{p} + p \rightarrow \bar{\Lambda} + \Lambda
\]

\[
\downarrow \quad \downarrow
\]

\[
\pi^+ + \bar{p} \quad \pi^- + p
\]

Fig. 18. — First production of anti-lambda.
Figure 18 shows this photograph, with the antiproton from the antilambda decay annihilating in a four-pion event. I believe that everyone who attended the 1959 High Energy Physics Conference in Kiev will remember the showing of this photograph—the first interesting event from the newly operating 72 in. chamber.

Hofstadter's classic experiments on the scattering of high energy electrons by protons and neutrons [63] showed for the first time how the electric charge was distributed throughout the nucleons. The theoretical interpretation of the experimental results [64] required the existence of two new particles, the vector mesons now known as the $\omega$ and the $\rho$. The adjective «vector» simply means that these two mesons have one unit of spin, rather than zero, as the ordinary $\pi$ and K mesons have. The $\omega$ was postulated to have $I$-spin = 0. and the $\rho$ to have $I$-spin = 1; the $\omega$ would therefore exist only in the neutral state, while the $\rho$ would occur in the $+, -, 0$ charged states.

Many experimentalists, using a number of techniques, set out to find these important particles, whose masses were only roughly predicted. The first success came to Bogdan Maglić, a visitor to our group, who analysed film from the 72 in. chamber's antiproton exposure. He made the important decision to concentrate his attention on proton-antiproton annihilations into five pions—two negative, two positive, and one neutral. KICK gave him a selected sample of such events; the tracks of the $\pi^0$ could not be seen, of course, but the constraints of the conservation laws permitted its energy and direction to be computed. Maglic then plotted a histogram of the effective mass of all neutral three-pion combinations. There were four such neutral combinations for each event; the neutral pion was taken each time together with all four possible pairs of oppositely charged pions. SUMX was just beginning to work, and still had bugs in it, so the preparation of the histogram was a very tedious and time-consuming chore, but as it slowly emerged, Maglic had the thrill of seeing a bump appear in the side of his phase-space distribution. Figure 19 shows the peak that signaled the discovery of the very important $\omega$ meson.

Although Bogdan Maglić originated the plan for this search, and pushed through the measurements by himself, he graciously insisted that the paper announcing his discovery [65] should be co-authored by three of us who had developed the chamber, the beam, and the analysis program that made it possible.

The $\rho$ meson is the only one from this exciting period in the development of particle physics whose discovery cannot be assigned uniquely. In our group, the two Frankensteins were being used full time on problems that the senior members felt had higher priority. But a team of junior physicists and graduate students, Anderson et al. [66], found that they could make accurate
Fig. 19. - Discovery of the \( \omega \) meson.
enough measurements directly on the scanning tables to accomplish a «Chew-Low extrapolation». Chew and Low had described a rather complicated procedure to look for the predicted dipion resonance now known as the $\rho$ meson. Figure 20 shows the results of this work, which convinced me that the $\rho$ existed and had its predicted spin of 1. The mass of the $\rho$ was given as about 650 MeV, rather than its now accepted value of 765 MeV. (This low value is now explained in terms of the extreme width of the $\rho$ resonance.)

The evidence for the $\rho$ seemed to me even more convincing than the early evidence Fermi and his co-workers produced in favor of the famous $3,3$ pion-nucleon resonance.

![Graph showing $\sigma \pi \pi$ (mb) vs $\omega^2 (m \pi^2)$ with data points and a fitted curve.](image)

**Fig. 20.** First evidence for the $\rho$ meson.

But one of the unwritten laws of physics is that one really has not made a discovery until he has convinced his peers that he has done so. We had just persuaded high energy physicists that the way to find new particles was to look for bumps on effective-mass histograms, and some of them were therefore unimpressed by the Chew-Low demonstration of the $\rho$. Fortunately, Walker and his collaborators [67] at Wisconsin soon produced an effective-mass ideogram with a convincing bump at 765 MeV, and they are therefore most often listed as the discoveres of the $\rho$.

Ernest Lawrence very early established the tradition that his laboratory would share its resources with others outside its walls. He supplied short-lived radioactive materials to scientists in all departments at Berkeley, and
he sent longer-lived samples to laboratories throughout the world. The first artificially created element, technetium, was found by Perrier and Segre [68], who did their work in Palermo, Sicily. They analysed the radioactivity in a molybdenum deflector strip from the Berkeley 28 in. cyclotron that had been bombarded for many months by 6 MeV deuterons.

We followed Ernest Lawrence's example, and thus participated vicariously in a number of important discoveries of new particles. The first was the $\eta$ found at Johns Hopkins, by a group headed by Aihud Pevsner [69]. They analysed film from the 72 in. chamber, and found the $\eta$ with a mass of 550 MeV, decaying into $\pi^+\pi^-\pi^0$. Within a few weeks of the discovery of the $\eta$, Rosenfeld and his co-workers [70] at Berkeley, who had independently observed the $\eta$, showed quite unexpectedly that $I$-spin was not conserved in its decay. Figure 21 shows the present state of the art with respect to the $\omega$ and $\eta$ mesons; the strengths of their signatures in this single histogram is in marked contrast to their first appearance in 72 in. bubble chamber experiments.

In the short interval of time between the first and second publications on the $\eta$, the discovery of the $Y_0^*(1520)$ was announced by Ferro-Luzzi, Tripp, and Watson [71], using a new and elegant method. Bob Tripp has continued to be a leader in the application of powerful methods of analysis to the study of the new particles.

The discovery of the $\Xi^*(1530)$ hyperon was accomplished in Los Angeles by Ticho and his associates [72], using 72 in. bubble chamber film. Harold Ticho had spent most of his time in Berkeley for several years, working tirelessly on every phase of our work, and many of his colleagues had helped prepare the high-energy separated $K^-$ beam for what came to be known as the K72 experiment. The UCLA group analysed the two highest-momentum $K^-$ exposures in the 72 in. chamber, and found the $\Xi^*(1530)$ just in time to report it at the 1962 High Energy Physics Conference in Geneva. (Confirming evidence for this resonance soon came from Brookhaven [73]).

Murray Gell-Mann had recently enunciated his important ideas concerning the «Eightfold Way» [74], but his paper had not generated the interest it deserved. It was soon learned that Ne'eman had published the same suggestions, independently [75].

The announcement of the $\Xi^*(1530)$ fitted exactly with their predictions of the mass and other properties of that particle. One of their suggestions was that four $I$-spin multiplets, all with the same spin and parity, would exist in a «decuplet» with a mass spectrum of «lines» showing an equal spacing. They put the Fermi 3, 3 resonance as the lowest mass member, at 1238 MeV. The second member was the $Y_1^*(1385)$, so the third member should have a mass of $(1385) + (1385 - 1238) = 1532$. The strangeness and the multiplicity
Fig. 21. – Present day histogram showing $\omega$ and $\eta$ mesons.

of each member of the spectrum was predicted to drop 1 unit per member, so the $\Xi^*(1530)$ fitted their predictions completely. It was then a matter of simple arithmetic to set the mass, the strangeness, and the charge of the
final member—the $\Omega^-$. The realization that there was now a workable theory in particle physics was probably the high point of the 1962 International Conference on High Energy Physics.

Since the second and third members of the series—the ones that permitted the prediction of the properties of the $\Omega^-$ to be made—had come out of our bubble chamber, it was a matter of great disappointment to us that the Bevatron energy was insufficient to permit us to look for the $\Omega^-$. Its widely acclaimed discovery [76] had to wait almost two years, until the 80 in. chamber at Brookhaven came into operation.

Since the name of the $\Omega$ had been picked to indicate that it was the last of the particles, the mention of its discovery is a logical point at which to conclude this lecture. I will do so, but not because the discovery of the $\Omega$ signaled the end of what is sometimes called the population explosion in particle physics—the latest list [77] contains between 70 and 100 particle multiplets, depending upon the degree of certainty one demands before «certification». My reason for stopping at this point is simply that I have discussed most of the particles found by 1962—the ones that were used by Gell-Mann and Ne'eman to formulate their $SU_3$ theories—and things became much too involved after that time. So many groups were then in the «bump-hunting business» that most discoveries of new resonances were made simultaneously in two or more laboratories.

I am sorry that I have neither the time nor the ability to tell you of the great beauty and the power that has been brought to particle physics by our theoretical friends. But I hope that before long, you will hear it directly from them.

In conclusion, I would like to apologize to those of my colleagues and my friends in other laboratories, whose important work could not be mentioned because of time limitations. By making my published lecture longer than the oral presentation, I have reduced the number of apologies that are necessary, but unfortunately I could not completely eliminate such debts.

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Recent developments in particle physics


Weak Interactions
and the Breaking of Hadron Symmetries.

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Introduction.

In a recent paper [1] we have proposed a dynamical relation among weak, strong, and e.m. interactions, on the basis of a self-consistency condition which requires that the description of hadron physics remains unchanged by the effect of leading weak corrections and e.m. tadpole effects. The main results of this approach were a relation between the weak interaction angle $\theta$ and the symmetry breaking parameters, and the appearance of a nonelectromagnetic breaking of isotopic spin.

The presentation in ref. [1] was centered on the application of the self-consistency condition itself. This condition is, to a certain extent, arbitrary at the present stage of the theory and a deeper understanding of the whole subject is certainly needed to acquire more confidence in its validity. In this paper we present a review of the whole theory putting a particular emphasis on those aspects which do not require explicitly the use of the self-consistency condition. These aspects are interesting on their own, and pose many open problems to future investigations.

In Sect. 1 we give an outline of hadron symmetries and their connection to weak interactions. Section 2 contains a phenomenological analysis of symmetry breaking. On this point we follow the analysis of Gell-Mann et al. [2] and of Glashow and Weinberg [3], and in addition we discuss the necessity, at a phenomenological level, of introducing a nonelectromagnetic isospin breaking. In Sect. 3 we discuss attempts to understand the structure
of symmetry breaking in terms of a purely strong interaction dynamics. In this connection we give a simple derivation of a result due to Michel and Radicati [4], indicating that the patterns of symmetry breaking preferred by strong interaction bootstrap are those which reduce \( SU_3 \otimes SU_3 \) to either \( SU_3 \) or \( SU_2 \otimes SU_2 \). Section 4 is devoted to an analysis of the consequences of the non-e.m.-isospin breaking previously introduced with respect to \( \gamma \rightarrow 3\pi \) decay and to the mass splittings within isospin multiplets.

In Sect. 5 we study the possible strong effects of weak interactions. In particular we show that if the explicit strong breaking of \( SU_3 \otimes SU_3 \) transforms as a \((3, \bar{3}) + (\bar{3}, 3)\) representation, these corrections do not produce a breaking of parity and strangeness at a strong level. We also show that this is not true if the explicit breaking transforms as a \((1, 8) + (8, 1)\).

Finally, in Sect. 6, we review our self-consistency requirement and its consequences.

1. – Investigations on symmetries in elementary particle physics have been pursued with an ever increasing degree of effort and sophistication in the last decade.

The well-established isospin symmetry acquired a new dimension with the Conservation of Vector Current (CVC) hypothesis of Feynman and Gell-Mann [5], which identified the charged isospin currents with a part of the vector current appearing in the weak coupling of leptons to hadrons. The neutral currents related to \( I_3 \) and to the hypercharge \( Y \) were already identified as components of the electromagnetic current. The CVC hypothesis thus accomplished the program of giving a physical role to the generators of the whole \( SU_3 \otimes U_{1Y} \), which at that time represented (apart from baryonic number and discrete symmetries) the full invariance group of strong interactions.

It was then natural to try the opposite approach, namely to give a symmetry role to all currents with a physical meaning, and in particular to the axial and the strangeness changing currents (both axial and vector) appearing in \( \beta \)-decays.

Consideration of the strangeness nonchanging axial current led to the extension of the \( SU_2 \) group of isospin into the chiral \( SU_2 \otimes SU_2 \) group. This interpretation of the axial current has been made possible by the use of an entirely new concept, i.e., that of a dynamically broken symmetry, introduced by Nambu and Jona-Lasinio [6]. The chiral \( SU_2 \otimes SU_2 \) symmetry, in fact, is not realized in the usual way, as this would require all hadrons to appear in degenerate parity doublets. Baryons not appearing in parity doublets should have a vanishing mass, a possibility even further from physical reality. That chiral \( SU_2 \otimes SU_2 \) symmetry can nevertheless be realized is made plausible by the following argument: A continuous symmetry group implies
the existence of a set of operators commuting with the Hamiltonian. One
of these operators, when acting on a single particle state, should turn it into
states of the same mass.

In the usual realization of a symmetry, these new states are also one
particle states, and this requires particles to appear in degenerate multiplets,
which form a basis for a representation of the group in question. The gen-
erators of a dynamically broken symmetry turn instead single particle into
multiparticle states, and in order for these states to have the same mass as
the original one, massless bosons must appear. In the case of chiral $SU_2 \otimes
SU_2$, these bosons are identified with the pions, and the symmetry is exact
to the extent that one can neglect the pion mass. An axial generator turns
then, e.g., a single nucleon state into « one nucleon plus many pions » states
and this avoids the parity doubling of nucleon states. It is clear that the
hypothesis of chiral $SU_2 \otimes SU_2$ symmetry for strong interactions does not
lead to new predictions on hadron spectrum, but provides a powerful tool
for relating processes involving low energy (soft) pions. The exploitation of
soft pion theorems, pioneered by Nambu, has received a great amount of
attention in the last few years, yielding numerous results in good agreement
with experiments [7]. The same results have also been obtained by an equiva-
 lent approach based on the hypothesis of Partial Conservation of the
Axial Current (PCAC), introduced by Gell-Mann and Lévy [8].

Only after the discovery of $SU_3$ as an approximate symmetry of hadrons,
it has been possible to complete the program of giving a symmetry inter-
pretation for the strangeness changing weak current. Since $SU_3$ is a symmetry
of the normal kind, akin to isotopic spin, it allows a classification of hadrons
in supermultiplets comprising multiplets of different isospin and strangeness.

The two extensions of $SU_2$, i.e., chiral $SU_2 \otimes SU_2$ and $SU_3$, both well
supported by experimental facts, can only coexist if they are subgroups of
a larger symmetry. The simplest possibility for the larger group is a chiral
$SU_3 \otimes SU_3$. Associated with this larger group is a set of eight vector and
eight pseudovector currents, $V_\mu^i(x)$ and $A_\mu^i(x)$ ($i = 1, \ldots, 8$). One can com-
bine these currents into two sets:

\begin{align}
(1) & \quad J_\mu^t = V_\mu^t + A_\mu^t, \\
(2) & \quad J_\mu^z = V_\mu^z - A_\mu^z,
\end{align}

whose charges generate two commuting $SU_3$.

It has been conjectured [9] that the weak current of hadrons is a com-
bination of these according to:

\begin{align}
(3) & \quad J^\text{weak}_\mu = \cos \theta (J_\mu^1 + iJ_\mu^2) + \sin \theta (J_\mu^4 + iJ_\mu^5),
\end{align}
Weak interactions and the breaking of hadron symmetries

\( \theta \) being a new, universal constant. Equation (3) has been used to give a simple description of all \( \beta \)-decay processes, which has been fully confirmed by the existing experimental data [10]. In particular \( \theta \) has been determined to be \( \simeq 0.22 \).

The structure of the theory can be presented in a very simple way, using the language of the quark model. In this model all hadrons are bound states of the three quarks \( p, n, \lambda \), and their antiparticles. The quarks themselves transform as the basic representation 3 of \( SU_3 \). The symmetry currents \( J^i_\mu \) and \( \bar{J}^i_\mu \) have the simple expression:

\[
\begin{align*}
J^i_\mu &= \bar{\psi} \gamma_\mu (1 + \gamma_5) \frac{\lambda_i}{2} \psi, \\
\bar{J}^i_\mu &= \bar{\psi} \gamma_\mu (1 - \gamma_5) \frac{\lambda_i}{2} \psi,
\end{align*}
\]

where \( \lambda_i \) (\( i = 1, \ldots, 8 \)) are the eight Gell-Mann matrices. The hadronic weak current, eq. (3), has the simple form:

\[
J^{\text{weak}}_\mu = \bar{p} \gamma_\mu (1 + \gamma_5) [\cos \theta n + \sin \theta \lambda] = \bar{\psi} \gamma_\mu (1 + \gamma_5) \lambda^+ \psi,
\]

where

\[
\lambda^+ = \begin{pmatrix}
0 & \cos \theta & \sin \theta \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

In this picture the meaning of the parameter \( \theta \) is particularly transparent. The weak current has a specially simple form in terms of the three fields: \( p, n' = \cos \theta n + \sin \theta \lambda, \lambda' = -\sin \theta n + \cos \theta \lambda \):

\[
J^{\text{weak}}_\mu = \bar{p} \gamma_\mu (1 + \gamma_5) n'.
\]

Thus \( \theta \) is seen to be the angle between the frame \( p, n, \lambda \) and \( p, n', \lambda' \) chosen in the quark internal space by the \( SU_3 \) breaking and by the weak interactions respectively. The current given by (5) and (7) displays exactly the same structure as the \((e\nu_e), (\mu\nu_\mu)\) pieces of the leptonic current, thus obeying the requirement of universality of weak interactions. The universality principle can be given a more abstract formulation in terms of current commutators [11], and can be proven to be satisfied by eq. (3) without any reference to the quark model.

\[
\begin{array}{c}
\text{SU}_3 \\
\text{SU}_2 \times \text{SU}_2 \\
\text{SU}_3 \times \text{SU}_3
\end{array}
\]

Fig. 1.
2. – In the previous section we have outlined the emergence of a chain of higher symmetries for hadrons, from $SU_2 \otimes U_{1Y}$ to $SU_3 \otimes SU_3$, according to the pattern of Fig. 1. We have also emphasized the deep connection between weak and e.m. interactions on one side and the symmetry structure of strong interactions on the other side. In this Section we will present a phenomenological study of the breaking of these symmetries following the works of refs. [2] and [3]. This discussion will be based on the usual assumption that hadron physics can be described by a hierarchy of interactions, where the main features of hadronic processes are determined by strong interactions, with small e.m. and weak perturbation. This is in fact a very strong assumption, which will probably be abandoned in the future developments of the theory. As we shall see later, weak interactions can give rise to large effects, strictly interwoven with what one would call proper strong effects.

With this proviso, let us write the hadron Lagrangian as:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_{\text{e.m.}} + \mathcal{L}_w.$$  \hspace{1cm} (8)

We shall also write $\mathcal{L}_s = \mathcal{L}_0 + \mathcal{L}_1$, where $\mathcal{L}_0$ is that part of the strong Lagrangian which is symmetric under the full $SU_3 \otimes SU_3$ group. We will refer to $\mathcal{L}_1$ as the explicit breaking term of $\mathcal{L}_s$. In fact, even in the limit $\mathcal{L}_1 = 0$ and neglecting weak and e.m. effects, one can still have a dynamical breaking, as discussed in the previous section for the case of spontaneous breaking of $SU_2 \otimes SU_2$. In this case a particular realization of $\mathcal{L}_0$ would be characterized by a subgroup $G$ of $SU_3 \otimes SU_3$, represented in the multiplet structure of hadrons. To each of the $SU_3 \otimes SU_2$ generators not belonging to $G$ it would correspond a scalar or pseudoscalar massless boson. Some possibilities are given in Table I. Only the first one appears to be near the physical situation. In fact the hadron spectrum clearly exhibits the $SU_3$ multiplet structure, and the eight pseudoscalar mesons are the least massive of known hadrons.

<table>
<thead>
<tr>
<th>Multiplet Structure</th>
<th>Massless Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU_3$</td>
<td>$\pi, K, \eta$</td>
</tr>
<tr>
<td>Chiral $SU_2 \otimes SU_3$</td>
<td>$K, \eta, K_s$</td>
</tr>
<tr>
<td>$SU_3 \otimes U_{1Y}$</td>
<td>$\pi, K, \eta, K_s$</td>
</tr>
</tbody>
</table>

Table I. – Some possibilities for the spontaneous breaking of $SU_3 \times SU_3$. $K_s$ indicates a scalar kappa meson.

A satisfactory description of $SU_3 \otimes SU_3$ breaking appears then to be one in which the symmetry of $\mathcal{L}_0$ is realized according to the first possibility of Table I. The explicit breaking term $\mathcal{L}_1$ would then cause a departure from
Weak interactions and the breaking of hadron symmetries

this situation, giving finite masses to the pseudoscalar mesons as well as mass splittings within $SU_3$ multiplets. This picture provides a rational basis to the observation that the masses of the pseudoscalar mesons are of the same order of magnitude as their mass splittings, and of mass splittings within other multiplets.

The fact that the pion mass is an extremely small quantity on the hadron scale ($m_{\pi}^2 \approx 0.019$ GeV$^2$) together with the considerations of Sect. 1 indicate that the structure of $\mathcal{L}_1$ is such as to give a relatively large breaking of $SU_3 \otimes SU_3$ into $SU_2 \otimes SU_2$, and a much smaller breaking of chiral $SU_2 \otimes SU_2$. The success of the Gell-Mann–Okubo mass formula moreover indicates that $\mathcal{L}_1$ is a superposition of $SU_3$ singlets and octets. All these requirements are met if we assume that $\mathcal{L}_1$ transforms as a component of a $(3, \bar{3}) + (\bar{3}, 3)$ representation of chiral $SU_3 \otimes SU_3$.

The basis for such a representation is a set of nine scalar and nine pseudo-scalar densities $\sigma^t$ and $\pi^t$ [12], obeying the equal-time commutation relations:

$$[V_1, \sigma_j] = if_{ijk}\sigma_k,$$
$$[V_1, \pi_j] = if_{ijk}\pi_k,$$
$$[A_t, \sigma_j] = id_{ijk}\sigma_k,$$
$$[A_t, \pi_j] = -id_{ijk}\pi_k.$$

In terms of these objects, (assuming parity and strangeness conservation) $\mathcal{L}_1$ has the simple form:

$$\mathcal{L}_1 = \epsilon_0\sigma_0 + \epsilon_8\sigma_8 + \epsilon_3\sigma_3.$$  

(9)

This picture has a simple transcription in quark language, where:

$$\sigma_t = \bar{\psi}\frac{\lambda_t}{2}\psi,$$
$$\pi_t = i\bar{\psi}\frac{\lambda_t}{2}\gamma_5\psi,$$

and the breaking term $\mathcal{L}_1$ can be interpreted as a quark mass term:

$$\mathcal{L}_1 = \alpha\bar{p}p + \beta\bar{n}n + \gamma\bar{\lambda}\lambda,$$

(10)

where:

$$\alpha = \sqrt{\frac{2}{3}}\epsilon_0 + \frac{\epsilon_8}{\sqrt{3}} + \epsilon_3,$$
$$\beta = \sqrt{\frac{2}{3}}\epsilon_0 + \frac{\epsilon_8}{\sqrt{3}} - \epsilon_3,$$
$$\gamma = \sqrt{\frac{2}{3}}\epsilon_0 - \frac{2}{\sqrt{3}}\epsilon_8.$$  

(11)
The requirement that $\mathcal{L}_1$ leads to a nearly exact $SU_3 \otimes SU_2$ symmetry can be stated as saying that $p$ and $n$ masses must be much smaller than the $\lambda$ mass, i.e.

$$\alpha, \beta \ll \gamma$$

or, equivalently,

$$\varepsilon_8 \approx -\sqrt{2}\varepsilon_0.$$

Detailed phenomenological analyses have been carried out in refs. [2] and [3], and indicate that:

(12) $$\varepsilon_8/\varepsilon_0 \approx -1.25.$$ 

In eq. (9) we have attributed to $\mathcal{L}_1$ also a term proportional to $\sigma_3$ which causes a breakdown of isospin symmetry, independent from the isospin breaking induced by the electromagnetic interaction $\mathcal{L}_{\text{e.m.}}$. The necessity of such a term is indicated by two different theoretical results. The first one is a theorem due to Bell and Sutherland [13], according to which in the soft pion limit the isospin violating decay $\gamma \rightarrow 3\pi$ cannot proceed through e.m. interaction. Bell and Sutherland have also shown that the observed decay rate is by at least two orders of magnitude larger than what could be obtained introducing the corrections for a finite pion mass. The second argument relies on a theorem by R. Dashen [14], which states that, if isospin breaking is of a purely e.m. origin the sum rule:

$$m_{K^+}^2 - m_{K^0}^2 = m_{\pi^+}^2 - m_{\pi^0}^2$$

should be verified up to terms of order $e^2\varepsilon_8$ and $e^2\varepsilon_0$. The failure of this sum rule, which reads:

$$-4000\text{ MeV}^2 = 1260\text{ MeV}^2$$

has to be compared with the good agreement of the Coleman and Glashow [15] sum rule:

$$\Xi^- - \Xi_0 = p - n + \Sigma^- - \Sigma^+$$

which is derived also neglecting terms of order $e^2\varepsilon_8$. The problems arising by the theorems of Bell and Sutherland and of Dashen can both be solved by introducing a $\sigma_3$ term in $\mathcal{L}_1$. Question arises whether one can fix the parameter $\varepsilon_3$ in such a way as to obtain a quantitative agreement in both cases. This question will be discussed in Sect. 4 where we will also analyze other possible consequences of the $\sigma_3$ interaction.
3. We want now to consider the question whether the structure of symmetry breaking which we have studied at a phenomenological level, and in particular the values for the parameters $\epsilon_0, \epsilon_8, \epsilon_3$, can be understood in the framework of current theories of strong interactions.

Our present understanding of strong interaction dynamics is based on the idea of bootstrap according to which the parameters of the theory should be determined by self-consistency conditions. The study of $SU_3$ breaking within a bootstrap theory has been pioneered by R. Cutkosky [16], who found that solutions of the bootstrap are favored, where the breaking reduces $SU_3$ to exact $SU_2$. Michel and Radicati [17] have produced an interesting geometrical insight into this result, and have recently extended [4] their study to the breaking of $SU_3 \otimes SU_2$. We shall state here their result, and discuss its limitations and its implications for our program. In Appendix I we shall give a simple derivation of the Michel-Radicati theorem.

Let us assume that the bootstrap condition can be expressed as a variational principle:

\begin{equation}
\delta G(\epsilon_3, \epsilon_8, \epsilon_0) = 0.
\end{equation}

Equation (13) will have to be completed by some other requirement, to be called the «stability condition» which will select a solution of eq. (13) as the physically relevant one.

We do not need to give here a specific form to these stability conditions; they could simply consist in the requirement that the stable solution be a minimum of $G$, or could be of a more complicated nature [18].

The result of Michel and Radicati then states that there exist always solutions of eq. (13) which correspond to:

i) exact $SU_3 \otimes SU_2$ ($\epsilon_0 = \epsilon_8 = \epsilon_0 = 0$);

ii) exact $SU_3$ ($\epsilon_3 = \epsilon_8 = 0; \epsilon_0 \neq 0$);

iii) exact chiral $SU_2 \otimes SU_2$ ($\epsilon_3 = 0; \epsilon_8 = -\sqrt{2}\epsilon_0$).

The significance of this result is limited by the consideration that one is not able to prove the nonexistence of other solutions which might be more «stable» than the previous ones.

The Michel-Radicati theorem can be taken as an indication that strong interactions favor solutions of the kind i), ii), iii), which are different from the one observed (even if only slightly in the case of solution iii)). We obtain therefore from their result some support to the idea that weak and e.m. interactions play a nonnegligible role in determining the structure of the breaking. In order to prove the existence of solutions ii) and iii), Michel
and Radicati restrict the range of variation of the $\varepsilon$'s to the unit sphere:

$$\varepsilon_3^2 + \varepsilon_8^2 + \varepsilon_0^2 = 1.$$  

As shown in Appendix I this restriction can be justified by the physical hypothesis that bootstrap equations are not able to fix the scale of hadron masses, i.e., that $G$ has the scale invariance property:

$$G(A\varepsilon_3, A\varepsilon_8, A\varepsilon_0) = G(\varepsilon_3, \varepsilon_8, \varepsilon_0).$$

4. Non-e.m. breaking of $I$-spin.

In this section we describe possible physical consequence of a non-e.m. breaking of isospin, as embodied in the term $\varepsilon_3 \sigma_3$ of $\mathcal{L}_1$.

Electromagnetism itself can give rise to corrections which imitate the effect of such a term. These corrections are the so-called tadpole contributions of the e.m. interactions. They arise from graphs of the kind shown in Fig. 2,

![Diagram](image)

Fig. 2.

which correspond to the annihilation into the vacuum of scalar mesons associated with the $\sigma_3$, $\sigma_8$, and $\sigma_0$ densities. We may separate this contributions from $\mathcal{L}_{\text{e.m.}}$, by adding to the Lagrangian a counter term $\delta_{\text{e.m.}}\mathcal{L}_1$, i.e., writing eq. (8) as:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{\text{e.m.}} + \mathcal{L}_{\text{weak}} =$$

$$= \mathcal{L}_0 + \mathcal{L}_1 + \delta_{\text{e.m.}}\mathcal{L}_1 + (\mathcal{L}_{\text{e.m.}} - \delta_{\text{e.m.}}\mathcal{L}_1) + \mathcal{L}_{\text{weak}}.$$

Therefore $\delta_{\text{e.m.}}\mathcal{L}_1$ is determined by the following condition on the matrix element for the transition of one scalar meson into the vacuum extrapolated to zero four-momentum:

$$\lim_{p, \mu \to 0} \langle 0 | (\mathcal{L}_{\text{e.m.}} - \delta_{\text{e.m.}}\mathcal{L}_1) | \sigma, p \rangle = 0.$$
In the following we will indicate by $\varepsilon_3 \sigma_3$ the whole isospin breaking term of $\mathcal{L}_1 + \delta^{\text{e.m.}} \mathcal{L}_1$, calling $\chi \varepsilon_3 \sigma_3$ it purely e.m. part:

\begin{equation}
\varepsilon_3 \sigma_3 = \varepsilon_3 (1 - \chi) \sigma_3 + \chi \varepsilon_3 \sigma_3.
\end{equation}

Any isospin violating amplitude can thus be split into two terms, \textit{i.e.}:

\begin{equation}
(\text{contribution of } \mathcal{L}_1 + \delta^{\text{e.m.}} \mathcal{L}_1) \mp (\text{non-tadpole e.m. contributions}).
\end{equation}

The non-tadpole contributions correspond to graphs different from Fig. 2, \textit{e.g.}, of the kind shown in Fig. 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Fig. 3.}
\end{figure}

We will first discuss the two cases, where, due to the theorems of Bell and Sutherland [13], and of Dashen [14], the non-e.m. contribution to isospin breaking can be uniquely identified.

These two effects are the violation of the Dashen sum rule for $K^+ - K^0$, $\pi^+ - \pi^0$ mass difference, and the $\gamma \rightarrow 3\pi$ decay. In these cases it is convenient to decompose as in eq. (16) the isospin breaking Lagrangian, and use the computational scheme:

\begin{equation}
(\text{isospin breaking effect}) = (\text{contribution of } \varepsilon_3 (1 - \chi) \sigma_3) + \\
+ (\text{total e.m. corrections}).
\end{equation}

To lowest order in $\mathcal{L}_1$, and neglecting e.m. effects, the masses of pseudoscalar mesons are:

\begin{align*}
\pi^0, \pi^\pm &= C \left\{ \frac{\varepsilon_3}{\sqrt{3}} + \varepsilon_0 \sqrt{\frac{2}{3}} \right\}, \\
K^\pm &= C \left\{ -\frac{\varepsilon_3}{2 \sqrt{3}} + \varepsilon_0 \sqrt{\frac{2}{3}} + \frac{\varepsilon_3 (1 - \chi)}{2} \right\}, \\
K^0 &= C \left\{ -\frac{\varepsilon_3}{2 \sqrt{3}} + \varepsilon_0 \frac{\sqrt{2}}{3} - \frac{\varepsilon_3 (1 - \chi)}{2} \right\}.
\end{align*}
\[ \eta = C \left\{ -\frac{\varepsilon_8}{\sqrt{3}} + \varepsilon_0 \sqrt{\frac{2}{3}} \right\}, \]
\[ \delta M_{\eta\pi}^2 = C \left\{ \frac{\varepsilon_8 (1 - \chi)}{\sqrt{3}} \right\}, \]

where particle symbols stand for the corresponding mass squared, \( C \) is the reduced matrix element \( \| \pi | \sigma | \pi \rangle \), and \( \delta M_{\eta\pi}^2 \) is the \( \eta - \pi^0 \) mixing matrix element induced by \( \sigma_3 \). The Dashen sum rule implies that the combination

\[ (K^+ - K^0) - (\pi^+ - \pi^0) = -5260 \text{ (MeV)}^2 \]

is not affected, up to terms of order \( e^2 \mathcal{L}_1 \), by the e.m. corrections. This allows a unique determination of the ratio \( \varepsilon_8 (1 - \chi) / \varepsilon_8 \) according to:

\[ \frac{\varepsilon_8 (1 - \chi)}{\sqrt{3} \varepsilon_8} = -\frac{(K^+ - K^0) - (\pi^+ - \pi^0)}{2(K^+ - \pi^-)} = 1.2 \times 10^{-2}. \]

Using eq. (18) and the Bell-Sutherland results one concludes that the \( \eta \rightarrow 3\pi \) amplitude is given by

\[ T = \langle 3\pi | (1 - \chi) \sigma_3 | \eta \rangle. \]

We note at this point that models of \( \eta \) decay based on the \( \sigma_3 \) interaction had already appeared in the literature. Only after Sutherland's result, however, it became clear that such an interaction could not be a consequence of electromagnetism.

Another class of models which have been advanced, is based on \( \eta - \pi \) mixing, and describes the decay as \( \eta \rightarrow \pi^0 \rightarrow 3\pi \) or \( \eta \rightarrow \eta \pi \pi \rightarrow \pi^0 \pi \pi \). Bell and Sutherland have shown that such models cannot work if e.m. interactions only are assumed. An insight into this result is given by the Dashen sum rule. Using \( U \)-spin invariance one obtains:

\[ \delta M_{\eta\pi}^2 = \frac{1}{\sqrt{3}} \{(K^+ - K^0) - (\pi^+ - \pi^0)\} \]

which, combined with the Dashen sum rule, shows that the mixing cannot be ascribed to electromagnetism.

In Appendix II we give a simple derivation of the \( \eta \rightarrow 3\pi \) amplitude based on eq. (20). The result of this calculation is:

\[ T(\eta \rightarrow 3\pi) = -\frac{2\varepsilon_8 (1 - \chi)}{F_{\pi} \varepsilon_8} \left( S - \frac{4}{3} m_{\pi}^2 \right) = \]
\[ = -\frac{2\varepsilon_8 (1 - \chi)}{3F_{\pi}^2 \varepsilon_8} (M_{\eta}^2 - M_{\pi}^2) \left( 1 - \frac{2Q M_{\eta}}{M_{\eta}^2 - M_{\pi}^2} \right), \]
where \( F_\pi = 2M_N g_A / g_{N\pi\pi} = 1.27M_\pi \) is the pion decay constant; \( S \) is the \( \pi^+\pi^- \) invariant mass, \( Q = M_\eta - 3m_\pi \) and \( y = (T_{\pi^+\pi^-} - T_{\pi^0}) / T_{\pi^+\pi^-} \). Equation (22) gives a slope of the Dalitz plot distribution in excellent agreement with the experimental one. From the previous determination of \( \epsilon_8(1 - \chi)/\epsilon_8 \), eq. (19), one obtains:

\[
\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 75 \text{ eV}.
\]

Using the experimental branching ratio for the \( \eta \rightarrow \pi^+\pi^-\pi^0 \) decay mode, one can transform eq. (23) into a prediction for the total \( \eta \)-width. One gets:

\[
\Gamma_{\eta} \approx 330 \text{ eV}.
\]

We do not try to attach an error to eq. (24). As an indication of the possible range of variation of \( \Gamma_{\eta} \), we may mention that a more refined determination of our parameters \( \epsilon_3, \epsilon_0, \epsilon_8 \), by G. Parisi and M. Testa [19], gives a values for \( \epsilon_8(1 - \chi)/\epsilon_8 \) as high as \( 1.6 \times 10^{-2} \), which would lead to

\[
\Gamma_{\eta} \approx 600 \text{ eV}.
\]

Our prediction disagrees by a factor \( 3 \div 6 \) from the value \( \Gamma_{\eta} = (2.1 \pm 0.5) \text{ keV} \) established [20] on the basis of the measurement of the \( \eta \rightarrow \gamma\gamma \) width by the Primakoff effect [21]. We note, however, that eq. (24) is in agreement with the width obtained from \( \pi^0 \) lifetime and the use of \( SU_3 \) to relate the \( \pi^0 \rightarrow 2\gamma \) and \( \eta \rightarrow 2\gamma \) rates [22]. A possible interpretation of the deviation of the experimental \( \eta \rightarrow \gamma\gamma \) rate from the \( SU_3 \) value invokes the effect of \( \eta \)-\( X^0 \) mixing, with an abnormally large \( X^0 \rightarrow \gamma\gamma \) coupling. We have not included in our calculation the effect of \( \eta \)-\( X^0 \) mixing, but a rough order of magnitude evaluation indicates that this effect should not change our prediction by more than \( 10 \div 30\% \) (which is the general order of magnitude of \( SU_3 \) breaking). We cannot exclude that also in our case the discrepancy can be explained by some abnormally large \( SU_3 \) breaking effect connected with \( \eta \)-\( X^0 \) mixing. A critical reconsideration of the experimental determination of \( \eta \rightarrow \gamma\gamma \) might also be useful.

We consider now the problem of the isospin breaking mass differences within the octet of stable, spin \( 1/2 \) baryons.

Using our previous consideration, and in particular eq. (19), we may compute the effect of the nonelectromagnetic term \( \epsilon_8(1 - \chi)\sigma_3 \). We can then split the experimentally observed mass differences into an electromagnetic and a nonelectromagnetic part (see Table II).

A first qualitative conclusion one can draw from Table II, is that the part ascribed to electromagnetism is either very small, or has the « natural »
sign, corresponding to charged particles heavier than neutrals. The analysis of the baryon mass differences can also be carried out along the first of the procedures indicated above, eq. (17). This corresponds to the well-known tadpole analysis of Coleman, Glashow and Socolow [23]. Their results appear in the last two columns of Table II. Comparing the second and fourth columns, one can obtain the value of \( (1 - \chi) \), i.e. the ratio of the non-electromagnetic tadpole to the total tadpole contribution. The result is:

\[
1 - \chi \approx 1.2.
\]

Again, we are not able to attach errors to this determination, which however should not exceed \( \approx 20\% \). The parameter \( \chi \), which determines the strength of the e.m. tadpole is then expected to be:

\[
\chi \approx -0.2.
\]

It emerges from Table II that actually the non-e.m. terms provide the main part of the isospin breaking mass differences. This suggests the possibility that one can obtain rough estimates of other \( \Delta I = 1 \) isospin breaking effects by considering the contribution of this interaction.

As an example let us consider the problem of corrections to \( \pi^-N \) scattering lengths.

We find that the charge independent relation:

\[
a(\pi^-p \to \pi^0n) = \frac{1}{\sqrt{2}} \{a(\pi^+p) - a(\pi^-p)\}
\]

after inclusion of the \( \varepsilon_3(1 - \chi)\sigma_3 \) term is modified into:

\[
a(\pi^-p \to \pi^0n) = \frac{1 + \delta}{\sqrt{2}} \{a(\pi^+p) - a(\pi^-p)\} ,
\]
where:

\[
\delta = \frac{(M_n - M_D)_{\text{non-e.m.}}}{2m_\pi} \approx 0.7 \times 10^{-2}.
\]

This correction is about one order of magnitude smaller than the present uncertainties on the measurements of the amplitudes involved, so that a test of eq. (29) is at present impossible.

5. – We have anticipated in Sect. 3 that weak interactions may lead to corrections to hadron processes competitive with strong and e.m. effects. The study of this poses a very complicated mathematical problem, which has been solved up to now only in part and for especially simple cases.

Let us assume weak interactions to be mediated by a vector boson coupled to the \(SU_3 \otimes SU_3\) hadron currents. The simplest model consistent with our present understanding requires only one charged boson, coupled to the current given in eq. (3).

The possibility of large corrections due to weak interactions is indicated by the appearance of highly divergent integrals in the computation of these effects with perturbation theory. If one introduces a regularization procedure through a cut-off \(\Lambda\), one finds at order \(2n\) divergent terms of order \((GA)^n\), where \(G\) is the Fermi constant, followed by less divergent terms of order \(G(GA)^{n-1}\), \(G(GA)^{n-1}\log \Lambda\), etc.

A way to extract information from the perturbation theory under such circumstances has been proposed by T. D. Lee [24]. It consists in resumming the series according to the order of divergence, and then letting \(\Lambda \to \infty\). This procedure would lead to the following expression, for the regularized \(S\)-matrix:

\[
S = S^{(0)}(GA^2) + GS^{(1)}(GA^2) + \ldots,
\]

where \(S^{(0)}(GA^2)\) contains all the terms of order \((GA^2)^n\), i.e., the leading divergent terms, \(S^{(1)}(GA^2)\) contains the terms of order \(G(GA)^{n-1}\) and so on. Going to the limit \(\Lambda \to \infty\), if \(S^{(0)}, S^{(1)}, \text{etc.}\) have finite limits, one would have obtained a new expansion of \(S\) in powers of \(G\) with finite coefficients. In particular the first term \(S^{(0)}(GA^2)\) gives the \(S\)-matrix a contribution completely independent from the Fermi constant.

This program is complicated by the presence of logarithmic divergences, which would probably require a separate treatment, and by the fact that only the first term of eq. (31) is in general unambiguous. The subsequent terms are expected to depend upon the way in which the theory is regularized. We will restrict our analysis to the first term in eq. (31), which is expected to contain all the possible «strong» effects due to weak interactions, and
leave open the important problem of the other terms, which are expected to
give the true, order \( G \) or higher, weak corrections.

Let us assume strong interactions to be described by a Lagrangian:

\[
\mathcal{L}_s = \mathcal{L}_0 + \mathcal{L}_1,
\]

where \( \mathcal{L}_0 \) is invariant under \( SU_3 \otimes SU_3 \), and \( \mathcal{L}_1 \) represents the symmetry
breaking. Hadronic weak interactions are assumed to be described by the
Lagrangian:

\[
\mathcal{L}_{\text{weak}} = g(W^\mu J_\mu + \text{h.c.}),
\]

where \( J_\mu \) is the current eq. (3), and \( g \) is related to the Fermi constant by:
\( G = \sqrt{2}g^2 M_w^{-2} \). Let us define:

\[
\begin{align*}
Q^+ &= \int d^4x J_0(x) \\
Q^- &= (Q^+)^\dagger.
\end{align*}
\]

It is instructive to consider a second order calculation of weak corrections to
any strong amplitude \( A(\alpha \to \beta) \), where \( \alpha \) and \( \beta \) are hadronic states. This is
given by:

\[
\delta^{(2)}A = -\frac{g^2}{2} \int d^4q \frac{i}{q^2 - M_w^2} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{M_w^2} \right) \cdot \int \exp \left[ iq(x) \langle \beta | T(J_\mu(x)J_\nu(0)) | \alpha \rangle + \text{c.c.} \right] d^4x.
\]

Assuming the validity of Bjorken's limit, quadratically divergent terms arise
only from the \( q_\mu q_\nu \) term, and are therefore connected with the nonconservation
of the weak current, \textit{i.e.}, to the symmetry breaking term \( \mathcal{L}_1 \). Introducing
a cut-off \( \Lambda \), by standard manipulations one finds the quadratically divergent
term to be \cite{25}:

\[
\delta^{(2)}A = -i\langle \beta | \delta^{(2)} \mathcal{L}_1 | \alpha \rangle
\]

with

\[
\delta^{(2)} \mathcal{L}_1 = -\frac{GA^2}{2} \left\{ [Q^+, [Q^-, \mathcal{L}_1]] + [Q^-, [Q^+, \mathcal{L}_1]] \right\}.
\]

From eq. (36) we see that \( \delta^{(2)}A \) is equivalent to the shift in \( A \) caused by
the addition of a piece \( \delta^{(2)} \mathcal{L}_1 \) to \( \mathcal{L}_1 \), when \( \delta^{(2)} \mathcal{L}_1 \) is treated perturbatively
to lowest order. If one goes to the fourth order weak corrections, one finds
a term which corresponds to the second order perturbation in $\delta^{(2)} \mathcal{L}_1$, as well as other terms which should be identified with a further shift, $\delta^{(4)} \mathcal{L}_1$, of $\mathcal{L}_1$, treated to lowest order.

This circumstance suggests that the leading weak corrections to any strong process might be equivalent to a modification of $\mathcal{L}_1$:

\begin{equation}
\mathcal{L}_1 \rightarrow \mathcal{L}_1 + \delta^{\text{weak}} \mathcal{L}_1,
\end{equation}

where:

\begin{equation}
\delta^{\text{weak}} \mathcal{L}_1 = \delta^{(2)} \mathcal{L}_1 + \delta^{(4)} \mathcal{L}_1 + \ldots.
\end{equation}

In the general scheme of weak interactions we have described till now, this is only a conjecture as yet unproven, and we have not even a complete analysis of $\delta^{(4)} \mathcal{L}_1$. There are however models where this conjecture can be proven, and the structure of $\delta^{\text{weak}} \mathcal{L}_1$ written down in detail. Two models of this kind were studied in ref. [1]. They are the free quarks model and models allowing nontrivial strong interactions, but where weak interactions are transmitted by a neutral vector boson. The neutral boson is coupled to the current $\vec{J}_\mu$, defined by the relation:

\begin{equation}
\vec{Q} = \int \! d^3x \vec{J}_\mu(x) = \frac{1}{2} \{Q^+, Q^- \}.
\end{equation}

The second model was analyzed under the assumption that the breaking term $\mathcal{L}_1$ transforms as a $(3, \bar{3}) \oplus (\bar{3}, 3)$ (which is necessarily the case for quarks). In both cases it was found that, if $\mathcal{L}_1$ is characterized by a $3 \times 3$ matrix $h$ (see Appendix I, eq. (I.1) and (I.3)), then $\mathcal{L}_1 + \delta^{\text{weak}} \mathcal{L}_1$ corresponds to the matrix $\vec{h}$:

\begin{equation}
\vec{h} = h + \delta^{\text{weak}} h = h - \xi \{\lambda^+, \lambda^-\} h,
\end{equation}

where $\lambda^+$ is the matrix defined in eq. (6), $\lambda^-$ its Hermitian conjugate, and $\xi$ is a real parameter. This result is the same that one would obtain at second order, eq. (36), if one put $\xi = GA^2$. In the quark model $\xi$ remains as a parameter as yet undetermined, whereas in the neutral case $\xi = 1$.

An important feature of this result is the following. Suppose we start from a parity and strangeness conserving $\mathcal{L}_1$, i.e. with a diagonal and real matrix $h$. Equation (40) then implies that $h$ is also a real matrix so that, by Theorem 1 of Appendix I, $h$ is equivalent to a real and diagonal matrix $h_D$:

\begin{equation}
\vec{h} = U^h h_D V.
\end{equation}

This means that the leading weak corrections do not cause a breaking of parity and strangeness, which should only arise at order $G$. This however
is a peculiar property of the \((3, \bar{3}) \oplus (\bar{3}, 3)\) behavior of the breaking \(\mathcal{L}_1\). In fact one can easily show that if \(\mathcal{L}_1\) contains a part which transforms as \((8, 1) \oplus (1, 8)\), this result is not valid, and strong parity violations arise. Proof of this is given in Appendix III, both using the second order calculation, eq. (36), and the neutral vector boson model, treated to all orders.

We have emphasized in Sect. 1 the remarkable connection between weak currents and strong interaction symmetries. The result we have just quoted together with the indication discussed in Sect. 2 that symmetry breaking is of a \((3, \bar{3}) \oplus (\bar{3}, 3)\) kind, adds a new piece of evidence for the strict dynamical intertwining of weak and strong interactions. Coming back to the case when \(\mathcal{L}_1\) belongs to a \((3, \bar{3}) \oplus (\bar{3}, 3)\) we note two facts.

First is that the transformation (41), which reinstates parity and strangeness, changes the value of the weak interaction angle. We will call \(\theta_0\) the uncorrected angle (i.e. the angle in the frame where \(h\) is diagonal) and \(\theta\) the one in the frame of \(h_D\), i.e. the physically observed one. The angle appearing in eq. (40) through \(\lambda^+\) and \(\lambda^-\), is obviously \(\theta_0\), and \(\theta\) is a function of \(\xi, h\), and \(\theta_0\), as discussed in Sect. 5 of ref. [1].

We finally note that even if \(h\) conserves isospin symmetry, \(h_D\) will in general not do so. Weak corrections thus seem to give a natural explanation for the origin of the non-e.m. isospin breaking which is required by the phenomenological analysis of Sects. 2 and 4, and which is not easily understood on the basis of strong interactions dynamics.

6. - We have studied till now the separate effects of e.m. and weak corrections on hadron dynamics. In this Section we will collect these different results, and introduce a self-consistency condition [1] which links at a dynamical level, the different interactions and allows a determination of the angle \(\theta\) in terms of other parameters.

Starting from the equation:

\[
\mathcal{L} = \mathcal{L}_5 + \mathcal{L}_1 + \mathcal{L}_{\text{e.m.}} + \mathcal{L}_w
\]

and following the results of Sects. 3 and 5, we separate the leading weak corrections and e.m. tadpole contributions according to:

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \delta^{\text{weak}} \mathcal{L}_1 + \delta^{\text{e.m.}} \mathcal{L}_1 +
\]

\[
(\mathcal{L}_{\text{e.m.}} - \delta^{\text{e.m.}} \mathcal{L}_1) + (\mathcal{L} - \delta^{\text{weak}} \mathcal{L}_1).
\]

The last two terms correspond to nontadpole e.m. corrections and to non-leading, i.e. order \(G\), weak corrections. As explained in ref. [1], Sect. 3, \(\delta^{\text{e.m.}} \mathcal{L}_1\) depends in a peculiar way on the explicit breaking.
If we define:

\[ \delta e.m. \mathcal{L}_{1} = \frac{1}{8} \text{Tr}(M^{\dagger} \delta h^{e.m.} + \delta h^{e.m.\dagger} M) \]

the result of ref. [1] is that \( \delta h^{e.m.} \) is diagonal in the same \( SU_3 \otimes SU_3 \) frame as the explicit breaking. The strength of \( \delta h^{e.m.} \) is, on the other hand, mainly determined by the dynamical breaking of \( SU_3 \otimes SU_3 \). Since this, according to the discussion of Sect. 2, reduces \( SU_3 \otimes SU_3 \) into a nearly exact \( SU_3 \), we expect \( \delta h^{e.m.} \) to be a nearly exact \( U \)-spin singlet.

We have seen that the effect of weak corrections is to change the explicit breaking from \( \mathcal{L}_1(h) \) to \( \mathcal{L}_1(h_D) \). Then \( \delta h^{e.m.} \) will still be diagonal with \( h_D \) and will be changed only slightly by this modification, as it is expected to depend mainly upon the dynamical breaking. The combined effect of weak and tadpole e.m. interactions then changes the explicit breaking Lagrangian \( \mathcal{L}_1(h) \) appearing in eq. (42) into \( \mathcal{L}_1(h_D + \delta h^{e.m.}) \).

The self-consistency condition of ref. [1] requires \( h \) to be stable under these effects, i.e.:

\[ h_D + \delta h^{e.m.} = h. \]

Equation (45) gives a relation among the parameters \( \alpha, \beta, \gamma, \xi \), and either \( \theta \) or \( \theta_0 \). We will not report here the complete structure of eq. (45), but simply give the results, valid in the case where \( |\beta/\gamma| \ll 1 \) and \( |\xi \beta/\gamma| \ll 1 \). One obtains the relations:

\[ \theta = \sqrt{\frac{\beta}{\gamma}} \frac{1 - \xi}{\sqrt{1 - \xi/2}} \]

(47) \[ \frac{\xi \alpha - \beta}{2} = \frac{\alpha^{e.m.} - \beta^{e.m.}}{2}. \]

Equation (47) implies that only a portion \( \xi \) of the isospin breaking contained in \( h_D + \delta h^{e.m.} \) is due to pure electromagnetism. The remaining fraction \((1 - \xi)\) is nonelectromagnetic, so that \( \xi \) has to be identified with the parameter \( \chi \) introduced phenomenologically in Sect. 4. Substituting into eq. (46) the result found there, eq. (27) and the value for \( \beta/\gamma \) deduced from eqs. (12), (22) and (26), one finds a prediction for \( \theta \):

\[ \theta \approx 0.25. \]

7. An outlook.

We have presented here a review of some recent developments in the study of the dynamical interplay of weak, electromagnetic and strong interactions.
The best established consequence of this approach is the natural explanation for a new kind of isospin breaking, not uniquely electromagnetic. This new term is required for the interpretation of experimental data concerning $\eta$-decay and the mass differences of pseudoscalar mesons.

A second result which might suggest ideas of future development is the realization of the special dynamical role of a symmetry breaking of the $(3, \bar{3}) \oplus (\bar{3}, 3)$ kind, which gives a fair description of experimental facts. Among «simple» breaking schemes this is the only one which allows parity to be conserved by the leading weak corrections.

Finally, the introduction of a new hypothesis, of a self-consistency condition among weak, electromagnetic, and strong interactions gives a relation between $\theta$ and other phenomenological parameters which yields an excellent prediction for $\theta$.

Although these problems have been partly clarified, there remains a great amount of work to be done on them, in particular in the treatment of higher order corrections with a $W^\pm$ boson. The model with a $W_0$ boson, although suggestive, is far from representing the real situation.

Other serious problems relating to higher order weak corrections have not be touched. The most serious one is that of the selection rules observed in weak decays ($\Delta S < 2$, $\Delta I < \frac{3}{2}$, no neutral currents, etc.). Simple computations at second order indicate a breakdown of each of these rules at order $G \times GA^2 \approx G$. In order to agree with experiment, the theory should compensate this breakdown, perhaps at higher orders.

On the other hand the solution of the parity problem at the $GA^2$ level leads some credibility to the hope that these harder problems will also be solved.

Appendix I.

In this Appendix we give a simple algebraic proof of the Michel-Radicati result.

The possible dependence of $G$ on $\varepsilon_8$, $\varepsilon_8$, $\varepsilon_0$ is restricted by symmetry considerations. To exploit them it is convenient to give a more general definition of the symmetry breaking Lagrangian.

Starting with the densities $\sigma_t$ and $\tau_t$ defined in Sect. 2, we define a $3 \times 3$ matrix $M$:

(I.1) \[ M = \sum_{t=0}^{8} (\sigma_t + i\tau_t) \lambda_t, \]

where $\lambda_t$ are the usual Gell-Mann matrices ($\lambda_0 = \sqrt{2/3}$). Under an element
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\((U, V)\) of \(SU_3 \otimes SU_3\), \(M\) transforms as:

\[(I.2) \quad M \rightarrow U M V^\dagger \quad \text{\(U, V = \text{unitary, uniodinal} \) 3\times3 \text{ matrices}}.\]

Since \(\mathcal{L}_1\) is an element of this \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation of \(SU_3 \otimes SU_3\), it can be identified by a \(3\times3\) matrix \(h\) according to:

\[(I.3) \quad \mathcal{L}_1(h) = \frac{1}{4} \text{Tr}(h^\dagger M + M^\dagger h).\]

Under a transformation \((U, V)\), \(\mathcal{L}_1\) transforms as:

\[(I.4) \quad \mathcal{L}_1(h) \rightarrow \mathcal{L}_1(h').\]

\[(I.5) \quad h' = U^\dagger h V\]

**Definition.** Any pair of matrices \(h\) and \(h'\), obeying eq. \((I.5)\) are called equivalent: \(h \sim h'\). In fact \(\mathcal{L}_1(h)\) and \(\mathcal{L}_1(h')\) have the same physical content, because they are related by a change of basis under which \(\mathcal{L}_0\) is invariant.

**Theorem 1.** If \(\det h\) is real, \(h \sim h_D\) where:

\[(I.6) \quad h_D = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}, \quad \alpha, \beta, \gamma \text{ being real numbers}.\]

A proof of this theorem is given in ref. [1]. In this language, the bootstrap condition which determines the breaking must have the form:

\[(I.7) \quad \delta G(h) = 0.\]

Since two equivalent matrices have the same physical content, one must have:

\[(I.8) \quad G(h') = G(h) \quad \text{if} \quad h' \sim h.\]

Then \(G(h)\) is a function of the three \(SU_3 \otimes SU_3\) invariants one can construct out of \(h\). These are:

\[(I.9) \quad x_1 = T_2(h h^\dagger), \quad x_2 = \text{Tr}[(h h^\dagger)^2], \quad x_3 = \det h.\]

For diagonal matrices \(h_D\), we have:

\[(I.10) \quad x_1 = \alpha^2 + \beta^2 + \gamma^2; \quad x_2 = \alpha^4 + \beta^4 + \gamma^4; \quad x_3 = \alpha \beta \gamma.\]

In the following we restrict our study to solutions which are parity conserving, *i.e.* such that \(\det h\) is real.
Theorem 2. Up to an equivalence, all the parity conserving solutions to
eq. (I.7) are obtained as solutions of the equation:

(I.11) \[ \delta G(x_1(x, \beta, \gamma), x_2(x, \beta, \gamma), x_3(x, \beta, \gamma)) = 0, \]

the variation being done in respect to \( x, \beta, \gamma \), assumed to be real numbers. This means that we can restrict in eq. (I.7) to real diagonal matrices \( h_D \), thus recovering the variational principle given in Sect. 3.

Proof. For any \( h \) we have:

\[ h = U^+h_DV \]

with suitable unitary, unimodular matrices \( U, V \), and a diagonal \( h_D \). Moreover:

\[ h + \delta h = U'^+(h_D + \delta h_D)V' \]

where \( U' \) and \( V' \) are infinitesimally different from \( U \) and \( V \) and \( \delta h_D \) is diagonal. One then has, using eq. (I.8)

\[ \delta G = G(h + \delta h) - G(h) = G[U'^+(h_D + \delta h_D)V'] - G(U^+h_DV) = \]
\[ = G(h_D + \delta h_D) - G(h_D). \]

Although \( x, \beta \) and \( \gamma \) can vary through the whole 3-dimensional space, \( x_1, x_2, x_3 \) are restricted to a definite domain \( D \). The boundary of \( D \) is composed by continuous surfaces, joined by edges which meet at singular points. The equations for the different elements of the boundary can be obtained as follows:

Define vectors \( H \equiv (x, \beta, \gamma) \) and \( X \equiv (x_1, x_2, x_3) \). To a variation \( \delta H \) there corresponds a variation

\[ \delta_H X = \frac{\partial X}{\partial x} \delta x + \frac{\partial X}{\partial \beta} \delta \beta + \frac{\partial X}{\partial \gamma} \delta \gamma. \]

The elements of the boundary are characterized by special properties of \( \delta_H X \).

I’1. Characterization of surfaces. – Denote by \( n(x) \) the normal to the surface pointing outwards from \( D \). The variation \( \delta_H X \) must be orthogonal to \( n \) for any choice of \( \delta H \):

(I.13) \[ n \cdot \delta_H X = 0. \]

Proof. For any \( \delta H, X \) must always remain in \( D \), i.e. \( n \cdot \delta_H X \leq 0 \). Since \( \delta_H X \) is linear in \( \delta H \), inequality can only be fulfilled as an equality.

I’2. Characterization of the edges. – Denoting by \( l(x) \) the unit vector tangent to the edge, by the same argument as before one must have:

(I.14) \[ \delta_H X \text{ parallel to } l. \]
I.3. Characterization of singular points. – Arguing as before, one gets:

\( \delta_H X = 0 \).

Conditions 1, 2, and 3 are equivalent to the requirements that the Jacobian matrix:

\[
J = \frac{\partial(x_1 x_2 x_3)}{\partial(x_2 x_3)} = \begin{pmatrix}
2x & 2\beta & 2\gamma \\
4x^2 & 4\beta^2 & 4\gamma^2 \\
\beta \gamma & \gamma x & \alpha \beta
\end{pmatrix}
\]

has a rank equal to two, one, and zero, respectively. One finds that, on the boundary, one must have:

\[
\det J = 8(x^2 - \beta^2)(\beta^2 - \gamma^2)(\gamma^2 - x^2) = 0.
\]

This implies that on the boundary of \( D \), two eigenvalues of \( h_D \) are equal (the solution, \( e.g., x, -x, \gamma \) is equivalent to the solution \( x, x, \gamma \)). The surfaces then correspond to \( SU_2 \otimes U_1 \). On the edges, the Jacobian matrix should be of rank one. This implies either:

i) \( x^2 = \beta^2 = \gamma^2 \), \( i.e. \) exact \( SU_3 \). These equalities have two inequivalent realization: \((x, x, x)\) and \((-x, -x, -x)\), corresponding to the two edges:

\[
x_2 = \frac{x_1^2}{3}; \quad x_3 = \pm \frac{1}{27} x_1^4;
\]

or

ii) \( x = \beta = 0, \gamma \neq 0 \) and permutations, \( i.e. \) exact \( SU_2 \otimes SU_2 \). Thus ii) corresponds to a single edge, where one has:

\[
x_2 = x_1^2; \quad x_3 = 0.
\]

There is only one singular point, where \( J = 0 \), \( i.e. \) the origin corresponding to exact \( SU_3 \otimes SU_3 \). Solving eq. (I.17) for \( x \), and substituting into eq. (I.10), one gets a parametric representation of the boundary. In Fig. 4 we report a section \( x_1 = \text{const.} \) of \( D \). Points \( A, B, C \) are the intersection of the edges with this plane. Points \( B \) and \( C \) correspond to eq. (I.18) and \( A \) to eq. (I.19). The following theorems establish the conditions for the existence of extremal points of \( G \) on the boundary of \( D \):

**Theorem 3.** If \( X \) is a point on the surfaces composing the boundary of \( D \), and is an extremal point with respect to the values assumed by \( G \) on this surface, then \( X \) is an extremal point for \( G \) in \( D \).

**Proof.** The condition for \( X \) to be an extremal point of \( G \) in \( D \) is:

\[
\nabla G \cdot \delta_H X = 0.
\]
If $X$ belongs to a boundary surface, $\delta_H X$ is tangent to it, by eq. (I.13), and the components of $\nabla G$ along the surface are zero by hypothesis.

![Diagram](image)

Fig. 4.

Analogously one can prove the following theorems:

**Theorem 4.** If $X$ is a point on an edge of $D$, and is an extremal point with respect to the values assumed by $G$ on this edge, then $X$ is an extremal point for $G$ in $D$.

**Theorem 5.** If $X$ is a singular point on the boundary of $D$, it is an extremal point for $G$ in $D$.

In spite of these theorems, we are still not in position to prove the Michel-Radicati result, apart from the existence of an extremal point corresponding to the case of no breaking. The reason is that, since the boundary of $D$ has an infinite extension, there is no guarantee for the existence of an extremal point on it. In order to conclude the proof of the theorem, a further assumption is needed, which permits the search of an extremal point in a region of finite extension.

We assume that the bootstrap conditions are not able to fix the scale of the masses, *i.e.* the scale of $\alpha$, $\beta$, and $\gamma$. This implies that $G$ depends only upon the ratios $x_2/x_3^2$, $x_2/x_4^2$ and is therefore constant along the edges of the boundary. Each point of this edge is therefore (by Theorem 4) an extremal point of $G$ on $D$. Moreover one can limit the search of extremal points of $G$ on the cross-section $x_1 = 1$, analogous to that given in Fig. 2. If $G$ is limited, it will have at least one point of minimum and one of maximum on this (finite) domain, which may or may not coincide with the extremal points $A$, $B$ and $C$. 
Appendix II.

We give here a derivation of the $\eta \rightarrow 3\pi$ amplitude given in Sect. 4 eq. (22), using the method of nonlinear realization of chiral symmetry given by Coleman, et al. [26]. These authors have shown that the results of current algebra and PCAC can be obtained by the use of phenomenological Lagrangians in the tree graphs approximation. They have also given rules for writing down these Lagrangians, and have shown the uniqueness of such a description.

The eight pseudoscalar mesons are represented in this formalism by a $3 \times 3$ traceless matrix:

$$\pi = \sum_{i} \lambda_{i} \pi_{i} \quad (i = 1, \ldots, 8) .$$

The $SU_{3} \otimes SU_{3}$ symmetric Lagrangian for amplitudes involving pseudoscalar mesons only is:

$$\mathcal{L}_{0} = -\frac{F_{\pi}^{2}}{2} \text{Tr}(p_{\mu}p^{\mu}) ,$$

where $p_{\mu}$ is the covariant derivative of the pseudoscalar fields, defined in ref. [26]. An expansion of $p_{\mu}$ in powers of $\pi$ is:

$$p_{\mu} = -\frac{1}{F_{\pi}} \partial_{\mu} \pi - \frac{1}{6F_{\pi}^{3}} [ [ \partial_{\mu} \pi, \pi ], \pi ] - \ldots .$$

The phenomenological form of the symmetry breaking $\mathcal{L}_{1}$ is:

$$\mathcal{L}_{1} = \varepsilon_{8}(1-\chi)S_{3} + \varepsilon_{8}S_{8} + \varepsilon_{0}S_{0} ,$$

where $S_{i}$ are functions of $\pi$-fields which transform according to the $(3, \bar{3}) \oplus (\bar{3}, 3)$ representation of $SU_{3} \otimes SU_{3}$. The choice of these functions is also unique, and they can be obtained from the formal definition:

$$S + i\gamma_{5}P = \exp \left[ \frac{2i\gamma_{5}}{F_{\pi}} \pi \right] .$$

A development of the right-hand side of eq. (II.5) in power series gives:

$$S = 1 - \frac{2}{F_{\pi}^{2}} \pi^{2} + \frac{2}{3F_{\pi}^{4}} \pi^{4} - \ldots$$

$$P = \frac{2}{F_{\pi}} \pi - \frac{4}{3F_{\pi}^{3}} \pi^{3} + \ldots .$$

Thus $\mathcal{L}_{1}$ contains terms bilinear in the $\pi$-fields. The coefficients of these terms give the mass matrix, including $\gamma_{5}\pi^{0}$ mixing. The elements of the mass matrix
agree with those given in eq. (18) of Sect. 4, provided one sets: $C = 4F^{-2}_\pi$. We can diagonalize this matrix, the lowest order in $\varepsilon_3(1 - \chi)/\varepsilon_8$ introducing two new fields $\tilde{\pi}$ and $\tilde{\eta}$ which describe the physical $\eta_0$ and $\pi_0$ fields:

$$\begin{align*}
\tilde{\pi} &= \pi_3 + \frac{\varepsilon_3(1 - \chi)}{2\varepsilon_8} \eta_8, \\
\tilde{\eta} &= \eta_8 - \frac{\varepsilon_3(1 - \chi)}{2\varepsilon_8} \pi_3.
\end{align*}$$

(II.8)

The amplitude for $\tilde{\eta} \to \tilde{\pi} \pi^+ \pi^-$ can then be calculated by the terms of $\mathcal{L}_0 + \mathcal{L}_1$ quadrilinear in $\pi$. There are in principle three such contributions. Two of them come, through the action of mixing, from $\mathcal{L}_0$ and from $\varepsilon_8 S_8 + \varepsilon_0 S_0$. Actually the second contribution vanishes in this case. The last contribution comes directly from the term $\varepsilon_3(1 - \chi)S_3$. The two nonvanishing contributions are:

$$\begin{align*}
\text{(II.9a)} & \quad -\frac{2\varepsilon_3(1 - \chi)}{3 F^2_\pi \varepsilon_8} (3S - P^2 - q^2 - k^2 - p^2) \\
\text{(II.9b)} & \quad -\frac{2\varepsilon_3(1 - \chi)}{3 \varepsilon_8 F^2_\pi} (M^2_\eta - m^2_\pi),
\end{align*}$$

where $P, q, k, p$ are the momenta of $\tilde{\eta}, \pi^+, \pi^-, \tilde{\pi}$, and $s = (q + k)^2 = (P - p)^2$. The total amplitude is then:

$$\text{(II.10)} \quad T = -\frac{2\varepsilon_3(1 - \chi)}{F^2_\pi \varepsilon_8} \left[ \left( S - \frac{4}{3} m^2_\pi \right) - \frac{1}{3} (P^2 - M^2_\eta) - \frac{1}{3} (q^2 + k^2 + p^2 - 3m^2_\pi) \right]$$

which reduces on the mass shell to eq. (22) given in Sect. 4. From the well-known relation:

$$\text{(II.11)} \quad \partial^\mu A^\mu = -i[Q^I_5, \mathcal{L}_I]$$

where $Q^I_5$ are the axial charges, and through the definitions, eqs. (II.4) and (II.5), we see that the divergences of axial currents turn out to be proportional to the $P_I$'s. This means in turn that PCAC is valid only up to terms trilinear (or higher) in $\pi$-fields.

Even in presence of the $\varepsilon_3(1 - \chi)$ breaking term, one can define suitable combinations of $A_3^\mu$ and $A_8^\mu$, whose divergences are proportional to the $\tilde{\pi}$ or $\tilde{\eta}$ fields, eq. (II.8), up to terms of higher order in pseudoscalar fields. In fact one has:

$$\begin{align*}
A^\mu_\pi &= A^\mu_3 + \frac{\varepsilon_3}{2\varepsilon_8} A^\mu_8 \\
A^\mu_\eta &= A^\mu_8 - \frac{\varepsilon_3}{2\varepsilon_8} A^\mu_3.
\end{align*}$$

(II.12)
and

\[
\begin{align*}
\partial_\mu A^\pi_\mu &= \frac{F_\pi m_\pi^2}{2} \tilde{\pi} + \ldots \\
\partial_\mu A^\eta_\mu &= \frac{F_\pi M^2_\eta}{2} \tilde{\eta} + \ldots.
\end{align*}
\]

(II.13)

This means that in processes involving less than three pseudoscalar mesons, and in the three graphs approximation, the \(\pi\)-fields can be treated as divergences of suitable currents. In particular they will satisfy the Adler consistency condition, and moreover the effect of the \(S_\pi\)-breaking reduces in this case to the effect of current mixing, eq. (II.12).

For processes with many pseudoscalar mesons the additional terms, present in axial current divergences in this scheme, cause a failure of the Adler consistency condition. Choosing a nonlinear realization different from the "standard one" given in ref. [26], one could restore the Adler zero's for pions. This however does not affect on the mass shell amplitudes since, as shown in ref. [26], these amplitudes are independent from the particular representation chosen.

We consider now \(\pi^-N\) scattering in the soft pion limit. One can treat this according to the method of Weinberg, using as interpolating fields the axial divergences:

\[
\frac{\sqrt{2}}{m_\pi^2 F_\pi} \partial_\mu A^\pm_\mu
\]

for \(\pi^+\) and \(\pi^-\), and eq. (II.8) for \(\pi^0\). One then easily finds that there is no modification to the Weinberg formula for \(A(\pi^+ p) - A(\pi^- p)\), whereas the charge exchange amplitude is given at threshold by:

\[
A_{\text{ch.ex.}} = \frac{A^+ - A^-}{\sqrt{2}} \left(1 - \frac{(M_p - M_n)_{\text{non-e.m.}}}{2m_\pi}\right),
\]

(II.14)

where \((M_p - M_n)_{\text{non-e.m.}}\) indicates the non-e.m. proton-neutron mass difference, as given in Table II, of Sect. 4.

Appendix III.

In this Appendix we consider the leading weak corrections to hadron processes in a fictitious model where nonleptonic weak interactions are ascribed to the coupling of a neutral vector boson \(W\) only. We assume strong interactions to be described by the Lagrangian

\[
\mathcal{L}_s = \mathcal{L}_0(\psi^i, \partial_\mu \psi^i) + \mathcal{L}_1(\psi^i),
\]

(III.1)

where \(\mathcal{L}_0\) is invariant under \(SU_3 \otimes SU_3\) and \(\mathcal{L}_1\), the breaking term, is assumed not to contain derivatives. \(\psi^i\) represent a set of hadron fields, having
definite transformation properties under $SU_3 \otimes SU_3$. If we call $F^a$ the sixteen
generators of $SU_3 \otimes SU_3$, we have:

\begin{equation}
[F^a, \psi^i] = i T^a_{ij} \psi^j,
\end{equation}

where the matrices $T^a_{ij}$ constitute a representation of the generators $F^a$.

Weak interactions with the neutral vector boson are assumed to arise from the minimal substitution:

\begin{equation}
\partial_\mu \psi^i \rightarrow \partial_\mu \psi^i + ig W_\mu T_{ij} \psi^j,
\end{equation}

where $T$ is the representative of the current to which $W$ is coupled. To connect this model to the realistic one discussed in Sect. 5, where weak interactions are mediated by charged vector bosons, we choose $T$ to correspond to the third component of weak isospin, i.e. $T$ corresponds to the charge $Q^3$ defined as:

\begin{equation}
Q^3 = \frac{1}{2} \{ Q^+, Q^- \}
\end{equation}

with $Q^\pm$ defined as in eq. (33) of Sect. 5. In the quark language:

\begin{equation}
Q^3 = \psi^* \lambda^3 (1 + \gamma_5) \psi
\end{equation}

\begin{equation}
\lambda^3 = \frac{1}{2} \{ \lambda^+, \lambda^- \} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos^2 \theta_0 & -\sin \theta_0 \cos \theta_0 \\ 0 & -\sin \theta_0 \cos \theta_0 & -\sin^2 \theta_0 \end{pmatrix},
\end{equation}

$\theta_0$ is the weak interaction angle in the $SU_3 \otimes SU_3$ basis in which $\mathcal{L}_1$ is parity and strangeness preserving. We also introduce minimal electromagnetic coupling, and write:

\begin{equation}
\mathcal{L} = \mathcal{L}_0(\psi^i, D_\mu \psi^i) + \mathcal{L}_1(\psi^i) + \mathcal{L}_{ph} + \mathcal{L}_W,
\end{equation}

$\mathcal{L}_{ph}$ and $\mathcal{L}_W$ are the free photon and W-meson Lagrangians, and we have defined $D_\mu \psi^i$ to be:

\begin{equation}
D_\mu \psi^i = \partial_\mu \psi^i + ig W_\mu T_{ij} \psi^j + ie A_\mu Q_{ij} \psi^j,
\end{equation}

$Q_{ij}$ being the matrix representing the electric charge.

To isolate the leading weak corrections, it is convenient to make use of the Stückelberg formalism [27], following the work of ref. [24]. This consists in decomposing $W_\mu$ into two terms:

\begin{equation}
W_\mu = \tilde{W}_\mu + \frac{1}{M_W} \partial_\mu \theta.
\end{equation}
With a suitable modification of $\mathcal{L}_w$, $\tilde{W}_\mu$ has the propagator:

\[(II.9)\quad \frac{-g_{\mu\nu}}{q^2 - M_w^2},\]

whereas the field $\theta$ is given the propagator:

\[(III.10)\quad \frac{1}{q^2 - M_\theta^2}.\]

The coupling of $\tilde{W}_\mu$ gives rise by itself to a renormalizable theory, provided the $\psi^i$ have spin $\frac{1}{2}$ or 0, similar to electrodynamics. All leading divergences, coming from the $q_\mu q_\nu$ term in the $W_\mu$ propagator, are now associated with a derivative $\theta$ coupling. We perform now the canonical transformation:

\[(III.11)\quad \psi^i = \left\{ \exp \left[ -i g \frac{\theta(x)}{M_w} \right] \right\}_{ij} \tilde{\psi}^j = U_{ij}(x) \tilde{\psi}^j.\]

One easily finds:

\[(III.12)\quad D_\mu \psi^i = U_{ij} \tilde{D}_\mu \tilde{\psi}^i,\]

where

\[(III.13)\quad \tilde{D}_\mu \tilde{\psi}^i = \partial_\mu \psi^i + i [g \tilde{W}_\mu T_{ij} + e A_\mu Q_{ij}] \tilde{\psi}^j,\]

$\mathcal{L}_0$ being invariant under chiral transformations, we have:

\[(III.14)\quad \mathcal{L}_0(U\tilde{\psi}, U\tilde{D}_\mu \tilde{\psi}) = \mathcal{L}_0(\tilde{\psi}, \tilde{D}_\mu \tilde{\psi}).\]

We have thus eliminated from $\mathcal{L}_0$ the dangerous field $\theta$. On the other hand $\mathcal{L}_1$ is not chiral invariant, so that:

\[(III.15)\quad \mathcal{L}_1(U(x)\tilde{\psi}) = \exp \left[ -i g \frac{Q^2(x)}{M_w} \right] \mathcal{L}_1(\tilde{\psi}) \exp \left[ i g \frac{Q^2(x)}{M_w} \right].\]

As explained in ref. [24], the leading weak corrections to hadron processes are obtained considering all the graphs where a $\theta$-line starts and ends at the same vertex. This amounts to calculate the expectation value of $\mathcal{L}_1(U(x)\tilde{\psi})$ in the vacuum state for $\theta$-mesons, thus defining:

\[(III.16)\quad \mathcal{L}_1 + \delta_{\text{weak}} \mathcal{L}_1 = \langle \mathcal{L}_1(U(x)\tilde{\psi}) \rangle_0.\]

To go further, we must specify the transformation properties of $\mathcal{L}_1$. The case where $\mathcal{L}_1$ transforms as a $(3, \overline{3}) \oplus (\overline{3}, 3)$ has been discussed in detail in ref. [1], and yields the result quoted in Sect. 5, eq. (40). With analogous techniques, one could treat the other possible forms for $\mathcal{L}_1$. 

Weak interactions and the breaking of hadron symmetries
We are here interested in the problem whether eq. (III.15) can give rise to a new breaking $\mathcal{L}_1 + \delta^{\text{weak}} \mathcal{L}_1$ equivalent to a parity conserving one, as was the case for the $(\bar{3}, \bar{3} \oplus (\bar{3}, 3))$.

Let us consider the case $(1, 8) \oplus (8, 1)$. A basis for such a representation is given by a set of eight scalar and eight pseudoscalar densities $d_l(x)$ and $d_l^5(x)$. We collect these densities into two, $3 \times 3$ matrices, according to:

$$D = \sum_{l=1}^{8} d_l(x) \lambda^l$$

$$D^5 = \sum_{l=1}^{8} d_l^5(x) \lambda^l.$$

The combinations:

\[(\text{III.17})\]

$$D^\pm = D \pm D^5$$

transform as the $(1, 8)$ and the $(8, 1)$ representations, respectively. A parity conserving $\mathcal{L}_1$, would have the form:

\[(\text{III.18})\]

$$\mathcal{L}_1 = 2 \text{Tr}(DH) = \text{Tr}(D^+H + D^-H),$$

$H$ being a numerical, $3 \times 3$ traceless Hermitian matrix. Because of the $1 + \gamma_5$ structure of the current, only the $D^+$ component of $\mathcal{L}_1$ is modified by weak corrections, and eq. (III.15) reduces to:

\[(\text{III.19})\]

$$\mathcal{L}_1(U)(x)D = \text{Tr} \left( D^+ \exp\left[ i \frac{g}{M_W} \lambda^3 \theta(x) \right] H \exp\left[ -i \frac{g}{M_W} \lambda^3 \theta(x) \right] + D^- H \right).$$

This means that the effect of the $\theta$-field is to impart the left-handed part of $\mathcal{L}_1$ an $SU_3$ transformation depending upon the quantized density $\theta(x)$. According to eq. (III.16), the modified breaking operator is to be obtained by averaging eq. (III.19) over the value of $\theta(x)$ in the state with no $\theta$ particles. One then finds:

\[(\text{III.20})\]

$$\mathcal{L}_1 + \delta^{\text{weak}} \mathcal{L}_1 = \text{Tr}(D^+H' + D^-H).$$

In order for parity to be conserved, it should be possible to reduce $H'$ to $H$ by an $SU_3$ transformation. A necessary condition for this is:

\[(\text{III.21})\]

$$\text{Tr}(H'H'^\dagger) = \text{Tr}(HH^\dagger).$$

However, one easily sees that:

\[(\text{III.22})\]

$$\text{Tr} \left( \langle U(x)HU^\dagger(x) \rangle_0 \langle U(x)H^\dagger U^\dagger(x) \rangle_0 \right) \leq \text{Tr} \langle UHU^\dagger UH^\dagger U \rangle_0 = \text{Tr}(HH^\dagger)$$

equality being attained if and only if

\[(\text{III.23})\]

$$[\lambda^3, H] = 0.$$
If $\lambda^3$ is given by eq. (III.5), eq. (III.23) can be satisfied either if $\theta_0 = 0$ (and $\theta = 0$), or if $H$ is proportional to the electric charge matrix $Q$ (where $H' = H$).

The analysis we have formally carried out in the neutral boson model can easily be applied to the second order computation eq. (36) of Sect. 5. One obtains again the result eq. (III.20) where:

$$H' = H - GA^2\{[\lambda^+, [\lambda^-, H]] + [\lambda^-, [\lambda^+, H]]\}.$$

Up to terms of order $(GA^2)^2$, one has:

$$(III.24) \quad \text{Tr}(H'H'^\dagger) - \text{Tr}(HH^\dagger) =$$

$$=-GA^2\{\text{Tr}[H'( [\lambda^+, [\lambda^-, H]] + [\lambda^-, [\lambda^+, H]])] + \text{h.c.}\} =$$

$$=-2GA^2\{\text{Tr}((H^\dagger, \lambda^+) [\lambda^-, H]) + \text{Tr}((H^\dagger, \lambda^-) [\lambda^+, H])\} =$$

$$=-2GA^2\{\text{Tr}([\lambda^-, H])^\dagger [\lambda^-, H] + \text{Tr}([\lambda^+, H])^\dagger [\lambda^-, H]<0.$$

We note that if $\theta_0 \neq 0$, the condition $[\lambda^\pm, H] = 0$ is necessary to have an equality in eq. (III.24), cannot be realized by a diagonal traceless matrix. This proves that if $\mathcal{L}_1$ has a component along $(8,1) \oplus (1,8)$, parity is violated at the order $GA^2$, contrary to the case of $(3,3) \oplus (3,3)$.

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The Role of Complexity in Nature.

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1. — A common aim of people interested in science is that of improving the comprehension of the phenomena that can be observed in the world. It happens that the variety of the panorama that falls under our senses is so great that the present degree of understanding is much diversified.

In attempting a classification, we tend to divide the various branches of science into two groups: the «exact» and the «nonexact», a subdivision that reflects the degree by which the language of mathematical analysis is used in the interpretation. Typical of the exact sciences are astronomy, physics, etc., while the nonexact are to a great extent the biological sciences, the more so the more they deal with the behaviour of living organisms.

This separation, however, is not merely methodological. It also reflects a more fundamental difference in the nature of the phenomena studied by the disciplines that are grouped under the two labels. The exact sciences have to deal with phenomena that, at our state of understanding, depend on a number of characteristic variables sufficiently small to make mathematical methods of analysis successful in predicting new results. The facts observed in nonexact sciences depend instead on such a complex interplay of causes, that mathematics is often of limited help.

Thus in this context, one can say that «exact» and «nonexact» are synonyms of «simple» and «complex», when these adjectives are used with the meaning specified above.

In classifying some scientific activities as exact (or simple), we also yeld, often unconsciously, to a sort of hope. The hope that many, possibly all, natural phenomena will eventually be mastered using mathematical tools. This hope runs all along the history of science, from the Greek philosophers who believed that the Gods were thinking in terms of arithmetic and geometry, to Newton and Einstein who tried to unify physics, astronomy, and cosmology under common laws, and to present-day scientists who use quantum theories to interpret all physical and chemical phenomena that fall under our experience.
In physics, we still hope that relativistic quantum mechanics and electrodynamics will be able to give a complete description of matter, from solid state to small and big molecules, from atoms to nuclei, from nucleons to subnuclear particles, i.e. hadrons, leptons, and their combinations made manifest by the study of the most energetic collisions produced either artificially or by means of cosmic rays. We still hope that, once a certain degree of understanding is reached in these fields, also the problems one meets in other fields, even in the nonexact sciences, will slowly fall under the powerful methods of analysis developed for the more exact disciplines. It is easy to find in the history of science several good examples of this trend.

However, there are also cases where disciplines believed to belong to the «simple» class were later found to be «complex». For the ancients, the animal and the vegetable worlds on Earth appeared as the products of a single act of creation, while now we know how complex the path of evolution must have been. The situation is similar for geology and astronomy, even through many still hope that a simple «big bang» is at the origin of the whole so-called Universe. These are several branches of science whose first principles we now understand much less than our predecessors thought they did.

The reason why I am talking about these matters here is that at the present moment several of the scientists involved in what is considered one of the most fundamental fields of research, the field of high-energy physics are beginning to ask themselves whether the complexities that become more and more manifest as the properties of matter are analyzed with higher spatial and temporal resolution could lead this branch of fundamental physics to a point where it becomes impossible to pretend that this discipline can be classified as exact.

Before further elaborating this point, I will try to qualify the meaning of the term «nonexact» attributed to a branch of science. A concise definition is the following: Complex disciplines are those obeying laws that evolve historically.

One must be aware of the fact that what makes our definition of complexity defy mathematical analysis is the word «historical». This is because history contains the idea of the «arrow of time», a concept absent in all fundamental laws of physics that can be expressed mathematically.

The classical example of historical evolution is that postulated by Darwin in order to understand the origin of life on Earth. We now know that all history of living organisms progressed via a string of processes, each a consequence, a chance consequence, of the previous one, and that a modification introduced in a step by a physical or chemical reaction had consequences that depended on the preceding history. In biological evolutions there is no means of going backwards to recreate the conditions that were present
at an earlier stage. This is the reason why the laws that govern life on Earth today are different from those that governed it yesterday and also from those that will govern it tomorrow.

It is illuminating to realize that in order to give an «exact» reason for the concept of the unique direction of time it is necessary to bring into the picture an evolving system. In physics, there is no way of understanding the existence of the arrow of time if one remains within the framework of the fundamental laws of quantum mechanics, which are time symmetric.

As pointed out by Gold, time asymmetry is created by the fact that the space around us acts, for the energy radiated into it, as a cold sink. Energy emanated from any reaction is lost into space, never to be returned. Time points in the direction corresponding to energy dump, a direction that is determined by the way in which the world around us evolved and is evolving.

In our context, the most important peculiarity of evolution is that, in the complex situations where it works, it has the possibility of creating at each instant of its development the laws that govern it.

For example, the way in which living matter organized itself in the early reducing atmosphere present on the Earth, was very different from the way in which it evolved later when the atmosphere became oxidizing because of the existence of life.

It is this possibility of self-regeneration, coupled with the chance that enters in each step, that makes historical evolution so powerful in creating new possibilities, actually new laws.

Now, the question is the following: can this kind of insight that we have gained from the nonexact disciplines be of help in developing further understanding of the «exact and simple» world of physics?

What is the consequence of evolution, of the arrow of time, in establishing the fundamental laws, the exact and simple laws of quantum physics? Those who think that this way of arguing is sacrilegious may indeed be right; however, they should be reminded that, when the arrow of time was not recognized to be operating in the biological sciences, the living world was considered static, dominated by fixed laws, apart from an act of creation that defied definition. And it made sense.

In physics now we are in about the same situation; we have some fundamental immutable laws to which all mechanical facts obey—conservation of energy, conservation of charge, constancy of velocity of light, etc., etc., and far back in time, ten billion years ago, some big event that put all this world into being. We accept the existence of an arrow of time, but we relate it to an inevitable emptiness of the space around us. Could it not be that this static conception is wrong, as was that of the biologists before Darwin?

In the following pages I will try to show that such a possibility exists, and
that complexity, far from bringing confusion, could in fact open vistas that our present culture has some tendency to dismiss.

2. – Having stated our aims, let us look at the causes of complexity and try to establish a correlation between complexity and the property of the simplest phenomena of which complexity is built. It is found that outstanding correlation exists between complexity and the sign of the energy balance of the elementary reactions. Complexity invariably arises when the simplest reactions become endothermic, *i.e.* when energy has to be supplied to the evolving medium. The worlds of molecules, of atoms, of nuclei, as well as that of planets, stars and galaxies, are «simple» worlds only when exothermic reactions take place in them.

Consider the case of nuclei. The building of heavier nuclei starting from lighter ones is exothermic only up to iron, then becomes endothermic, if the Coulomb repulsion is not circumvented. As a consequence, the formation of nuclei up to iron is easily achieved in the laboratory via collisions of two bodies and in nature through the slow burning of light nuclei in stars, as in the sun. However, the formation of heavier nuclei, like lead, uranium and beyond, asks for a lot of ingenuity on the part of nature. We now believe that it can be achieved through the very intense neutron bombardment (to overcome Coulomb repulsion) that can take place in rapidly evolving matter, but it is not yet clear whether these conditions can be found in a collapsing supernova or in the big bang (again!) that presumably took place ten billion years ago. In the laboratory, the heaviest nuclei have been obtained, so far, only via the bombardment of already existing heavy nuclei; never by bringing light nuclei together.

Also the astronomical world is simple, beautifully simple, only when bodies at great distances from one another, placed there by a still unknown mechanism, slowly dissipate their gravitational energy to form planetary systems, streams of stars rotating into a galaxy, or nebular matter on the way to condensing into proto stars. The endothermic process that brings all this into being is certainly not simple, be it the initial big bang or the continuous creation of matter of the steady state.

An even better story is presented by molecules, whose behaviour we are able to follow without interruption, from the simplest exothermic two-body reactions of inorganic chemistry, to the great organic constructions kept alive by the continuous flux of energy that, on Earth, comes from the Sun.

These examples, besides illustrating how complexity needs an energy source, also show that, for building elaborate structures, it is necessary to have a medium where many-body reactions can take place within times
short in comparison with the disintegration lifetimes. On Earth, the molecules had the opportunity of arranging themselves into the long chains that eventually led to DNA, in the warm broth imagined by Oparin, where the products of the reactions occurring in the atmosphere and on the surface could meet and react; a process that is going on even today, within the living organisms, and is made manifest by mutations.

These considerations are of importance for high-energy physics because the most characteristic property of elementary particles is that they are all produced via endothermic reactions. In classical nuclear physics the energetic particles produced by accelerators were used either to make stable structures or to overcome Coulomb repulsive barriers in order to add some nucleons to already existing nuclei, all exothermic processes. Today in high-energy physics the accelerators are used in a different way, namely to supply to the known stable particles the energy necessary to build more complex, heavier states, which require energy to be created. The so-called elementary particles obtained in this way, mesons and baryons, are thus the product of endothermic reactions, are heavier than the sum of the masses of their constituents and, consequently, have in general very short disintegration lifetimes, down to $10^{-22}$ s. Though the number of these particles is already quite large—more than one hundred—and their masses are as high as about three proton masses, we do not have any indication that particles of much higher mass cannot exist, albeit for short times. The present limitation in the number of particles reflects only the present maximum energy of the accelerators. High-energy physicists are convinced that as soon as larger accelerators are built, particles of larger masses will be discovered, having disintegration lifetimes not shorter than those already known today.

In these circumstances, it becomes legitimate to ask whether situations could occur capable of bringing to the most fundamental branch of physics developments resembling those so far observed only in nonexact sciences, the developments produced by complexity.

Thus, there are similarities between the world of elementary particles and that of molecules; both depend on the occurrence of endothermic processes and for both there is the possibility of building complex structures using simpler building blocks. How far one can build with elementary particles is not known, but so far no limits have been felt, either. Actually, as long as we create particles as we do now, one by one, by bringing together two simpler particles and hoping that by hitting each other at great speed they will somehow coalesce, there is a drastic limit to the complexities that can be built. In the case of molecules, the progress would have been very limited indeed if chemists could produce compounds only by shooting together a pair of atoms or simple molecules. The building of
more elaborate structures requires the myriad of collisions taking place in a gas, in a solution, and, very important, the presence of catalysts that can keep alive intermediate steps. One needed the primitive oceans and billions of years in order to have the chance of creating the molecules of life. Of course, one also needed the practically inexhaustible possibilities of the carbon bonds, but can we say now that equally far reaching possibilities do not exist in structures composed of elementary particles?

This doubt makes it legitimate to speculate in which circumstances the complexities of elementary particles, if virtually possible, could become manifest.

In the world as we know it today, it appears that the spontaneous evolution of matter we are talking about can be induced only by gravity. Gravity is the only known force that does not saturate, \textit{i.e.} does not cease to operate when many field sources are brought together, contrary to what happens for electromagnetic forces, weak coupling forces and molecular forces. The pull of gravity steadily increases as more and more matter is brought together. Just as nuclear reactions start when enough matter is condensed in a star, it is conceivable that the exothermic reactions of hadronic and leptonic matter will fully develop their possibilities only when large enough masses will compress matter to the point where the characteristic energy per particle is well above the giga electronvolt. In matter consisting of baryons compressed to degeneracy by gravity, the characteristic momentum per particle is $p c \approx 10^{3.8} \text{geV}$, where $\rho$ is the density in grams per cubic centimeter. Already at the density of neutron stars ($\rho \approx 10^{15} \text{g cm}^{-3}$) momenta up to $\sim 1 \text{ GeV}$ are available; these are also the densities of matter in nuclei. The places where these densities can be exceeded are those where matter condenses even further: in the collapsing neutron stars and possibly in galactic centres. It is not surprising by chance that at present we have no way of telling what the behaviour of matter is in these bodies.

In the laboratory, present knowledge about the interactions of nuclei with giga electronvolt particles derives exclusively from the study of two-body interactions and it can be easily predicted that much more complex situations, if they exist, will not be observed as long as we continue to use this simple channel.

A practical way of partially circumventing this limitation, short of creating the concentration of energy possible for astronomical bodies, is perhaps to use the collisions between particles and nuclei, and between nuclei and nuclei at effective energies (\textit{i.e.} centre-of-mass energies between composite structures) of several giga electronvolts. Until now, particle-nuclei interactions have been studied at high energy only to gather information about the so-called coherent or diffraction dissociation reactions, where the nucleus acts as a single particle. The complex reactions we are looking for should be observed in the incoherent background, where the complex structures built by the collision with
The role of complexity in nature

the first nucleon have the chance of interacting further before leaving the nucleus.

With the present accelerators, the energies available are not yet sufficiently high. Only with the new generation of 100 to 500 GeV accelerators will one obtain second-order collisions within the same nucleus with total c.m. energies of many giga electronvolts. The best conditions in this respect will undoubtedly be obtained with colliding beams of nuclei.

Large Lorentz factors for the complex structures built in these collisions are necessary not only to make available the energy needed for carrying out the endothermic reactions, but also for bringing together these unstable structures before they decay. Even if widths of the order of a few 100 MeV continue to be characteristic, mean decay paths equal to nuclear dimensions demand Lorentz factors larger than ten.

3. – It is appealing to think that in the realm of high energies, situations could develop similar to those possible for molecules, and that subtle and apparently insignificant details of some interactions could have unimaginable and radical consequences in the historical evolution of matter.

Clearly, the only justification today for this kind of science fiction is the observation of a great variety of endothermal high-energy reactions, a phenomenon that has some parallel with the molecular case.

It should be realized that if such a possibility really exists, our conceptions of the physical world would be greatly affected.

The immutable laws of physics could become as "ephemeral" as those of organic life, immutable only for observations limited in space and in time, and even more exotic, the evolution of these laws would depend on history, a history that has followed a path that, to a great extent, must have been determined by chance. Seen from this point of view, even the Heisenberg uncertainty principle could be considered a temporary consequence of laws establishing themselves in an undeterminable manner.

Another kind of life, the life of the physical world, would then be developing around us, in parallel with that we are accustomed to call the real life, that on Earth, of the organic world.

Is the history of organic life a subset of the more general history of the physical world? Has the evolution of organic life the possibility of interfering with the evolution of the physical laws—and vice versa?

I realize that I have carried the argument to its extreme consequences and beyond, into the metaphysical sphere. But to me, it seemed useful to emphasize how complexity can have peculiar ways of manifesting itself through the possibility of developing momentous consequences from details apparently of no importance.

The high-energy phenomena, as we know them now, seem to have properties that could give rise to complexities of this kind.
Channeling of Ultrarelativistic Charged Particles in Crystals.

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When very high energy charged particles (usually electrons) or gamma rays pass through crystals, the coherent interaction with the lattice gives rise to several interesting phenomena, which have been studied both theoretically and experimentally in recent years [1].

There is an aspect of these phenomena, which until now has not been investigated experimentally, because it becomes important at energies higher than those actually at hand, and which, as far as I know, has not received either much theoretical attention, with the exception of a stimulating paper by Schiff [2]. We have in mind the fact that, in appropriate circumstances, the interaction between lattice and charged particles becomes so strong that the Born approximation, usually quite valid in the electrodynamic case at very high energies, has to break down completely [3]. This breakdown can be attributed to two circumstances: First, the fact that a description of the electrons wave functions by means of plane waves becomes very inadequate; and, second, the fact that the coherent emission of many quanta, or even a «coherent cascade» may become important.

The resolution of the problem of finding a more adequate solution for the motion of very high energy electrons in crystals is preliminary dependent upon the investigation of the «coherent cascade». This short note contains a few results obtained in trying to solve this preliminary problem.

What we have in mind to discuss particularly is the possibility of «channeling» of very high energy electrons, or, better, positrons, around certain directions in a crystal.

We shall consider, for simplicity, a cubic lattice, having the principal crystallographic axes along \( x, y, z \) and a particle traveling around a direction making a very small angle with the \( x \) axis. More precisely, if \( p_x, p_y, p_z \) are the component of the momentum of the incoming (free) particle, we consider...
the case in which

\[ p_x \gg p_y \gg p_z \simeq 0. \]

Then, it might be shown [4] that the wave function of the electron inside the crystal can be suitably approximated by:

\[ \Phi = A \exp \left[ ipx \right] \sqrt{\frac{2E'}{E' - \overline{V}(yz)}} \exp \left[ i \sqrt{2p} \int_0^y \sqrt{E' - \overline{V}(yz')} \, dz' \right] \chi_a(z), \]

where

\[ E' = E - p - \frac{m^2}{2p} \quad c = 1; \quad \hbar = 1, \]

\( E \) is the energy of the particle, \( p \) is the component of the Bloch momentum along \( x \) (\( p \simeq p_x \)), \( m \) is the mass of the particle, \( \overline{V}(y, z) \) is the average of the potential of the crystal, taken along the classical path in a cell, and the function \( \chi_a(z) \) satisfies the equation

\[ \chi''_a(z) + 2p(E_a - \overline{V}(z))\chi_a = 0, \]

where \( \overline{V}(z) \) is the average of \( V(x, y, z) \) in the \( xy \) plane. Inside the crystal, in place of eq. (1) we shall have

\[ p \gg \sqrt{2pE'} \gg \sqrt{2pE_a}. \]

If \( a \) is the pitch of the lattice, \( \overline{V}(z) \) might be quite well represented by

\[ \overline{V}(z) = V_0(\exp [-z/s_0] + \exp [-(a-z)/s_0]) \quad 0 < z < a \]

where

\[ s_0 = a_0/Z, \]

\( a_0 \) is the atomic Bohr radius and \( Z \) is the atomic number of the element which is supposed to form the crystal.

In the case of the positron (which is the most interesting for the reason that we shall see) the channeling is important when

\[ E_a \ll V_0. \]

In this case we see immediately, considering eqs. (2) and (3), that the positron can be trapped between two nets of equilibrium positions of the nuclei of the crystal. The planes \( \pi_1 \) and \( \pi_2 \) of these nets are obviously orthogonal
to \( z \). In this case it is quite clear that the particle might be channeled in a small neighbourhood of directions belonging to these planes. This channelling however can be disturbed by several effects, as the interaction with the electrons of the crystal, the effect of the zero point, and thermal motion of the atom, and so on.

If \( E_a \) is not much less than \( V_0 \), the most disturbing effect will be, in general, the interaction with the zero point and thermal motion. It is this effect which we have particularly tried to estimate.

To arrive at expression (2) for the wave function of the positron one has essentially to use the fact that the potential of the lattice is periodic, and that, for the particular solution which we are considering, the frequencies of the components of the forces along \( x \) and \( y \) are very different. In this way, for instance, the forces along \( y \) can be considered as an adiabatic perturbation for the motion along \( x \).

These circumstances allow us to separate approximately the variables in the solution of the equation of motion, and to arrive at expressions (2) and (3).

If, however, one takes into account the zero point motion, the perturbation due to the displacement of the lattice atoms will not be almost periodic, and consequently the separation of the variables will be no longer possible.

A preliminary estimate, however, shows that if condition (5) is satisfied, the effect of the atom motion is very small, in such a way that it can be considered as produced by a small perturbation \( \Delta V \). This perturbation \( \Delta V \) will be a functional of the operators \( u_1(x_n, y_m, u_n, y_m) \) which describe the displacement of the two nets \( \tau_1, \tau_2 \). \( (x_n \text{ and } y_m \text{ are the co-ordinates of the equilibrium positions of the atoms belonging to } \tau_1) \) (*)

\[
\mathbf{u}_1 = \frac{1}{\sqrt{2NM}} \sum \frac{p_{qj}}{\omega_q} \left( \exp[iqx] a_{qj} + \exp[-iqx] a^*_{qj} \right),
\]

where \( M \) is the atom mass and \( N \) is the number of atoms for unit volume. Only the atoms, however, which are nearest to the classical trajectory can contribute appreciably to \( \Delta V \). For a given classical trajectory and for any \( x_n \) there will be in general one (or in exceptional cases, that we shall disregard, at most two) of such atoms belonging to \( \tau_1 \); and similarly, for any \( x_n \), one belonging to \( \tau_2 \). The «effective» displacement operators \( u_1 \) and \( u_2 \) therefore, might be considered as functions of a parameter only. Furthermore, the

(*) \( a_{qj} \) and \( a^*_{qj} \) in the \( u_1 \) expression are the usual annihilation and creation operators
correlations of the orthogonal component of the displacement will not be important for determining $\Delta V$ when eq. (5) is valid.

For these reasons we might use as well a two-dimensional, instead of a three-dimensional lattice, for studying the effect of the random motion of the atoms on the channeling. The wave equation then will be

$$
(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial z^2})\psi(x,z) + \left[ E^2 - m^2 - 2E(\overline{V}(z) + \Delta V(x,z)) \right]\psi(x,z) = 0 ,
$$

where

$$
\Delta V = V_0 \left( \exp \left[ -\frac{z}{s_0} \right] \frac{u_1(x)}{s_0} - \exp \left[ -\frac{a-z}{s_0} \right] \frac{u_2(x)}{s_0} \right) .
$$

$\Delta V$ is obtained by expanding the potential $V$ in powers of the displacements $u_1$ and $u_2$.

The solution will be written in the form

$$
\psi = \psi_a + \Delta \psi ,
$$

where

$$
\psi_a = \exp \left[ ip_a x \right] \chi_a(z) ,
$$

$$
\Delta \psi = \sum c_b(x) \psi_b ,
$$

$$
E = p_a + \Delta E_a = p_b + \Delta E_b ,
$$

$L$ being a normalization length.

We can then solve our problem in the Born approximation with respect to the perturbation $\Delta V$, in order to find the probability of transition for unit path, from any state $\psi_a$ to any state $\psi_b$. This probability of transition is given in the usual form

$$
P_{ab}(u_1 , u_2) = 2\pi \left| \left( \psi_a | \Delta V \psi_b \right) \right|^2 \rho ,
$$

where $\rho = L/2\pi$ is the density per unit energy of the states $\exp [ipx]/\sqrt{L}$.

We are of course interested in the average value (with respect to the motion of the atom) of this probability transition

$$
P_{ab} = \langle \left( \psi_a | \Delta V \psi_b \right) \rangle L .
$$

The evaluation of $\langle \left( \psi_a | \Delta V \psi_b \right) \rangle$ can immediately be performed, and one
obtains:

$$
\langle \psi_a | \Delta V \psi_b \rangle |^2 = \frac{2V_0^2}{L^2 s_0^2} \int_0^a \chi_a^* \exp \left[ -\frac{z}{s_0} \right] \chi_b \exp \left[ \frac{z}{s_0} \right] \, dz \cdot
\frac{1}{2NM} \sum_{\omega_q} \left\{ \sin \left( \frac{|i(p_b - p_a) + q|}{kT} \right)^2 \frac{1}{1 - \exp \left[ -\omega_q/kT \right]} + \sin \left( \frac{|i(p_b - p_a) - q|}{kT} \right)^2 \frac{1}{1 - \exp \left[ -\omega_q/kT \right]} \right\},
$$

where \( i \) is a unit vector parallel to \( x \).

Using the Debye approximation, in the case in which \( T = 0 \) we obtain

$$
P_{ab} = \begin{cases} \frac{\pi}{16} & \frac{V_0}{s_0} \int_0^a \chi_a^* \exp \left[ -\frac{z}{s_0} \right] \chi_b \frac{\exp \left( \frac{v}{s_0 kT_D} \right)^2 kT_D / v \left| p_b - p_a \right|}{kT_D M} \\ 0 & \text{if } \left| p_b - p_a \right| < \frac{kT_D}{v} \end{cases}
$$

(8)

(\( v \) is the sound velocity and \( T_D \) the Debye's temperature).

Before using the result (8) we have to discuss the consistency of this result and the conditions under which it is valid. The undisturbed particle classically travels between \( \pi_1 \) and \( \pi_2 \), oscillating along \( z \). During a complete oscillation it travels along \( x \) a length \( \bar{x} \left( \bar{x} \simeq p \frac{\pi}{2} \int_{z_0}^{z_0 + \frac{\pi}{2}} \right) \), where \( z_0 \) is the classical amplitude of oscillation.

For the consistency of our results it is necessary that

$$
\sum_b P_{ab} \bar{x} \ll 1.
$$

(9)

The condition (9), together with the condition that the quantum numbers of \( \chi_a \) and \( \chi_b \) are not too small, is also sufficient.

In fact, if we follow our particle for a path \( x > \bar{x} \), due to the randomness of the perturbation \( V \), the states will no longer be a coherent superposition as in (6'), but a statistical superposition of incoherent states.

Let \( p_b \) be the probability of any pure state \( \psi_b \). Then, in place of the wave equation (6) we have to consider the diffusion equation

$$
\frac{dp_a}{dx} = \sum_b P_{ab}(p_b - p_a),
$$

(10)
which may be considered valid with $P_{ab} = P_{ba}$ given by eq. (8), until the condition (9) is respected for all the states for which $p_a$ is appreciably different from zero.

We can give some order of magnitude: If

\[
p = 10^{10} \text{ eV,} \\
V_0 = 150 \text{ eV,} \\
V_0/M = 0.6 \times 10^{-9}, \\
v = 10^{-5}, \\
kT_D/v = 3 \times 10^3 \text{ eV/c,} \\
a = 3 \times 10^{-8} \text{ cm,} \\
2z_{0a} = 0.6a, \\
2z_{0b} = 0.75a,
\]

one has:

\[
P_{aa}\bar{x} \simeq 10^{-3}, \quad P_{ab}\bar{x} \simeq 10^{-5}, \quad \sum_c P_{ac}\bar{x} \simeq 10^{-2}.
\]

Leaving the other data unchanged, and taking $z_{0a} = 0.25a$, the mean free path for the diffusion of a particle which is initially in the state $a$ will be of the order of one centimeter or more. In these conditions the scattering against the electrons might become more effective in destroying the channeling than the zero point motion. Obviously, in the case of incoming electrons, on the contrary, the zero point motion will be always very effective in destroying the channeling, because they would be trapped around the equilibrium position net of the nuclei, and not between two nets of equilibrium positions as it happens for positrons.

Returning to positrons, of a beam impinging on the crystal, one half will have a « collision parameter » such that $z_{0a} < 0.25a$. The condition on the incidence angle $\theta$ with the $xy$ plane is much more restrictive: for $p = 10$ GeV, $\theta \simeq 10^{-8}$ rad.

The resulting restriction in the phase space of the incoming particle is however largely compatible with the indetermination principle.

One can now ask whether our results can be generalized to cases in which the condition (1') is not satisfied. One might investigate whether, for instance, the trapping can arise between parallel crystallographic planes different than those which we have previously considered.

Now, an immediate generalization of the preceding analysis shows that a necessary condition for the smallness of the zero-point motion perturbation is that the distance between neighbouring planes which are supposed to trap the particle is great compared to $\sqrt{1/(8kT_D M)}$. This condition rules
out almost all the possibilities, with the exception of the planes for which the Mill indices can be very small numbers.

When the very severe restriction for trapping are not satisfied, the scattering will not be very different in a crystal or in a corresponding amorphous material.

Concluding, we remark that it is perhaps possible either to use the channeling phenomenon which we have discussed for collimating very high energy positive particles, if one can obtain suitable crystals, or to use the channeling for studying very rare imperfections in almost perfect crystals by means of very high energy positive particles.

REFERENCES

[1] A rather complete list of references about this topic can be found in the report by U. Timm: Coherent Bremsstrahlung of Electrons in Crystals, DESY, 69/14 (March 1969).
[4] See, for instance, the notes of the lectures given in 1969 at CERN, about The Bremsstrahlung in Crystals.
Experimental Work on Coherent Scattering of High-Energy Hadrons by Light Nuclei.

G. Fidecaro and M. Fidecaro

CERN-Trieste High-Energy Group - Geneva and Trieste

1. - Introduction.

In the last few years, several measurements [1-19] have been performed in order to study the coherent scattering of high-energy hadrons from light nuclei. We intend to present here the results so far obtained, including those contributed, in the case of the deuterons, by the CERN-Trieste High-Energy Group. A summary is given in Tables I-IV.

Various techniques (emulsions, bubble chambers, counters, and spark chambers) have been used, as expected from the fact that the cross-sections involved vary by some orders of magnitude when the momentum transfer to the target particle is increased. Typically, in the pd case, at 13 GeV/c, the cross-section decreases from 0.3 b/(GeV)² in the forward direction, to 0.1 μb/(GeV)² for |t| = 1.8 (GeV)². Moreover, a good precision in the determination of the kinematics of the events is of great importance for this type of experiments: an accurate reconstruction of the events in fact, on one hand helps to select the collisions which leave the target nucleus in the fundamental state, on the other hand it provides the t-resolving power required to search for a possible structure in the differential cross-section.

All the experiments but one [3] cover a limited angular region in the forward or backward direction and, correspondingly, two different kinds of phenomena are studied.

Concerning the forward scattering region, in the first place these experiments give information on the structure of light nuclei. The use of strong interacting particles which have a non negligible probability of colliding at least twice with nucleons when traversing a nucleus, allows one to obtain results complementary to those obtained in the case of electron scattering, where the scattering amplitude depends predominantly on the single-particle


**Table I.** Coherent (elastic) scattering of high-energy hadrons by light nuclei: a) protons.

<table>
<thead>
<tr>
<th>Momentum transfer (GeV/c)</th>
<th>Momentum transfer (GeV/c)^2</th>
<th>Technique used</th>
<th>Authors</th>
<th>( \theta_{\text{c.m.}} ) (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pd</td>
<td></td>
<td>Emulsion to record the slow recoil deuterons from a target of deuterated polyethylene - ( \Delta E/E \approx 2.5 \pm 5% )</td>
<td>N. Dalkhazhav <em>et al.</em> [1], JINR</td>
<td>4.1 ( \pm ) 15.6</td>
</tr>
<tr>
<td>2.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.85</td>
<td>0.003 ( \leq - t \leq 0.2 )</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis and TOF measurement in the d-branch</td>
<td>E. Coleman <em>et al.</em> [2], Cosmotron</td>
<td>3.1 ( \pm ) 15.6</td>
</tr>
<tr>
<td>6.89</td>
<td>0.44 ( \leq - t \leq 1.54 )</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis and TOF measurement in the d-branch</td>
<td>E. Coleman <em>et al.</em> [2], Cosmotron</td>
<td>2.4 ( \pm ) 9.3</td>
</tr>
<tr>
<td>8.89</td>
<td>0.026 ( \leq - t \leq 3.44 )</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis and TOF measurement in the d-branch</td>
<td>E. Coleman <em>et al.</em> [2], Cosmotron</td>
<td>1.4 ( \pm ) 12.6</td>
</tr>
<tr>
<td>10.90</td>
<td>0.026 ( \leq - t \leq 3.44 )</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis and TOF measurement in the d-branch</td>
<td>E. Coleman <em>et al.</em> [2], Cosmotron</td>
<td>1.4 ( \pm ) 8.9</td>
</tr>
<tr>
<td>12.8</td>
<td>0.2 ( \leq - t \leq 1.8 )</td>
<td>Scintillation counters and wire spark chambers, ( \Delta t/t \approx 2% )</td>
<td>CERN-Trieste [4]</td>
<td>1.2 ( \pm ) 8.2</td>
</tr>
<tr>
<td>p^4He</td>
<td>1.2</td>
<td>p-branch: scintillator telescope, ( \Delta E \leq 20\ MeV )</td>
<td>H. T. Boschitz <em>et al.</em> [5], Virginia University</td>
<td>29.0 ( \pm ) 55.0</td>
</tr>
<tr>
<td>1.7</td>
<td>0.007 ( \leq - t \leq 0.47 )</td>
<td>p-branch: scintillator telescope, ( \Delta E \leq 20\ MeV )</td>
<td>H. Palevsky <em>et al.</em> [6], Cosmotron</td>
<td>10.0 ( \pm ) 170.0</td>
</tr>
<tr>
<td>p^12C</td>
<td>1.7</td>
<td>p-branch: magnetic spectrometer (wire spark chambers), ( \Delta E = 3\ MeV ), and TOF measurement</td>
<td>H. Palevsky <em>et al.</em> [6], Cosmotron</td>
<td>6.9 ( \pm ) 58.1</td>
</tr>
<tr>
<td>p^16O</td>
<td>1.7</td>
<td>p-branch: magnetic spectrometer (wire spark chambers), ( \Delta E = 3\ MeV ), and TOF measurement</td>
<td>H. Palevsky <em>et al.</em> [6], Cosmotron</td>
<td>6.0 ( \pm ) 18.0</td>
</tr>
</tbody>
</table>

density. This is equivalent to saying that by using a hadron probe it is possible to study nucleon-nucleon correlations inside a nucleus [20].

There is also a specific interest which concerns the hadron-nucleon elementary interaction. At high energy (\( \geq 1\ GeV/c \)) the probability of having a single hadron-nucleon elastic collision decreases strongly when the momentum transfer is increased, while, as it will be seen later, the probability for a double collision decreases less fast; that is, it is easier to obtain a large transfer of momentum by means of two subsequent collisions than by means of only one; and there exists a \( t \)-interval for which the two probabilities are nearly equal. This region, in which the two scattering amplitudes interfere, is sensitive to the phase difference between the single and the double scat-
**Table II.** - Coherent (elastic) scattering of high-energy hadrons by light nuclei: b) pions, kaons, deuterons.

<table>
<thead>
<tr>
<th>Momentum (GeV/c)</th>
<th>Momentum transfer (GeV/c)^2</th>
<th>Technique used</th>
<th>Authors</th>
<th>θ_{e.m.} (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>π^+d</td>
<td>3.65 0.05 &lt; -t &lt; 0.90</td>
<td>20 in. deuterium bubble chamber</td>
<td>H. C. Hsiung et al. [7], BNL CERN-Saclay [8]</td>
<td>8.0 ± 33.5</td>
</tr>
<tr>
<td></td>
<td>6.0 0.03 &lt; -t &lt; 0.21</td>
<td>81 cm deuterium bubble chamber</td>
<td></td>
<td>4.5 ± 12.0</td>
</tr>
<tr>
<td>π^-d</td>
<td>2.01 0.262 &lt; -t &lt; 0.878</td>
<td>Scintillator hodoscopes for π^-d angular correlation; TOF window in the d-branch, sweeping magnets in π and d branches to decrease the background</td>
<td>R. C. Chase et al. [9], Argonne</td>
<td>26.1 ± 48.9</td>
</tr>
<tr>
<td></td>
<td>3.77 0.282 &lt; -t &lt; 0.898</td>
<td></td>
<td></td>
<td>18.1 ± 32.7</td>
</tr>
<tr>
<td></td>
<td>5.53 0.291 &lt; -t &lt; 1.232</td>
<td></td>
<td></td>
<td>14.7 ± 32.6</td>
</tr>
<tr>
<td></td>
<td>0.895 0.165 &lt; -t &lt; 0.940</td>
<td>Wire spark chambers for π^-d angular correlation, Δt/t ~ 2%; d-TOF correlated (or not) with pion angle keeps down the background</td>
<td>CERN-Trieste [10]</td>
<td>37.2 ± 99.0</td>
</tr>
<tr>
<td></td>
<td>0.994 0.17 &lt; -t &lt; 0.46</td>
<td></td>
<td>CERN-Trieste [11]</td>
<td>34.8 ± 70.5</td>
</tr>
<tr>
<td></td>
<td>0.13 0.20 &lt; -t &lt; 2.3</td>
<td></td>
<td>CERN-Trieste [12]</td>
<td>9.3 ± 21.0</td>
</tr>
<tr>
<td></td>
<td>13.0 0.20 &lt; -t &lt; 0.57</td>
<td></td>
<td></td>
<td>7.5 ± 12.5</td>
</tr>
<tr>
<td></td>
<td>15.2 0.20 &lt; -t &lt; 1.02</td>
<td></td>
<td></td>
<td>7.0 ± 15.8</td>
</tr>
<tr>
<td>K^-d</td>
<td>3.0 0.27 &lt; -t &lt; 0.175</td>
<td>81 cm deuterium bubble chamber</td>
<td>W. Hoogland et al. [13], CERN</td>
<td>6.5 ± 16.5</td>
</tr>
<tr>
<td>dd</td>
<td>2.2 0.05 &lt; -t &lt; 1.9</td>
<td>Deuterium bubble chamber</td>
<td>M. Bazin et al. (preliminary results - private communication)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.9 0.05 &lt; -t &lt; 0.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table III.** - Coherent (elastic and quasi elastic) scattering of high-energy hadrons by light nuclei.

<table>
<thead>
<tr>
<th>Momentum (GeV/c)</th>
<th>Momentum transfer (GeV/c)^2</th>
<th>Technique used</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>pd</td>
<td>1.29 7.0 × 10^{-4} &lt; -t &lt; 8.0 × 10^{-3}</td>
<td>Magnetic spectrometer, with sonic spark chambers, Δp/p = 0.5%</td>
<td>L. M. C. Dutton and H. Buan van der Raay [14]</td>
</tr>
<tr>
<td></td>
<td>1.39 8.0 × 10^{-4} &lt; -t &lt; 9.5 × 10^{-3}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.54 9.5 × 10^{-4} &lt; -t &lt; 12.0 × 10^{-3}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.69 11.0 × 10^{-4} &lt; -t &lt; 14.0 × 10^{-3}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>19.3 1.6 × 10^{-3} &lt; -t &lt; 0.1</td>
<td>Magnetic spectrometer, with sonic spark chambers, Δp/p = 0.5%</td>
<td>G. Bellettini et al. [15a]</td>
</tr>
<tr>
<td>p^6Li</td>
<td>19.3 -t &lt; 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>^7Li</td>
<td>19.3 -t &lt; 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>19.3 -t &lt; 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>21.5 -t &lt; 0.12</td>
<td></td>
<td>G. Bellettini et al. [15b]</td>
</tr>
</tbody>
</table>
TABLE IV. – Coherent (elastic) scattering of high-energy hadrons by light nuclei (backward).

<table>
<thead>
<tr>
<th>Momentum (GeV/cm)</th>
<th>Technique used</th>
<th>Authors</th>
<th>$\theta_{e.m.}$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pd 1.70</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis and TOF measurement in the d-branch</td>
<td>E. Coleman et al. [2]</td>
<td>152.1–120.0</td>
</tr>
<tr>
<td>2.03</td>
<td></td>
<td>BNL-Cosmotron</td>
<td>153.5–117.3</td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td></td>
<td>154.0–120.0</td>
</tr>
<tr>
<td>1.41</td>
<td>Scintillator telescopes for both the proton and the deuteron; range analysis (optical spark chambers) in the p-branch; magnetic analysis in the d-branch</td>
<td>N. G. Birger et al. [16]</td>
<td>152</td>
</tr>
<tr>
<td>1.70</td>
<td></td>
<td></td>
<td>151</td>
</tr>
<tr>
<td>2.40</td>
<td></td>
<td></td>
<td>149.4</td>
</tr>
<tr>
<td>2.65</td>
<td></td>
<td>Yu. D. Bajukov et al. [17]</td>
<td>147.5</td>
</tr>
<tr>
<td>1.37</td>
<td></td>
<td></td>
<td>160</td>
</tr>
<tr>
<td>1.70</td>
<td></td>
<td></td>
<td>159.5</td>
</tr>
<tr>
<td>2.78</td>
<td>Yu. D. Bajukov et al. [18]</td>
<td></td>
<td>162</td>
</tr>
<tr>
<td>4.50</td>
<td>ITEF Proton Synchrotron (internal beam, CD$_2$ target)</td>
<td></td>
<td>165.5</td>
</tr>
<tr>
<td>4.25</td>
<td></td>
<td>J. Banaigs et al. [19], Saturne</td>
<td></td>
</tr>
<tr>
<td>1.9</td>
<td>Scintillator telescopes for both the proton and the deuteron; magnetic analysis ($\Delta p/p = \pm 2%$) and TOF selection in the d-branch. (The reaction studied is dp $\rightarrow$ dp)</td>
<td>G. W. Bennett et al. [3], BNL-Cosmotron</td>
<td>174.5–180</td>
</tr>
<tr>
<td>1.7</td>
<td></td>
<td></td>
<td>$&lt; 170$</td>
</tr>
</tbody>
</table>

Scattering amplitudes; if, as it happens generally at high-energy, both the amplitudes are purely imaginary (high absorptive process), the interference is destructive and the differential cross-section goes down to zero (*). 

If the ratio between the real and the imaginary part of the scattering amplitude is different from zero, the cross-section will show a more or less pronounced minimum; one could hope to evaluate from the depth of this minimum the dependence of the real part of the scattering amplitude on the momentum transfer, which at high energy cannot be obtained in any other direct way [21]. This is a case in which nuclear physics comes to help elementary particle physics.

It is perhaps worth recalling also that attempts have been made to interpret the forward scattering of hadrons by nucleons in terms of a model

(*) A similar situation arises when one compares the probability of a scattering of order $n$, with the probability of a scattering of order $(n-1)$. This leads to a typical diffraction pattern.
which treats the target particle as a composite system. The nuclear cross-sections show indeed a \( t \)- (or \( u \)-) dependence very similar to the one observed when the scatterer is an elementary particle [22].

Concerning the backward scattering region, the present tendency is to try models in terms of Feynman diagrams with exchange of baryons. However, the phenomenology is in a much more rudimentary stage and the experimental work still very much incomplete.

The progress made in the field of hadron scattering by light nuclei comes both from development of the Glauber model [23] of multiple scattering which tends to explain satisfactorily the forward scattering, and from the new technical developments which have allowed experimental physicists to perform measurements of very small cross-sections, thus making possible the continuous comparison and the improvement of the theory. The existence of these new techniques, in particular, will certainly shed light and induce progress in the phenomenology of backward scattering.

2. – The forward scattering region.

The formulae given below have been derived [23, 24] in the limit of the high-energy approximation and of very small forward angles, \( i.e. \), one assumes that the wavelength of the incident particle is small in comparison to its range of interaction and that the angles considered correspond to the angular region near the forward diffraction peak.

In this approximation the elastic scattering amplitude for the case of two colliding particles is given by:

\[
\frac{f(k_1, k_2)}{2\pi} = \int \exp \left[ i(k - k_1) \cdot b \right] \left[ 1 - \exp \left[ i\chi(b) \right] \right] d^3b,
\]

where \( k \) and \( k_1 \) are, respectively, the initial and the final momentum of the incoming particle in the laboratory system, and \( b = \hbar(l + \frac{1}{2}) \) is the impact parameter. The term \( \chi(b) \) is a complex phase shift which in the case of spherically symmetric interaction is related to the better-known phase shifts of the partial wave analysis through the formula

\[
\chi(b) = \chi \left( \frac{l + \frac{1}{2}}{k} \right) = 2\delta_l.
\]

Formula (1) is valid for a spin-independent interaction of an arbitrary shape. It has been obtained by using the approximation \( P_l(\cos \theta) \to J_0(b \cdot \sqrt{-t}) \) where \(-t\) is the four-momentum transfer, and by replacing the sum over \( l \) with an integral over \( b \).
The next step is to consider a system of $A$ particles bound to form a nucleus. The approximation is here made that the single nucleons are frozen in their instantaneous position $r_1, \ldots, r_A$ during the time that the incoming particle goes through the nucleus. The generalization of expression (1) is

\begin{equation}
F(q) = \frac{k}{2\pi i} \int \exp \left[ i q \cdot b \right] d^2 b \left< \psi | 1 - \exp \left[ \chi(b, r_1, \ldots, r_A) \right] \right| \psi \right>,
\end{equation}

where $q = k - k^\dagger$ and $|q|^2 = -t$. It is here that the most critical hypothesis of the model arises; that is, the phase factor $\chi(b, r_1, \ldots, r_A)$ is assumed to be the sum of the individual phase factors

$$
\chi(b, r_1, \ldots, r_A) = \sum_{j=1}^{A} \chi_j(b - s_j),
$$

$s_j$ being the component of $r_j$ along the incident beam. As a result,

\begin{equation}
F(q) = \frac{ik}{2\pi} \int \exp \left[ i q \cdot b \right] d^2 b \int d^3 r_1 \ldots d^3 r_A \psi^*(r_1, \ldots, r_A) \delta^3 \left( \frac{1}{A} \sum r_j \right)

\left[ 1 - \prod_{j=1}^{A} \left( 1 - \frac{1}{2\pi ik} \int \exp \left[ -i q(b - s_j) \right] f_j(q') d^2 q' \right) \right] \psi(r_1, \ldots, r_A).
\end{equation}

If one expands $\prod_{j=1}^{A}$, the scattering amplitude $F$ can be represented as a polynomial in the hadron-nucleon scattering amplitude $f_j(q)$. This polynomial is interpreted as a sum of terms originated by multiple scatterings: first, second, ... order term corresponds to a single, double, ... scattering; the highest term is of order $A$, which is a consequence of the fact that the Glauber model takes into account the multiple collisions just by adding the phase shifts.

Various attempts [25-28] have been made to improve the Glauber model by dropping some of the approximations. However, it appears to be a delicate matter to introduce new corrections; for instance, in the case of the deuteron, Harrington [29] has shown that in some cases the off-shell contributions cancel the sum of all the higher order terms. On the other hand it is a fact that the experimental values for the cross-sections obtained until now, with the exception of the ones in a very backward direction, are fitted in a reasonable way if the scattering amplitude is given by eq. (3), with $(d \sigma/d \Omega)_{t.s.} = |F(q)|^2$.

In most of these fits [30] the hadron-nucleon scattering amplitude which
appears in formula (3) has been parametrized as

\[ f(q) = \frac{ik\sigma}{4\pi} (1 - i\alpha) \exp \left[ -\frac{\beta^2 q^2}{2} \right] \]

as suggested by the available experimental data, where \( \sigma \) is the hadron-nucleon total cross-section, and \( \alpha \) is the ratio between the real and the imaginary part of the scattering amplitude. In some cases more accurate amplitudes were used, when available from phase-shift analysis of experimental data [27] or from extrapolation to high energy via finite energy sum rules [27, 31].

Concerning the wave function \( \psi(r_1, ..., r_A) \), the experimental information is rather accurate in the cases where \( A < 3 \), at least for values of \( r \) not too small, while for \( A > 4 \) particular models have to be taken.

Czyz and Lesniak [32] have computed formula (3) by using an independent particle model and a Gaussian dependence on \( r \) for the single particle density. The nuclear scattering amplitude is well approximated by

\[
F(q) = \frac{ik}{2\pi} (R^2 + 2\beta^2) \exp \left[ \frac{q^2 R^2}{4A} \right] \sum_{j=1}^{A} \left( \begin{array}{c} A \\ j \end{array} \right) (-1)^{i+1} \frac{1}{j} (1 - i\alpha)^j \cdot \left( \frac{\sigma}{2\pi(R^2 + 2\beta^2)} \right)^j \exp \left[ -\frac{(R^2 + 2\beta^2)q^2}{4j} \right],
\]

\( R \) being the width of the single-particle density distribution. This formula shows that if the hadron-nucleon amplitude is purely imaginary (\( \alpha = 0 \)), the nuclear amplitude \( F(q) \) is also purely imaginary, and the double scattering term (\( j = 2 \)) has opposite sign to the single scattering term (\( j = 1 \)), and half its slope; for the value of the momentum transfer at which the two terms become equal (in absolute value) the cross-section goes to zero, as mentioned in Sect. 1. This happens, when

\[-t_0 = \frac{4}{(R^2/2 + \beta^2)} \ln \left( \frac{16\pi \left( \frac{R^2}{2} + \beta^2 \right)}{\sigma \left( \frac{R^2}{2} + \beta^2 \right)} \right).\]

In a similar way other \( (A - 2) \) minima would arise. If \( |x| \neq 0 \) the dip will be filled up fast when \( |x| \) is increased, while the other parts of the curve are rather insensitive to this parameter.

3. The hadron-deuteron scattering.

Among the experiments already mentioned, the hadron-deuteron scattering should be the simplest one from the point of view of the analysis,
because only two nucleons can be involved as scatterers. This advantage is partly counterbalanced by the fact that the deuteron is a spin-one particle. For unpolarized deuteron targets, the observed cross-section is the average of that found for the three states of polarization [33]:

\[
\left( \frac{d\sigma}{d\Omega} \right) = \frac{1}{3} \sum_{m_m'} |\langle m' | F(q, S) | m \rangle|^2.
\]

The operator \( F(q, S) \) is a linear functional \( F(q, S) = F_1(q, S) + F_2(q, S) \) of the form factor operator \( S \), which is a linear combination of the scalar form factor \( S_0 \) and of the quadrupole form factor \( S_2 \). Operators \( F_1 \) and \( F_2 \) represent the single and double scattering, respectively,

\[
F_1(q, S) = f_1(q) S(q/2) + f_2(q) S(-q/2),
\]

and

\[
F_2(q, S) = \frac{i}{4\pi k} \int S(q') \left\{ f_1 \left( \frac{q}{2} + q' \right) f_2 \left( \frac{q}{2} - q' \right) + f_2 \left( \frac{q}{2} - q' \right) f_1 \left( \frac{q}{2} + q' \right) - C_1 \left[ f_1 \left( \frac{q}{2} + q' \right) - f_2 \left( \frac{q}{2} + q' \right) \right] \left[ f_1 \left( \frac{q}{2} - q' \right) - f_2 \left( \frac{q}{2} - q' \right) \right] \right\} d^2 q',
\]

where \( f_1 \) and \( f_2 \) are the elastic scattering amplitudes for collisions between the hadron and particles 1 and 2 of the deuteron. The coefficient \( C_1 \) is 1 if the incident hadron has isospin \( \frac{1}{2} \), and is \( \frac{1}{2} \) for an incident hadron of unit isospin.

By choosing, for instance, a polarization axis along \( q \), one observes by developing the above formula that only the double scattering can contribute to the spin-flip transition \( \Delta m = \pm 2 \) term, so that the dip in the differential cross-section, mentioned at the end of the previous section, is missing; as regards to the nonspin-flip transitions, both the cross-sections that correspond to an initial state \( m = 0 \) and \( m = \pm 1 \) show pronounced minima brought about by the destructive interference of the single and double scattering amplitude. The position of these two minima occur at rather different values of \( q \) since in that range of momentum transfer \( S_2(q/2) \) is nearly equal to \( S_0(q/2) \); thus the single scattering term, in the \( m = 1 \) state, which is weighted by the factor \( (S_0 - S_2) \), decreases faster than in the \( m = 0 \) state, where the weight factor is \( (S_0 + \frac{1}{2} S_2) \). As a result in the case of unpolarized deuterons, the differential cross-section in the interference region is not very sensitive to the ratio between the real and the imaginary part of the scattering amplitude; the differential cross-section is instead very sensitive to the quadrupole form factor and therefore to the \( d \)-wave percentage included in the wave function of the deuteron.
Fig. 1a. - Differential cross-section for π-d elastic scattering at 0.895, 9.0, and 15.2 GeV/c. The experimental data are from refs. [10] and [12]. The 0.895 curve (●) (ref. [27]) was computed by using the pion-nucleon amplitudes derived from the CERN phase-shift analysis. For the 15.2 GeV/c curve (○) (ref. [27]) the amplitudes were those of Barger and Phillips (ref. [21]). In both cases the Gartenhaus-Moravcsik deuteron wave function was used. For the 9.0 GeV/c curve (▲), the scattering amplitude was parametrized as
\[ f(q) = \frac{(ikq/4\pi)(1-i\alpha)}{(-\beta_q^2/2)}, \]
with \( \sigma_p = 26.9 \text{ mb}, \sigma_n = 25.3 \text{ mb}, \alpha_p = -0.13, \alpha_n = -0.23, \beta_p^2 = \beta_n^2 = 8.5 (\text{GeV})^{-2}; \) the Humberston wave function was used for the deuteron.
Fig. 1b. – Differential cross-section for p-d elastic scattering at 10.9 GeV/c (α) and 12.8 GeV/c (●). The experimental data are from ref. [1] and [4]. For the nucleon-nucleon scattering amplitude it was assumed that $\sigma_p = 39.2$ mb, $\sigma_n = 40.2$ mb, $\alpha_p = \alpha_n = -0.33$, $\beta_p^3 = \beta_n^3 = 10$ (GeV)$^{-2}$; for the deuteron the Humberston wave function was used.
In the case of polarized deuterons, if \( q_d \) is the position of the minimum, the cross-section at \( q_d \) is strongly sensitive to the real part of the hadron-nucleon scattering amplitude at the value \( q = q_d / 2 \) and \( q = q_d \).

In Fig. 1 we show a sample of data for pions and protons. There is no dip and the agreement with the theoretical curves is rather good.

The coherent quasi-elastic scattering can be described in a similar way, starting from formula (3). The agreement with the experimental data (see Table IV) is again rather good.

4. – The \( A > 2 \) case.

A similar analysis has been carried out for He and other light nuclei. Here the information exists only for protons and is rather limited. The experimental data for \(^4\)He are compared in Fig. 2 with the curves foreseen by the model. At low momentum transfer the agreement is good for what concerns the size and slope of the diffraction peak as well as the size and position of the first minimum.

The situation changes at larger angles. It seems that a better representation of the data is obtained by decreasing the scattering amplitude with a change of the nuclear density, in such a way that the term of order \( n \) is decreased less than the term of order \( (n-1) \).

Because, roughly speaking, the multiple scattering amplitude is an expansion in \( \sigma \cdot r_{ij}^{-2} \), where \( r_{ij} \) is the distance between two nucleons in the nucleus, one can obtain the above effect by increasing the average separation of the nucleons themselves. This is the case when there are nucleon configurations which are preferred or if there is a correlation such that the two nucleons cannot approach each other beyond a certain limit. In this scheme Czyz and Lesniak [32] and Bassel and Wilkin [30] have modified the nuclear density function, given by the independent particle model.

In Fig. 2 the results of Bassel and Wilkin are reported. They tried to fit the experimental data of ref. [6] either by using a double Gaussian as density function for the single particle, or by introducing a correlation function. The fit of these data (as well as of the data for e-\(^4\)He elastic scattering) is good in the first case (continuous curve of Fig. 2), but not in the second one (*).

(*) A critical analysis of the kind of interpretation has been made recently by Cromer [34].
Fig. 2. – Differential cross-section for p-\(^4\)He elastic scattering at 1.7 GeV/c. The experimental data are from ref. [6]. The curves are from ref. [30], and were computed by assuming for the single-particle density a Gaussian distribution of width \(R^2 = 1.87\) fm\(^2\) (---), or a double Gaussian distribution (---); for the nucleon-nucleon scattering amplitude it was assumed that \(\sigma = 44\) mb, \(\alpha = -0.3\) and \(\beta^2 = 5.4\) (GeV/c\(^2\)) (---), or \(\sigma = 40.4\) mb, \(\alpha = -0.5\) and \(\beta^2 = 5.4\) (GeV\(^{-2}\)) (---). Curve (1) represents the contribution of single scattering (impulse approximation), curve (2) single plus double, etc.

The results of a similar analysis for the \(^4\)He data at 1.2 GeV/c [5], and for \(^{12}\)C and \(^{16}\)O at 1.7 GeV/c [6], are reported in Figs. 3, 4 and 5.

For the heavier nuclei, \(A \gg 1\), the scattering amplitude which results from the sum over the various terms tends to the same form that one would find by using an optical model of the nucleus; this, in fact, is already true for the \(^{16}\)O case [35].
5. The backward region.

The backward scattering has been studied only in the pd case, and in a rather limited momentum interval. The important experimental fact is

![Graph showing differential cross-section for p-^4He elastic scattering at 1.2 GeV/c. The experimental data are from ref. [5]. The curves are from ref. [35b] and were computed by assuming for the single-particle density a Gaussian distribution of width R = 1.56 fm (- - -) or the one obtained from the e-^4He elastic scattering (-----); for the nucleon-nucleon scattering amplitude it was assumed that σ = 39 mb, α = −0.43 (-----), and α = −0.5 (-- --), β = 4.3 (GeV/c)^{−2}.]
Fig. 4. – Differential cross-section for p$_{12}^C$ elastic scattering at 1.7 GeV/c. The experimental data are from ref. [6]. The curve is from ref. [35b] and was computed by assuming for the single-particle density a Gaussian distribution of width $R^2 = 2.5$ fm$^2$, as obtained from electron scattering; for the nucleon-nucleon scattering amplitude it was assumed that $\sigma = 44$ mb, $\alpha = -0.28$, $\beta^2 = 5.4$ (GeV/c)$^{-2}$.

the strong enhancement of the cross-section toward the largest angles, as it is shown in Fig. 6.

In order to extend to the large angles the mechanism that describes forward scattering, let us refer to the pictorial representation of the
Fig. 5. – Differential cross-section for $p^{16}O$ elastic scattering at 1.7 GeV/c. The experimental data are from ref. [6]. The curve is from ref. [35b] and was computed by assuming for the single-particle density a Gaussian distribution of width $R^2 = 2.92 \text{ fm}^2$; for the nucleon-nucleon scattering amplitude it was assumed that $\sigma_p = 47.5 \text{ mb}$, $\sigma_n = 40.0 \text{ mb}$, $\alpha = -0.4$, $\beta^2 = 4.7 \text{ (GeV/c)}^{-2}$.

Glauber model that is indicated in Fig. 7. If the graphs are interpreted as Feynman diagrams and certain approximations are made, one obtains the same values for the cross-section as those obtained from the multiple scattering model.
Fig. 6. – Differential cross-section for p-d elastic scattering in the backward direction at 1.7 (●), 2.0 (○), and 2.25 (△) GeV/c. The experimental data are from ref. [2]. The curves are from ref. [38] and were computed from the triangle diagram of Fig. 7e.

There have been attempts [36, 37] to use this technique for the backward scattering, but the computed cross-section is an order of magnitude lower than the experimental data.

An attempt [18] to explain the data on the basis of the exchange diagram of Fig. 7e has also been unsuccessful, because the cross-section does not decrease fast enough when the momentum of the incoming proton is increased.
Very recently, Craigie and Wilkin [38] related the proton-deuteron backward scattering to the case of a pp collision in which a fast deuteron is produced together with a slow pion; the pion (dashed line in Fig. 7f) is afterwards absorbed by the spectator nucleon. The predicted cross-sections are too low, but the angular dependence is in good agreement with the experimental data.

6. Conclusions.

The experimental results reported in this paper tend to indicate a fairly good agreement of the experiments with the Glauber model of multiple scattering. As mentioned in the introduction, this progress has been made possible by new technical developments. These can still be exploited for more refined measurements in the forward scattering region and for new
measurements at larger angles, including the backward scattering region. In particular, it seems to be of great interest to perform experiments of this kind on polarized deuterons, as one could check the model in detail through simplified experimental conditions. Unfortunately the technique of polarized targets has not yet reached such a point of refinement.

The success so far obtained does not imply, however, that the theory is already perfect: much more systematic work of comparison with experiments is needed in order to clarify the exact meaning of the various assumptions and approximations, and to find out the limit of validity of the present ideas.

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The Determination of the Axial-Vector Coupling for Strangeness Nonchanging Currents.

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1. - Introduction.

According to our present knowledge, weak interaction processes can be described using a phenomenological Lagrangian density of the type

$$\mathcal{L} = \frac{G}{\sqrt{2}} J_\lambda J_\lambda^* + \text{h.c.},$$  \hspace{1cm} (1)

where

$$J_\lambda = j_\lambda^{(e)} + j_\lambda^{(\mu)} + J_\lambda^{(h)}$$  \hspace{1cm} (2a)

$$j_\lambda^{(e)} = i\bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_e$$  \hspace{1cm} (2b)

$$j_\lambda^{(\mu)} = i\bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5) \psi_\mu$$  \hspace{1cm} (2c)

are the local leptonic currents, associated with the electronic and muonic field, respectively. The hadronic current $J_\lambda^{(h)}$ has, in general, a more complicated structure: Following Cabibbo’s theory, the strangeness conserving and strangeness nonconserving parts add up to give

$$J_\lambda^{(h)(+)} = \cos \theta_v V_\lambda^{(+)} + \cos \theta_A A_\lambda^{(+)} + \sin \theta_v V_{SA}^{(+)} + \sin \theta_A A_{SA}^{(+)} ,$$  \hspace{1cm} (3)

where $V$, $A$, are the vector and axial vector parts of the strangeness nonchanging currents and $V_{SA}$ and $A_{SA}$ the corresponding parts of the strangeness changing currents. The $(+)$ suffix is a remainder of the fact that the current is a charge-raising operator (whereas its conjugate $J^*$ is a charge-lowering
operator). Cabibbo’s angles $\theta_{V,A}$ have the values [1, 2]

$$\theta_V = 0.232 \pm 0.013$$
$$\theta_A = 0.250 \pm 0.018.$$

$G$ is the universal Fermi constant

(4) \quad G = (1.4350 \pm 0.0011) \times 10^{-49} \text{ erg cm}^3.

The operation indicated by the asterisk is defined by the equation

(5) \quad J_k^* = J_k^\dagger, \quad J_4^* = -J_4^\dagger,

where $^\dagger$ indicates Hermitian conjugation. The change of sign of the fourth component puts this component on the same position as the others with respect to transformation under charge- and $G$-conjugation operations.

The strangeness-conserving vector current $V_\lambda$ is supposed to behave like a «conserved current», $i.e.$, to satisfy

(6) \quad \partial_\lambda V_\lambda = 0,

whereas the axial vector part is certainly not conserved. According to the «isotriplet hypothesis» $V_\lambda, V_\lambda^*$, and the isovector part of the electromagnetic current $(1/e)J^{T,V}_\lambda$ form the three components of the same isotriplet.

The most general matrix element of $J = V + A$ between two single nucleon states, satisfying the requirements of invariance of the Lagrangian density under proper Lorentz transformation has

(7) \quad V_\lambda = i\bar{u}_p(f_3\gamma_\lambda + f_2\sigma_{\lambda\mu} q_\mu + f_3 q_\lambda) u_n\exp[-iqx],

(8) \quad A_\lambda = i\bar{u}_p(g_3\gamma_\lambda\gamma_5 + g_2\sigma_{\lambda\mu} q_\mu\gamma_5 + g_3 q_\lambda\gamma_5) u_n\exp[-iqx].

The $f_i$ and $g_i$’s are (in general complex) functions of the four-momentum transfer squared

(9) \quad q^2 = (p_p - p_n)^2.

If the conserved vector current hypothesis is valid, then

(10a) \quad f_3(q^2) = 0.

Moreover, if the isotriplet hypothesis is accepted, then

(10b) \quad f_1(q^2) = F_{1,m}^e(q^2)

(10c) \quad f_2(q^2) = -\frac{\mu_p - \mu_n}{2M} F_{2,m}^e(q^2),
where $F_1$ and $F_2$ are Hofstadter's nucleon form factors normalized to unity for $q^2 \to 0$; $\mu_p$ and $\mu_n$ are the anomalous magnetic moments of the proton and neutron, respectively in units of Bohr magnetons ($\Delta \mu = 3.69$); and $M$ is the nucleon mass $M_n = M_p = M$.

Invariance under time reversal operation imposes

$$g_1 = g_1^*, \quad g_2 = g_2^*, \quad g_3 = -g_3^*,$$

whereas charge symmetry requires

$$g_1 = g_1^*, \quad g_2 = -g_2^*, \quad g_3 = -g_3^*.$$

Thus $g_1$ is real; $g_2 = 0$, and $g_3$ pure imaginary. At variance with the corresponding form factors of the vector part, the $g$'s are not normalized to unity when $q^2 \to 0$. Putting

$$G_V = G \cos \theta_V, \quad G_A = G_A^0 \cos \theta_A, \quad G_p = G_p^0 \cos \theta_p,$$

$$\lim_{q^2 \to 0} g_1(q^2) = G_A/G_V, \quad \lim_{q^2 \to 0} g_3(q^2) = i(G_p/G_V),$$

the entire matrix element for $J^{(b)}_x$ reads

$$\langle p|J^{(b)}_x|n\rangle = i \frac{G_V}{\sqrt{2}} \bar{u}(p) \left\{ F_1 \gamma_\lambda - \frac{\Delta \mu}{2M} F_2 \sigma_\mu q_\mu + \right. \right.$$

$$+ \frac{G_A}{G_V} F_A \gamma_\lambda \gamma_5 + \frac{i G_p}{G_V} F_p q_\lambda \gamma_5 \left. \right]\right. \left. \bar{u}(n) \exp \left[-i q x \right],$$

where

$$F_1(0) = F_A(0) = 1.$$

\[2. \text{ The determination of } G_A \text{ in experiments at low momentum transfer.}\]

In the limit of low $q_1$ the matrix elements (11) reduces to

$$\langle p|J_x|n\rangle = \frac{i}{\sqrt{2}} \bar{u}_p \gamma_\lambda (G_V + G_A \gamma_5) u_n.$$

Equation (12) can be used to predict the values of relevant parameters and distributions accessible to experimental determination. From the total decay rate of the neutron, one deduces

$$\left[ \frac{3}{4} |G_V|^2 + \frac{3}{4} |G_A|^2 \right]^{\frac{1}{2}} = (1.63 \pm 0.02) \times 10^{-49} \text{ erg cm}^3,$$
\( G_V \) can be deduced independently from a pure Fermi \( 0 \to 0 \) transition. On the basis of seven transitions \([3](^{14}O, ^{26}Al, ^{34}Cl, ^{42}Sc, ^{46}V, ^{50}Mn \text{ and } ^{54}Co)\), the value of \( G_V = (1.4149 \pm 0.0022) \times 10^{-49} \text{ erg cm}^3 \) was obtained. Thus

(14) \[ G_A/G_V = 1.18 \pm 0.02. \]

This is the best determination of \( G_A/G_V \) for strangeness nonchanging currents. All other experiments, though not as precise, have given results consistent with eq. (14) (*).

3. - The physical interpretation of the experimental result \( G_A/G_V = 1.18 \).

The physical interpretation of this result received a first contribution when Goldberger and Treiman discovered that

(15) \[ \frac{G_A}{G_V} \approx \frac{f_\pi g_{\pi N N}}{M}, \]

where \( g_{\pi N N} \) is the renormalized pion nucleon constant and \( f_\pi \) the pion decay constant.

Their deduction was based on complex arguments of dispersion theory; however it was later rederived starting from different assumptions and using simpler methods \([4]\).

Let the pion field be defined by the equation

(16) \[ \vec{\partial}_\lambda A_\lambda = c_\pi \vec{q}_\pi, \]

c_\pi being a constant (**) . The matrix elements of (16) between two nucleon states is

(17) \[ \langle p|\vec{\partial}_\lambda A_\lambda|n\rangle = iN[2MG_A F_A(q^2) + q^2G_p F_p(q^2)]\vec{u}_p \gamma_5 \vec{u}_n, \]

\( N \) being a normalization coefficient. For \( q^2 \to 0 \) this reduces to

(18) \[ \langle p|\vec{\partial}_\lambda A_\lambda|n\rangle = iN2MG_A \vec{u}_p \gamma_5 \vec{u}_n. \]

We must now calculate the matrix element of the pion field between the


(**) We omit here isospin indices which are not essential.
same states. Using the field equation

\begin{equation}
(\square - m_\pi^2) \varphi_\pi(x) = J^{(a)}(x)
\end{equation}

we obtain

\begin{equation}
\langle p | \varphi_\pi(0) | n \rangle = -\frac{1}{q^2 + m_\pi^2} \langle p | J^{(b)}(0) | n \rangle = -\frac{D(q^2)}{q^2 + m_\pi^2} \sqrt{2} g_{\pi,N^*} iN\bar{u}_p \gamma_5 u_n,
\end{equation}

where \( q \) is the pion momentum and \( D(q^2) \) is, by hypothesis, a slowly varying function of \( q^2 \) in the interval \( 0 \gg q^2 \gg -m^2 \) and accounts both for the renormalization of the free propagator and the free vertex. The \( \sqrt{2} \) factor arises from the isospin pion-nucleon coupling. Then \( D(q^2) \) is intended to be normalized to 1 for \( q^2 = -m^2 \), i.e.

\[ D(-m^2) = 1. \]

At \( q^2 = 0 \) we have, using (16),

\[ 2MG_A = -\sqrt{2} \frac{g_{\pi,N^*}}{m_\pi^2} D(0) c_\pi. \]

We can deduce \( c_\pi \) from the measured \( \pi \)-decay rate. This is in fact determined by the matrix element

\[ \langle 0 | A_\lambda | \pi \rangle = ic_\pi \frac{q_\lambda}{m_\pi^2} \langle 0 | \varphi_\pi | \pi \rangle \]

and one obtains

\[ c_\pi = \sqrt{2} f_\pi m_\pi^2, \]

where \( f_\pi \) is the (experimental) decay constant of the pion (\( \sqrt{2} f_\pi \simeq 133 \text{ MeV} \times G_V [5] \)).

Thus

\begin{equation}
G_A = \frac{g_{\pi,N^*}f_\pi}{M} G_V D(0).
\end{equation}

If \( D(q^2) \) does not change appreciably moving from \( q^2 = -m_\pi^2 \) to \( q^2 = 0 \), then (21) coincides with (15). Putting the measured values of the constants in (15), one obtains

\[ G_A/G_V \simeq 1.4. \]
The difference between this value and the experimental one (eq. (14)) is \( \sim 15\% \). The origin of such difference may well be due to the \( (q^2 = -m_{\pi^2}^2 \to q^2 = 0) \) extrapolation of the form factor \( D(q^2) \) (see Gell-Mann and Lévy\cite{4}).

The hypothesis formulated in eq. (16) is often referred to as Partially Conserved Axial-Vector Current hypothesis (PCAC). It may be expressed in a different way, namely by stating that the matrix elements of the divergence of the axial-vector current satisfy unsubtracted dispersion relations and that the imaginary part of this amplitude is essentially determined by the one-pion pole at \( q^2 = -m_{\pi^2}^2 \). Thus, although no extrapolation of the form factor is involved here the result is not exact because of the somewhat arbitrary neglect of the background contribution beginning at \( q^2 = -m_{\pi^2}^2 \).

A major step toward our understanding of the origin of the renormalization of the axial-vector coupling was made independently by Adler\cite{5} and by Weisberger\cite{6}.

Adler’s deduction which is based on a method proposed by Fubini and Furlan\cite{7}, starts from the following assumptions:

\( a) \) That the pion field is defined by the equations

\[
\partial_\lambda A_\lambda^I = C_i \frac{M m_{\pi^2}}{g_{\pi N \pi N}} \frac{G_A}{G_V} q_{\pi^2}^I,
\]

\( \langle i \rangle \) being an isotopic spin index and \( C_i \) an isospin Clebsch-Gordan coefficient.

\( b) \) That the fourth components of the axial-vector currents satisfy Gell-Mann’s equal-time commutation relations

\[
[A_\lambda^I(x), A_\lambda^J(y)]_{\pi^2} = -\delta(x - y) \epsilon_{ijk} V_k^I(x)
\]

and obtains the following equation relating \( G_V/G_A \) to the off-mass-shell pion-nucleon cross-sections:

\[
1 - \left( \frac{G_V}{G_A} \right)^2 = \frac{2 M^2}{g_{\pi N \pi N}^2} \frac{1}{\pi} \int \frac{dW^2}{W^2 - M^2} \left[ \sigma_{\text{off}}(\pi^+ - p) - \sigma_{\text{off}}(\pi^- - p) \right].
\]

The numerical evaluation of (24) requires some care since the off-mass-shell cross-sections are not the experimental ones. He carries out the extrapolation assuming that the 3-3 \( p \)-wave resonance predominates and obtains

\[ G_A/G_V = 1.24. \]

Weisberger, using a different formulation which does not imply off-mass-shell extrapolations (nor the Goldberger and Treiman relation) obtains

\[ G_A/G_V = 1.15. \]
in excellent agreement with the experimental value (14). Other methods to
to obtain (24) have been indicated (Adler [5], Weisberger [6] Fubini and
Furlan [7]) which the reader may find in the literature quoted here.

It is interesting, in this connection to mention that Weinberg [8] has
obtained a sum rule which differs from (24) only in terms of the order $O(m_n^2/M_N^2)$
starting from the Goldberger-Myizawa-Oehme sum rule for the pion-nucleon
scattering lengths $a_1 - a_4$. Turning the other way around one deduces a
relation between $a_4 - a_4$ and $G_A/G_V$.

4. The $q^2$-dependence of the axial-vector coupling.

4'1. Neutrino experiments. - The most direct way of investigating the
dependence of the axial-vector coupling on $q^2$ is the study of elastic reactions
produced by energetic neutrinos, i.e., processes of the type

\begin{equation}
\nu + n \rightarrow l^- + p
\end{equation}

\begin{equation}
\bar{\nu} + p \rightarrow l^+ + n,
\end{equation}

where $l$ is a lepton and $\nu$ the neutrino associated with the corresponding
lepton field.

The theoretical distribution $d\sigma/dq^2 = f(E_\nu, q^2), E_\nu$ being the primary neutrino
energy, can be deduced from eq. (11). Neglecting terms proportional
to the lepton mass, one obtains

\begin{equation}
\left(\frac{d\sigma}{dq^2}\right)_\nu = \frac{G_F^2}{32 \pi M^2} \frac{1}{E_\nu} [A(q^2) \pm x B(q^2) + x^2 C(q^2)],
\end{equation}

the $\pm$ sign before $B(q^2)$ referring to $\nu$ and $\bar{\nu}$ reactions, respectively. Moreover,
putting $\Delta \mu = \mu_p - \mu_n = 3.7$ B.N.M.

\begin{equation}
A = 4 M^2 q^2 \left(\frac{G_A}{G_V}\right)^2 \left(F_A^2 - F_1^2\right) + q^2 \left[F_1^2 + 4 \Delta \mu F_1 F_2 + F_2 \Delta \mu + \left(\frac{G_A}{G_V}\right)^2 F_A^2\right]
\end{equation}

\begin{equation}
B = \frac{G_A}{G_V} F_A (F_1 + \Delta \mu F_2) q^2
\end{equation}

\begin{equation}
C = F_1^2 + q^2 \left(\frac{\Delta \mu F_2}{2 M}\right)^2 + \left(\frac{G_A}{G_V}\right)^2 F_A^2.
\end{equation}

Experiments to measure the functions $A(q^2), B(q^2), C(q^2)$, and hence $F_A(q^2)$
have been performed at CERN and Argonne National Laboratory, using
spark chambers and heavy liquid bubble chambers. The analysis was
carried out assuming $F_1$ and $F_2$ were identical with the electromagnetic form
factors; and $F_A(q^2)$ was represented by the parametric form

$$F_A(q^2) = \left(1 + \frac{q^2}{M_A^2}\right)^{-2}$$

with $M_A$ to be determined from fitting the experimental data. The results are:

a) at CERN:
- Spark chamber exp. [19] $M_A = 0.65^{+0.45}_{-0.33} \text{ GeV/c}^2$
- Bubble chamber:
  - (freon filled) [10] $0.9^{+0.35}_{-0.25} \text{ GeV/c}^2$
  - (propane) [11] $0.7 \pm 0.2 \text{ GeV/c}^2$

b) at Argonne National Laboratory:
- Spark chamber exp. [12] $1.05 \pm 0.2 \text{ GeV/c}^2$.

The errors quoted here are largely due to nuclear effects which make
it difficult both to select genuine elastic events and also to determine precisely
the relevant kinematical parameters. In fact, reaction (25) takes place on
neutron target, namely inside nuclei. Also using propane, i.e., carbon targets
which are comparatively small nuclei, nuclear effects such as scattering of
protons, Fermi motion, pion absorption, cannot be neglected. The size of
such effects has been estimated using simple models by Montecarlo meth-
ods [13] and also by more sophisticated nuclear models [14]. Two typical
experimental distributions of $d\sigma/dq^2$ (integrated over the neutrino spectrum)
are shown in Fig. 1, 2 and 3.

If the cross-sections $d\sigma/dq^2$ is measured for processes (26) as well, the
axial form factor can be directly computed.

In fact, from (27) and (28)

$$\left(\frac{d\sigma}{dq^2}\right)_{\nu+n} - \left(\frac{d\sigma}{dq^2}\right)_{\bar{\nu}+p} = \frac{G_N G_A}{4\pi} \frac{q^2}{M^2 E_\nu} (4M E_\nu - q^2) (F_1 + \Delta \mu F_2) F_A(q^2).$$

Experiments with antineutrinos have been attempted but, so far, have
not yielded substantial results. In fact it is much more difficult to produce
a clean beam due to the unfavorable ratio $\pi^-/\pi^+$ in meson production processes
by protons. Moreover, the process (26) produces only one charged particle
in the final state, i.e. a $\mu^+$ or an $e^+$. No kinematical fit can be made to
establish the nature of the event which has to be assumed a priori.

A less direct way of measuring $F_A(q^2)$ is given by the neutrino inelastic
events

$$\nu+N \to l^- + N' + \pi,$$
where $N, N'$ indicate a nucleon, either a proton or a neutron. The hadronic current is, in this case, given by a much more complicated expression than (11). It can be seen that the most general matrix element is formed by 8 vectors and 8 pseudovectors. Conservation of the vector current reduces the number of terms from 16 to 14 and the «isotriplet vector current hypothesis» determines the vector current directly from electro- and photoproduction data.

The axial-vector part can be calculated with the help of dispersion relation technique. Let $M_t$ be the projection of the axial-vector amplitude on the
Axial-vector coupling

$i$-th multipole and let us assume that $M_i$ satisfies the dispersion relation

\begin{equation}
M_i(W) = M_i^B + \frac{1}{\pi} \int \frac{\text{Im} M_i(W')}{W'-W} dW',
\end{equation}

where $M_i^B$ is the contribution of the Born terms to the same multipole.

The solution of eq. (32) is proportional to $M_i^B$ and thus is a linear function of $F_A(q^2)$. Thus (*) an analysis of «single pion» events gives in the end an estimate of $F_A(q^2)$.

This analysis is much more elaborate and perhaps less valid than that on the elastic events, due to the various assumptions and approximations.

(*) The induced pseudoscalar term, which is proportional to the lepton mass, does not contribute appreciably to the axial vector matrix element.
Fig. 3. The $q^2$ distributions of neutrino events observed in the CERN spark chamber [9] and selected according to the criterion: $E_\nu \simeq 1.4$ GeV; $\cos \theta \simeq 0.8$.

The curves give the theoretical distributions estimated for the elastic events + inelastic background for different values of $M_A$.

which are involved in the calculations. However, if it is assumed to be valid and the theoretical curves thus obtained are fitted on the experimental distributions, one obtains (assuming $F_A(q^2)$ to be as in eq. (29)):

a) experiment using the C$_3$H$_8$-filled bubble chamber:

for the reactions $\nu+p \rightarrow \mu^-+\pi^+ + p$, $M_A = 1.250 \pm 0.350$ GeV/c$^2$ for the reactions $\nu+N \rightarrow \mu^-+\pi^-+N'$, $M_A = 0.850 \pm 0.250$ GeV/c$^2$

b) experiment using the freon-filled bubble chamber

for the reaction $\nu+N \rightarrow \mu^-+\pi^-+N'$, $M_A = 0.900 \pm 0.250$ GeV/c$^2$

where $N'$ or $N''$ indicate a nucleon, either a proton or a neutron. Thus within these large limits of uncertainty it appears that the «axial-vector radius» of the nucleon does not differ appreciably from the vector form factor.

Theoretical predictions on the form of $F_A(q^2)$ are rather vague at present. Sum rules, connecting the nucleon form factors (and hence $F_A(q^2)$) to the inelastic structure factors have been obtained by Adler [5]. «Structure factors» are the quantities $\alpha(q^2, W)$, $\beta(q^2, W)$, $\gamma(q^2, W)$ in the expression of the cross-section

$$
\frac{d^2\sigma}{d\Omega dE_1} (\nu+N \rightarrow \mu^-+H) = \frac{G^2 \cos^2 \theta_C}{(2\pi)^2 E_\nu} E_1 \left[ q^2 \alpha + 2E_\nu E_1 \cos \frac{q^2}{2} \beta - (E_\nu + E_1) q^2 \gamma \right].
$$
where \( l \) indicates a lepton, \( q^2 = (p_\nu - p_l)^2 \), \( E_\nu \) the neutrino energy, and \( \varphi \) the angle of emission of the charged lepton with respect to the neutrino; \( W \) is the mass of the final hadronic system \( H \), \( \theta_C = \theta_A \approx \theta_\nu \). However \( \alpha, \beta, \gamma \) are far less known than \( F_\lambda(q^2) \) and also less easy to be determined over a wide range of \( q^2 \) and \( W \). Thus these rules—which are in fact tests of local commutation relations—are in general of little help to determine the axial-vector coupling.

4.2. Low-energy single pion electroproduction. - Symmetry considerations involving weak and electromagnetic interactions, suggest the existence of similarities between process (31) and the process

\[
(33) \quad e + \pi^0 \rightarrow e + \pi^0 + \pi,
\]

so that one has to expect to be possible to express the amplitudes for both processes in terms of common form factors, namely the vector and axial vector form factors.

Under the hypothesis of a single photon exchange between the electron and the hadronic system, process (33) is equivalent to a single pion photo-production process induced by an off-mass-shell photon so that one has to expect the existence of both transverse and longitudinal amplitudes [15]. Furthermore, for given initial and final electron four-momenta, the e.m. radiation possesses a well-defined polarization state described by the polarization parameter

\[
(34) \quad \mathcal{E} = \left[ 1 + 2 \frac{|k|^2}{k^2} \tan^2 \frac{\psi}{2} \right]^{-1},
\]

which measures the transverse linear polarization of the virtual photon. Here \( k_\mu \) is the photon four-momentum and \( \psi \) the laboratory electron scattering angle.

The differential cross-section for scattering into the electron solid angle \( d\Omega_t \) measured in the laboratory and into the pion solid angle \( d\Omega_\pi \) measured in the \( \pi^-\pi^0 \) c.m., is given by

\[
(35) \quad \frac{d^3\sigma}{dE' d\Omega_t d\Omega_\pi} = \frac{\alpha}{2\pi^2 \frac{E'}{E}} \frac{E'}{k_0^2} \frac{|k|^2}{(1 - \mathcal{E})^{-1}} \frac{d\sigma_\psi}{d\Omega_\pi},
\]

where \( E, E' \) are the initial, final laboratory electron energies; \( |k|, k_0 \) are the laboratory photon 3-momentum and energy \( (k_0 = E - E') \); \( d\sigma_\psi/d\Omega_\pi \) is the \( \pi^-\pi^0 \) c.m. differential cross-section for pion production by a virtual photon.
and can be written as

\begin{equation}
\frac{|k|}{|q|} \frac{d\sigma_\varphi}{d\Omega_\pi} = A_T + \epsilon_L A_L + B \cos 2\varphi + C \cos \varphi,
\end{equation}

where $|q|$ is the pion 3-momentum; $\varphi$ is the angle between the planes of initial and final electrons and initial electron and final pion; $\epsilon_L = (k^2/k_0^2)\epsilon$. The first term represents the cross-section for pion production by an unpolarized, transverse virtual photon; the second term is the cross-section for pion production by a longitudinal photon; the third term arises from the interference between transverse states while the fourth from interference between longitudinal and transverse states.

That is all can be inferred by the hypothesis of a single photon exchange and by the use of the properties of the electromagnetic field. An evaluation of the $A_T$, $A_L$, $B$, $C$ coefficients implies dynamical considerations on the interaction of the e.m. field with the hadronic system and we have to expect, on general grounds, that they will depend on $k^2$, $q^2$, and $\theta_\pi$ (the $\pi-N$ c.m. angle between photon and pion). If eq. (35) is integrated over the $\theta$ and $\varphi$ variables one obtains

\begin{equation}
\frac{d^2\sigma}{dE'd\Omega_t} = \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{|k|}{k^2} (1 - \epsilon)^{-1} [\sigma_T(k^2, W) + \epsilon_L \sigma_L(k^2, W)],
\end{equation}

where

$$
|k| \sigma_T = \pi \int_0^\pi A_T d(\cos \theta_\pi), \quad |k| \sigma_L = 2\pi \int_0^\pi A_L d(\cos \theta_\pi),
$$

$$
W = \sqrt{m_\pi^2 + |q|^2} + \sqrt{M^2 + |q|^2},
$$

and $\sigma_T$, $\sigma_L$ measure the total absorption cross-section for transverse and, respectively, longitudinal virtual photons. In the following (37) will be referred as «total cross-section».

By using the PCAC and current algebra hypotheses, it is possible to obtain definite predictions on the single pion electroproduction amplitude for the pion four-momentum $q_\mu \rightarrow 0$. Under this condition, that for external pions implies $m_\pi \rightarrow 0$, Adler and Gilman [16] and Riazuddin and Lee [17], have been able to obtain sum rules through the comparison of a standard dispersion calculation with the PCAC-current algebra approach.

In general, an evaluation of a zero pion mass amplitude runs through the following steps [8]:
a) An off-mass-shell amplitude is defined and by using the PCAC hypothesis a reduction formula is obtained as a sum of two terms: for process (33) one term contains the fourth-component of the axial current-vector current commutator (equal-time commutator) and the other contains the axial current-vector current time-ordered product.

b) Current algebra commutators can then be used to obtain the term due to the equal-time commutator while, when \( q_\pi \to 0 \), in the time-ordered product, only the single nucleon pole survives to which continuum contributions beginning at \( q^2 = -9m^2_\pi \) have to be added. For elastic \( \pi^-N \) scattering, for instance, in the \( q_\pi \)-complex plane, the single nucleon pole is at \( \text{Re}q_0 = -m^2_\pi/2M \) while the threshold unitarity cut starts at \( \text{Re}q_0 = m_\pi \). When \( m_\pi \to 0 \) both the single nucleon pole and the threshold go toward \( q_0 \to 0 \) so that one can expect that the amplitudes evaluated for \( q_\pi \to 0 \) (including only the single-nucleon pole contribution) represent a good approximation to the low energy physical amplitudes. It has to be noticed that the pole terms obtained in the limit \( q_\pi \to 0 \) do not contain the pion pole.

c) An estimate of the low energy amplitude on the mass shell is then performed extrapolating from the zero-pion-mass expression.

The first step, as a definition, implies a certain amount of arbitrariness. The problem of the extrapolation of the zero-pion-mass result to physical pions is not trivial and the procedure to solve it is not unique. In general, the used procedures are all based on the assumption that the off-mass-shell amplitude is a smooth function of \( q \) as would be expected in a perturbation expansion, based on a Lagrangian field theory for which the PCAC hypothesis holds.

We quote here two extrapolation methods which allow to obtain definite predictions for process (33):

**Method I.** Balachandran et al. [18], following a proposal of Sugawara [19] and Suzuki [20], obtain local statements about physical quantities from current algebra and are able to approximate the physical amplitude in terms of the value of the off-mass-shell amplitude and its derivatives evaluated at an appropriate unphysical point.

**Method II.** To perform the extrapolation to physical pions, Fubini and Furlan [21] propose the use of mass dispersion relations and give definite prescriptions on the path along which it is convenient to extrapolate, namely a path along which the amplitudes are almost constant. The Fubini-Furlan method gives predictions for physical amplitudes in defined points of the physical region: for process (33) this is done at the nucleon Breit-threshold.

Method I predicts the pion-nucleon scattering length with a precision
of $\sim 10\%$. Method II predicts the same physical magnitudes with a precision of 10 to 20\%, while applied to threshold single pion photoproduction predicts the $\pi^+$ threshold matrix element to better than 10\% and remarkably well the threshold $\pi^-/\pi^+$ ratio.

Method I has been used by Gleeson et al. [22] to evaluate the positive pion electroproduction amplitude at the physical threshold. At the physical threshold, we have

$$
\left[ \frac{k}{q} \sigma_\perp^{+n}(q^2, q = 0) \right]_{q=0} = |E_{0+}^{+n}(q^2, q = 0)|^2
$$

$$
\left[ \frac{k}{q} \sigma_\perp^{+n}(q^2, q = 0) \right]_{q=0} = |L_{0+}^{+n}(q^2, q = 0)|^2,
$$

$E_{0+}, L_{0+}$ being the transverse and longitudinal electric dipole $J = \frac{1}{2}$ transition amplitudes.

The Gleeson et al. predictions are

$$
E_{0+}^{+n}(k^2, q = 0) = \sqrt{2} \Omega \left\{ \mu_n G_\perp^{+n}(k^2)[2\Delta F_1(k^2) - F_A(k^2)] \left( 1 + \frac{2M^2}{k^2} \right) \right\}
$$

$$
L_{0+}^{+n}(k^2, q = 0) = \sqrt{2} \Omega \left\{ \left[ 1 + \frac{k^2}{4M^2} (\mu_p - \mu_n) \right] G_\perp^{+n}(k^2) - \Delta F_1(k^2) \left( 1 + \frac{k^2}{4M^2} \right) + \left[ \frac{5}{2} \Delta F_1(k^2) - 2F_A(k^2) \right] \left( 1 + \frac{2M^2}{k^2} \right) \right\},
$$

where

$$
\Delta F_1(k^2) = F_1^{+}(k^2) - F_1^{-}(k^2)
$$

$$
\Omega = \frac{k^2}{c} \frac{\mu^2}{2M^2} A^{+} \frac{(4M^2 + k^2)^{\frac{3}{2}}}{2M^2 + k^2}
$$

$$
G_\perp^{+n}(k^2) = F_\perp^{+}(k^2) - \frac{k^2}{4M^2} F_\perp^{-}(k^2)
$$

and $c$ is defined through the PCAC relation $\bar{c}_\mu A_\mu^+ = \sqrt{2}|c_+|\bar{c}^+$. The authors claim that, within the method, the amplitudes are correct to better than 15\%.

Method II has been used by Furlan et al. [23] to evaluate the single $\pi^+$ electroproduction amplitude at the nucleon Breit-threshold. For $k^2 \ll 10^4$ fm$^2$ the $|q|$ values corresponding to the nucleon Breit-threshold range from $\sim 10$ to $\sim 35$ MeV/c so this prediction can be easily used, by only introducing kinematical factors, to evaluate the physical threshold amplitude. Their result depends on $F_A$ and $F_p$, but using the relation between them given by
the pion pole dominance hypothesis it can be written as

\[ E_{0+}^\pi(n, q = 0) = \frac{E}{W} \left[ \frac{1}{f_\pi} \left( \frac{1}{2} F_A(t) + \frac{t}{8E^2} G_M^V(t)G_A(0) \right) + \delta \right] \]

\[ L_{0+}^\pi(n, q = 0) = \frac{\sqrt{-k^2}}{W} \frac{M}{\mu} \left\{ \frac{1}{2} \frac{g_{\pi\eta\eta}}{1 - t/m_\pi^2} + \gamma \right\}, \]

where \( F_A(t) \) is the axial-vector form factor of the nucleon; \( G_M(t) = F_Y^V + F_2^V \) is the Sachs nucleon form factor; \( t = k^2 - m_\pi^2 - 2m_nk \) is the nucleon four-momentum transfer at threshold; \( \delta \) and \( \gamma \) are corrections to the main soft pion term and the authors are able to give a precise recipe to evaluate them. Both the above formulas, at the limit \( k^2 \to 0 \), reproduce the features of the Kroll-Rudermann theorem.

For the reaction \( e+p \to \pi^0+n+e \), Amaldi et al. [24] have recently published an experimental result of the threshold amplitude at \( k^2 = 5\text{fm}^{-2} \). The experiment was a measurement of total cross-sections in the interval of \( |q| \) from 30 to 80 MeV/c. The experimental apparatus is shown in Fig. 4. It consists of an electron magnetic channel through which \( k^2 \) and \( q \) are determined for fixed incident energy. By detecting the coincidences of electrons with the protons of the concomitant processes

\[ e+p \xrightarrow{\gamma} \pi^0+e+p \]

it is possible to evaluate what fraction of the single electron arm rate is attributable to process (33). As a result of this subtraction method, the authors obtain the data of Fig. 5 already corrected for radiative corrections. By fitting the data with a fourth-order \( |q| \) polynomial, the authors obtain, at \( 5\text{fm}^{-2} \)

\[ \lim_{|q| \to 0} \left[ \frac{1}{|q|} \frac{d^2 \sigma}{d \Omega dE'} \right] = (4.9 \pm 0.7) \times 10^{-31} \frac{\text{cm}^2}{\text{sr (GeV)}^2/c}, \]

which is proportional to \( |E_{0+}^\pi(n, |q| = 0)|^2 + \delta_L|L_{0+}^\pi(n, |q| = 0)|^2 \). A comparison of this experimental result with the Gleeson et al. predictions where use is made of the axial form factor parametric representation

\[ F_A = \left(1 + \frac{k^2}{M_A^2}\right)^{-2} \]

gives

\[ M_A = (1.03 \pm 0.07) \text{ GeV}. \]
The fit made by using the Furlan et al. amplitude gives a similar value of $M_A$.

To decide the most appropriate extrapolation procedure, a comparison of the predictions with the threshold electroproduction amplitude is more efficient than with low energy pion-nucleon scattering and threshold single pion photoproduction. As a matter of fact, the electroproduction physical threshold amplitude is a function of the virtual photon mass and a more complete experimental investigation of the threshold region could indicate the best representation of the amplitude and, in the mean time, give an efficient way to measure $F_A(k^2)$. The extension of the results to a range of $k^2$ is now in progress by using an electron-neutron coincidence method.
Fig. 5. – Experimental results on $\pi^+$ electroproduction ($e+ p \rightarrow e+ n+ \pi^+$) near threshold referring to two different settings of the primary electron energy. 

($\bullet$): $E = 800$ MeV; ($\circ$): $E = 780$ MeV.

Furthermore, it would be useful to allow a still more detailed comparison with the predictions, obtaining separate experimental informations on the threshold transverse and longitudinal amplitudes. This could be accomplished by performing measurements, at the same $k^2$ value, for different values of $\sigma$ but unfortunately this method encounters serious counting rate difficulties. Another method could consist in measuring the c.m. angular distributions of pions for low values of $|\mathbf{q}|$. In fact, for fixed $k^2$, $|\mathbf{q}|$ and $\theta_\pi$, the $q$ distributions gives $A = A_T + \delta_L A_L$, $B$, and $C$ separately. Limiting ourselves to $s$ and $p$ waves we obtain for $A$, $B$, and $C$ expressions of the type

$$ |\mathbf{q}|^{-1}A = A_0 + A_1 \cos \theta_\pi + A_2 \cos^2 \theta_\pi $$
$$ |\mathbf{q}|^{-1}B = \sin \theta_\pi (B_0 + B_1 \cos \theta_\pi) $$
$$ |\mathbf{q}|^{-1}C = C_0 \sin^2 \theta_\pi , $$

where $A_0$, ... are only functions of $k^2$ and $|\mathbf{q}|^2$. At threshold only $A_0$ is different from zero. Of the other coefficients can be measured the slopes at
threshold. Having obtained these experimental informations, one can see by using a multipole expansion that it is possible, within reasonable hypotheses, to separate the threshold longitudinal and transverse amplitudes. An experiment is now in progress at NINA (Daresbury) to measure low energy angular distributions of the pion.

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Old Problems and New Ideas in Elementary Particle Physics.

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After twenty years of elementary particle physics we are still very far from having a satisfactory theory accounting for the different phenomena in this field.

Although one might feel that this situation is somewhat justified by our experimental knowledge of the spectrum and of the interactions of elementary particles, I think that the main reason is indeed theoretical.

In our field we are dealing with an extremely relativistic problem, where the binding energies are of the same order of the masses, so that creation and destruction of particles is the most usual phenomenon. This has not as yet allowed us (and probably never will) to isolate from the general structure some less complicated system for which to construct a simple self-consistent theory.

As an example we can consider a two-body process:

(I) \[ A + B \rightarrow C + D. \]

The relativistic nature of the problem requires that the same amplitude represents at the same time the «crossed» processes

(II) \[ A + \bar{C} \rightarrow \bar{B} + D \]

and

(III) \[ A + \bar{D} \rightarrow \bar{B} + C, \]

where \( \bar{A}, \bar{B}, \bar{C}, \bar{D} \) are the antiparticles of \( A, B, C \) and \( D \).

We denote by \( p_A, p_B, p_C, p_D \) the four momenta of the particles and define
the invariant quantities:

\[
\begin{align*}
    s &= (p_A + p_B)^2 \\
    t &= (p_A - p_C)^2 \\
    u &= (p_A - p_D)^2
\end{align*}
\]  

(1)

which represent the square of the c.m. energies in channels (I), (II), and (III), respectively.

We recall the kinematical relation

\[ s + t + u = \sum M_i^2. \]  

(2)

The situation can be thus represented by the triangular plot of Fig. 1, in which \( s \), \( t \) and \( u \) are represented by the distance of our point from the \( s \), \( t \) and \( u \) axes.

In the case of equal masses the three physical regions for processes (I), (II) and (III) are represented by the cross-hatched regions in the plot. Those regions are indeed disconnected. Since we are dealing with analytic functions

Fig. 1. – See text for explanation of symbols.
of $s$, $t$, and $u$, continuation between one physical region and another is possible. One of the fundamental problems in elementary particle physics is to construct a scattering amplitude which can be analytically continued from one region to the other and satisfies all fundamental physical constraints (like unitarity) in all regions.

The triangular plot in Fig. 1 provides us with a simple way of recognizing how much of relativistic dynamics is present in one problem. This is achieved by comparing the size of the fundamental $(s, t, u)$ triangle with some characteristic energy of the problem, for example the average distance (or better $M_1^2 - M_2^2$) between energy levels.

In questions of low energy nuclear physics the size of the triangle is of the order of the square of the mass of the nucleus, whereas the average level spacing is of the order of mega electronvolt. In this case one can live happily without worrying about the existence of crossed channels. On the other extreme, for pion-pion scattering, the size of the triangle ($\sim 4m_\pi^2$) is of the same order (and even smaller) as compared to $m_\rho^2 - m_\pi^2$. We thus face the much more difficult problem of providing an amplitude which is simultaneously «reasonable» in all three channels.

In particular, if we want to discuss a model in which the main effect is due to the exchange of resonant states in all channels we have to look for a crossing invariant generalization of the Breit-Wigner formula.

During the last few years a new point of view has been developed in this respect. This has led to the so-called «duality principle» which (in the case of two-body collisions) requires that the sum of all resonant contributions in the $s$ channel does automatically represent the sum of contributions to the crossed $t$ channel.

A simple, beautiful realization of duality is given by the Veneziano representation in which the two-body scattering amplitude has the form

\begin{equation}
A(s, t, u) = A_I(s, t) + A_{II}(t, u) + A_{III}(u, s),
\end{equation}

where

\begin{equation}
A_I(st) = \frac{1}{\alpha(s) - 1} \int x^{-\alpha(s) - 1} (1 - x)^{-\alpha(s) - 1} \, dx
\end{equation}

and analogous expressions for $A_{II}$ and $A_{III}$.

The exponent $\alpha(s)$ appearing in eq. (4) is a linear Regge trajectory

\begin{equation}
\alpha(s) = as + b.
\end{equation}
Each integer intercept \( \alpha(s) = n \) corresponds to the mass of one (or more!) resonance. Equation (5) represents the case in which all resonances are taken in the limit of zero width.

Let us go back to eq. (4); substituting \( x = 1 - y \) we obtain for \( A_f(s, t) \) the reciprocal form:

\[
A_f(s, t) = \int_0^1 (1 - y)^{-\alpha(s)-1} y^{-\alpha(t)-1} \, dy .
\]

If we now expand the term \( (1 - x)^{-\alpha(t)-1} \) (in eq. (4)) and in powers of \( x \) we obtain:

\[
A_f(s, t) = \sum_{\alpha=0}^{\infty} \frac{c_\alpha(t)}{\alpha(s) - n} .
\]

\( A_f(s, t) \) is thus written as an infinite sum on \( s \) channel resonances.

On the other hand, if we expand the term \( (1 - y)^{-\alpha(s)-1} \) (in eq. (6)) in powers of \( y \), we obtain for \( A_f(s) \) the completely equivalent form

\[
A_f(s, t) = \sum_{\alpha=0}^{\infty} \frac{c_\alpha(s)}{\alpha(t) - n}
\]

as an infinite sum of \( t \) channel resonances.

Besides the perfect duality property exhibited in eqs. (7) and (8) the Veneziano formula has many appealing features. The asymptotic behaviour follows the Regge law in all three channels; moreover all constraints due to super-convergence and finite energy sum rules are automatically satisfied.

A recent wonderful development has been the generalization of the expression for the two-body amplitude to processes with any number of external lines. Those amplitudes do again satisfy duality (which is now a much more stringent requirement because of the larger number of crossed channels) and exhibit the well-known multi-Regge behaviour in all channels.

All this might suggest that we are dealing, not only with a beautiful model for scattering amplitudes but, maybe, with the starting point of a new general theoretical scheme for elementary particle physics. Time will tell. At present the weakest point is the absence of unitarity. The resonant dual amplitudes look very much like generalized Born approximations. Until now, attempts of introducing unitarity in a systematic way have met with great difficulties.

Waiting for some new ideas which might get us out of this deadlock; some interest has been devoted to the more modest question of understanding the nature of the resonant states appearing in the dual models. It has been
found that the different intercepts:

(9) \[ \alpha(s) = n \]

do not correspond to single resonances, but to very degenerate states. For large \( n \) the degree of degeneracy increases as \( \exp\left[cn\right] \), i.e., \( \exp\left[c'\sqrt{s}\right] \). It appears that this rapid growth of the number of levels with energy is needed in order to satisfy all constraints related to duality.

This somewhat unexpected feature of the level structure of dual resonant models is by no means unreasonable. It tells us that the concept of a single resonances is a useful one only at sufficiently low energies (in the giga electron-volt range). At larger energies the number of levels for energy intervals becomes so great that we shall practically have to deal with a continuum in which the single levels will lose their individuality.

Thus we can use the dual resonant models as starting points of statistical considerations about average properties of such levels.

Let me close this short survey by an optimistic note. Although the still unsolved questions look formidable and many years of hard work might still be needed, I think that the new approach based on duality is a modest step in the right direction. This may finally lead to a satisfactory theoretical treatment of strongly interacting elementary particles.
High-Energy $e^+e^-$ Annihilation into Hadrons.

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1. In 1961 the Italian Institute for Nuclear Physics, under the Presidency of Amaldi, took the important decision to start the construction of Adone, an $e^+e^-$ storage ring designed to reach a total energy of 3 GeV. The machine has recently been successfully operated [1]. In the meantime spectacular developments in the field have taken place at Novosibirsk and at Orsay [2] thus encouraging the hope of further interesting experimental results.

Investigation of the theoretical aspects of $e^+e^-$ collisions had started very early in Rome and in Frascati [3, 4], leading to the conclusion that $e^+e^-$ storage rings, when available, would be of foremost importance to the development of high-energy physics. In the last Section of the paper in ref. [4], Cabibbo and I discussed two points that have appeared to be of some interest recently [5-8]: a) the connection of $e^+e^-$ cross-sections to hadronic contributions to vacuum polarization; b) the possible asymptotic behaviours of the cross-section. In this note I shall further develop such points essentially with the aims to provide a classification of cross-section behaviours and related sum rules, and to illustrate the possible underlying physical interpretation. In relation to the latter point we shall find interesting connections to the concept of compound field algebra (CFA). CFA has appeared of interest in an investigation of leading divergences in weak interactions relevant to a theory of the Cabibbo angle [9].

Using the Yang-Mills theory [10] and the developments by Lee, Weinberg and Zumino [11] to provide a frame for the discussion, we shall here suggest the classification summarized in Tables I, II and III. We hope that higher energy colliding beam experiments may bring decisive information in choosing among the alternatives presented in the Tables.

2. Here I shall briefly summarize some of the older results by Cabibbo and myself [4] which are relevant to the present discussion.
High-energy e⁺-e⁻ annihilation into hadrons

Table I. - Finite field algebra (FFA).

\[ \int d\sigma^2 \frac{\sigma^2}{g^2} < \infty \quad m_0^2 < \infty \quad \frac{Z}{Z_0} > 0 \]

\[ \int d\sigma^2 q(\sigma^2) < \infty \quad g_0 \neq 0 \quad Z > 0 \]

\( A'_{\mu\nu}(0) \mathcal{P} < \infty \)

(\( A'_{\mu\nu}(q) \mathcal{P} \) is the Feynman propagator for the gauge particle.)

Asymptotic behaviour: \( \sigma^2 q(\sigma^2) \to 0 \).

\( \sigma(s) \) vanishes more rapidly than \( s^{-6} \) (\(^*\)). (For instance \( \sigma(s) \sim s^{-6} (\log s)^{-3} \), etc.)

e.m. mass differences expected to be infinite (i.e., uncalculable); finite Schrödinger term in \( \mathcal{D}_0, \mathcal{J}_i \); finite e-number term in \( \{ \delta_{ij}, \mathcal{J}_i, \mathcal{J}_j \} \).

\(^*\) In c.m., \( s = 2E \).

We assumed that the analysis could be reasonably limited to lowest electromagnetic order, at least for a finite range of energies. One calls \( F \) a set of hadronic final states produced according to

\[ e^+ + e^- \to F \]

and \( \sigma_F(E) \) the cross-section for such a process at energy \( E \) of \( e^+ \) in c.m. (total energy in center of mass = 2\( E \)). The set \( F \) will contribute a term \( \pi_F(K^2) \) to the absorptive part, \( \pi(K^2) \), of the photon propagator. Here \( K^2 \) is the virtual photon momentum

\[ K^2 = -4E^2. \]

It was stressed in ref. [4] that, for any set of final states \( F \), \( \pi_F \) and \( \sigma_F \) are related through

\[ \sigma_F(E) = \frac{\pi_2}{E^2} \pi_2 (-4E^2) \]

and that the existence of such a relation was indeed one of the most interesting aspects of the theory of e⁺-e⁻ annihilation. Relation (3) is equivalent to the relation

\[ \sigma_F(4E^2) = \frac{E^4}{\pi^2 \alpha^2} \sigma_F(E), \]

between \( \sigma_F(E) \) and the contribution from the set \( F \) to the spectral function
Table II. – Divergent field algebra (DFA).

\[
\int \frac{d\sigma^2 \varrho(\sigma^2)}{\sigma^2} < \infty \quad m^2_0 \rightarrow \infty \quad \frac{Z}{Z_0} > 0 \\
\int d\sigma^2 \varrho(\sigma^2) = \infty \quad g_0^2 \rightarrow \infty \quad Z \rightarrow 0 \quad (Z_0 \rightarrow 0) \\
A'(\sigma^2)_0 < \infty \\
\varrho(\sigma^2) \rightarrow 0
\]

\(\varrho(\sigma^2)\) vanishes more rapidly than \(s^{-4}\) (for instance \(\varrho(\sigma) \sim s^{-4} (\log s)^{-2}\), etc.).

e.m. mass differences infinite; finite Schwinger term in \([j_0, j]\); infinite c-number term in \([\bar{\partial}_i j_i - \partial_i \bar{j}_i, \bar{j}_i]\).

\(\varrho(\sigma^2)\) which appears in the Lehman-Källen representation [12] for the vacuum expectation value of the e.m. current commutator

\[
\langle [j_{\mu}^{e.m.}(x), j_{\nu}^{e.m.}(0)] \rangle_0 = i \int_0^{\infty} d\sigma^2 \varrho(\sigma^2) \left( \delta_{\mu\nu} - \frac{1}{\sigma^2} \bar{\partial}_{\mu} \bar{\partial}_{\nu} \right) \Delta(x, \sigma^2).
\]

We also pointed out how different assumptions on the finiteness or lack of finiteness of relevant integrals involving \(\pi(K^2)\) (or equivalently, involving the spectral function \(\varrho\)) would lead to statements on the asymptotic behaviour of \(\varrho(E)\). In particular, some observable effects are known to depend on integrals

\[
\int_{a}^{\infty} \frac{\pi(-a)}{a^2} da.
\]

If one wants them to be finite, the integral (6) must be convergent or, equivalently,

\[
\int_0^{\infty} \frac{\varrho(\sigma^2)}{\sigma^6} d\sigma^2 < \infty,
\]

or, in terms of \(\varrho(E)\)

\[
\int_{0}^{\infty} dEE^{-1} \varrho(E) < \infty.
\]

Equation (6) implies a decreasing cross-section, a very weak statement presumably. (Under such conditions the hadronic vacuum polarization corrections to \(g - 2\) of the electron or of the muon, for instance, are finite.)
TABLE III. – Compound field algebra (CFA).

\[
\int d\sigma^2 \frac{\varrho(\sigma^2)}{\sigma^2} = \infty \quad \left( \frac{m_0^2}{g_0} \right)^2 \to \infty \quad \frac{Z}{Z_0} \to 0
\]

\[
\int d\sigma^2 \varrho(\sigma^2) = \infty \quad \left( \frac{m_0^2}{g_0} \right)^2 m_0^8 \to \infty \quad Z \to 0
\]

\[\Lambda'_{\mu}(0)_F = \infty\]

We consider what we call the « standard realization » of CFA: \( m_0^2 \to \infty \) and \( g_0 \to \text{constant} \).

In this case \( \int d\sigma^2 \frac{\varrho(\sigma^2)}{\sigma^2} \sim m_0^8 \) and \( \int d\sigma^2 \varrho(\sigma^2) \sim m_0^4 \).

\( \varrho(\sigma^2) \sim \sigma^2 \) asymptotically: \( \sigma(s) \sim 1/s^3 \).

e.m. mass differences finite: infinite Schwinger term in \([i_0, j_i]\); infinite c-number term in \([\partial_0 j_i - \partial_i j_0, j_i]\).

A more stringent statement was derived in ref. [4] from the assumption that

\[
\int_0^\infty \frac{\pi(-a)}{a} \, da ,
\]

be finite. Such an assumption is connected to the possible finiteness of the hadronic contributions to charge renormalization. From eqs. (3) and (4), one would obtain

\[
\left(7'\right) \quad \int_0^\infty \frac{\varrho(\sigma^2)}{\sigma^4} \, d\sigma^2 < \infty ,
\]

or equivalently

\[
\left(7''\right) \quad \int dE E \varrho(E) < \infty .
\]

The cross-section in this case would have to decrease faster than \( E^{-2} \), say, \( \sim E^{-2}(\log E)^{-2} \), etc.

3. – The spectral representation of eq. (5), taken at equal times, gives directly

\[
\left(8\right) \quad \delta(x_0) \langle [j_{\mu}^{a.m.}(x), j_{\nu}^{a.m.}(0)] \rangle_0 = (\delta_{\mu 4} \delta_{\nu 4} + \delta_{\nu 4} \delta_{\mu 4}) \partial_4 \delta(x) \int_0^\infty \frac{d\sigma^2}{\sigma^2} \varrho(\sigma^2) .
\]

Equation (8) is one of a class of sum rules [13] that have been intensively exploited during last years. Equation (8) can be taken as a proof for the
existence of Schwinger terms in the commutator between a space component and the time component of a local current. The conservation of the current is irrelevant to such a proof. For a conserved current, Schwinger's original argument \[14\], makes the argument quite transparent: The limit

\[
\lim_{x \to 0} \langle [j_0(0), [H, j_0(x)]] \rangle_0 \delta(x_0),
\]

vanishes only if \( j_0(x) \mid 0 \rangle = 0 \), \( i.e., \) for a vanishing current; for a conserved current the limit is

\[
\lim_{x \to 0} \partial_1 \langle [j_0(0), j_1(x)] \rangle_0 \delta(x_0)
\]

and its nonvanishing contradicts the naive calculation of the commutator with currents taken as bilinear forms of fields at the same space-time point. One solution is to redefine the currents by introducing an infinitesimal space-like separation \( \varepsilon_{\mu} \) between the arguments of the fields, to formally compute the commutators, and to let \( \varepsilon_{\mu} \to 0 \) isotropically (\( \varepsilon \)-limit procedure). This procedure (which may still be deceptive because it is based on the unrenormalized fields; see, however, our discussion later) suggests a quadratically divergent vacuum expectation value for the Schwinger term

\[
\delta(x_0) \langle [j_0(x), j_1(0)] \rangle_0 \sim \frac{1}{\varepsilon^2} \frac{\partial}{\partial \varepsilon} \delta(x).
\]

We also recall that in quantum field theory Schwinger terms are seen to be related to the so-called seagulls \[15\]. Such a relation follows here directly from gauge invariance.

4. – Sum rules for higher moments of the spectral function

\[
\int d\sigma^2 \sigma^{2N} \varrho(\sigma^2),
\]

can formally be obtained in terms of multiple commutators

\[
\langle [[[Q_\mu(0), P_\alpha] P_\beta] \ldots P_\psi j_\nu(0)] \rangle_0,
\]

where \( P_\mu \) is the total four-momentum operator and \( Q_\mu(t) = \int j_\mu(x) d^4x \). Generally such sum rules will be divergent. Even so they may be of value in suggesting the asymptotic behaviour of \( \varrho(\sigma^2) \) or, equivalently, of the cross-sections. To illustrate the derivation consider, for instance,

\[
C_{\mu \alpha \beta \gamma} \delta(x_0) = \langle [[[Q_\mu(0), P_\alpha] P_\beta] P_\gamma j_\nu(0)] \rangle_0 \delta(x_0) =
\]

\[
= -i(2\pi)^3 \delta(x_0) \delta_{\alpha \beta} \delta_{\gamma \mu} \langle j_\mu(0) H^3 \delta(P) j_\nu(0) + j_\nu(0) H^3 \delta(P) j_\mu(0) \rangle_0, \quad (P_4 = iH).
\]
Inserting the spectral representation, in the form in eq. (5), one can write

\begin{equation}
C_{\mu \nu} \delta(x_0) = \frac{1}{2\pi} \int \frac{d^4x}{d^4p} \delta(\vec{p} \cdot \vec{x}) \left\langle \delta_j^{\mu \nu}(x) \right| \delta_j^{\mu \nu}(0) \right\rangle_0
\end{equation}

which is the desired sum rule.

5. – The spectral function \( g(\sigma^2) \) can be obtained from eq. (5)

\begin{equation}
0(q) \propto (-q^2) q^2 = \frac{1}{2\pi} \int d^4x \exp \left[ -i q \cdot x \right] \left\langle \frac{\partial j_\mu^{\text{em}}(x)}{\partial x_\nu} \right| \frac{\partial j_\mu^{\text{em}}(0)}{\partial x_\nu} \right\rangle_0
\end{equation}

In the c.m. frame

\begin{equation}
0(q) \propto (-q^2) q^2 \delta(q) = \frac{1}{3} \frac{1}{2\pi} \int d^4x \exp \left[ -i q \cdot x \right] \left\langle j_\mu^{\text{em}}(x) j_\mu^{\text{em}}(0) \right\rangle_0
\end{equation}

which, among other things, exhibits clearly the relation written in eq. (4) for each set of intermediate states \( F \).

In applying the sum rules one may note that, in the lowest order electromagnetic approximation that we are adopting here, one can separately treat the isovector and the isoscalar terms in \( j_\mu^{\text{em}} \). The sum rule in eq. (8) then gives, calling \( 2E = s \),

\begin{equation}
\delta(x_0) \left\langle [j_0^{\text{isov}}(x), j_0^{\text{isov}}(0)] \right\rangle_0 = \frac{1}{16\pi^2} \frac{i}{2\pi} \delta(x) \left\langle S^{\text{isov}} \right\rangle_0 = \frac{s}{16\pi^2 x^2} \delta(x) \int_0^\infty ds^2 s^2 \sigma(s)^{\text{isov}}
\end{equation}

and a completely similar equation with « isovector » substituted by « isoscalar ». In eq. (18) the Schwinger term has been called \( S \). On the basis of (asymptotic) \( SU_3 \) one can try to use the relation: \( 3 \left\langle S^{\text{isov}} \right\rangle_0 = \left\langle S^{\text{isov}} \right\rangle_0 \) or

\begin{equation}
\int_0^\infty ds^2 s^2 [\sigma^{\text{isov}}(s) - 3\sigma^{\text{isov}}(s)] = 0
\end{equation}

Equation (19) suggests a persistent oscillatory character of \( \Delta \sigma = \sigma^{\text{isov}} - 3\sigma^{\text{isov}} \), or a sharp decrease of such a difference. From the view point of saturation with resonances it would not be unrealistic to think of a persistent simultaneous occurrence of \( T = 1 \) and \( T = 0 \) vector mesons with relations among residue such that eq. (19) is satisfied.

The model for the currents we had examined in Sect. 4, in terms of bilinear expressions in spin \( \frac{1}{2} \) fields evidently suggests a quadratic behaviour in energy of the divergent integral in eq. (18). This means \( \sigma(s) \propto s^{-2} \) for large \( s \).
(remember, however, we are only including one-photon exchange). The same result would obtain when including currents formed out of spin zero bosons. An alternative argument for such a behaviour follows from the requisite of gauge-invariance on the vacuum polarization tensor $\pi_{\mu\nu}(q)$. The gauge invariant form is notoriously \[16\]

\begin{equation}
\pi_{\mu\nu}(q) = \int \left[ \frac{d^4x}{(2\pi)^4} \right] \text{exp}[iqx] \langle Tj_{\mu}(x)j_{\nu}(0) \rangle_0 - i\delta_{\mu\nu}q_0 \int \frac{d\sigma^2}{\sigma^2} g(\sigma^2)
\end{equation}

and gauge invariance requires

\begin{equation}
q_0 \int \left[ \frac{d^4x}{(2\pi)^4} \right] \text{exp}[iqx] \langle Tj_{\mu}(x)j_{\nu}(0) \rangle_0 = \delta_{\mu\nu}q_0 \int \frac{d\sigma^2}{\sigma^2} g(\sigma^2)
\end{equation}

The left-hand side of eq. (21) can easily be calculated in the renormalized quantum electrodynamics of spin $\frac{1}{2}$ fermions and seen to be quadratically divergent. A behaviour $g(\sigma^2) \propto \sigma^2$ is equivalent to $\sigma(s) \propto s^{-2}$ (recall that $s = 2E$). The lack of covariance (besides gauge invariance) of the left-hand side of eq. (21) is an example of a frequent situation with divergent Feynman integrals. In the notation in eq. (18) higher moment sum rules are of the form

\begin{equation}
\langle [H, Q_1^{\text{isov}}], j_1^{\text{isov}}(0) \rangle_0 = \frac{1}{16\pi^2} \int_0^\infty ds^2 s^4 \sigma(s)^{\text{isov}},
\end{equation}

\begin{equation}
\langle [H[H, Q_1^{\text{isov}}], j_1^{\text{isov}}(0) \rangle_0 = \frac{1}{16\pi^2} \int_0^\infty ds^2 s^6 \sigma(s)^{\text{isov}},
\end{equation}

and quite similar equations with «isoscalar» substituted for «isovector». When applied to a model of bilinear currents from spin $\frac{1}{2}$ fermions these equations appear all consistent with the above behaviour, $g(\sigma^2) \propto \sigma^2$. (Of course the result may not hold when singular interactions are present.)

6. – A rigorous Lagrangian scheme of vector dominance, including proper treatment of gauge-invariance, is that of Kroll, Lee and Zumino \[17\]. It rests on the idea of field-current identity. To illustrate the main point let us limit ourselves to the $\varphi$-meson and its strong interaction. The Lagrangian density is supposed to be of the form

\begin{equation}
-\frac{1}{2} m^2 \varphi_{\mu} e_{\mu} + \mathcal{L}',
\end{equation}

where $\mathcal{L}'$ is invariant under $\varrho_\mu \to \varrho_\mu + g^{-1} \varrho_\mu A$ and a corresponding transformation on the matter fields. The (gauge-invariant) prescription is $\varrho_\mu \to \varrho_\mu + (e/g) A_\mu$ inside $\mathcal{L}'$ (of course only isovector photons are included). The equations of motions

\begin{align}
\frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{e}{g} \frac{\partial \mathcal{L}'}{\partial \varrho_\mu} &= 0, \\
-m_\varrho^2 \varrho_\mu + \frac{\partial \mathcal{L}'}{\partial \varrho_\mu} &= 0,
\end{align}

establish the required field-current identity. One performs a wave-function renormalization $\varrho_\mu^0 = \sqrt{Z} \varrho_\mu$ and introduces $Z_0 = (m/m_0)^2$. The renormalized and unrenormalized sources $J_\nu$ and $J^0_\nu$ for the $\varrho$-meson are directly related

\begin{align}
J^0_\nu &= J_\nu + (1 - Z_0) \frac{1}{g} \frac{\partial G_{\nu\mu}}{\partial x_\mu} = - \frac{m_\varrho^2}{g} \left( \varrho_\nu - \frac{1}{m_0^2} \frac{\partial G_{\nu\mu}}{\partial x_\mu} \right),
\end{align}

where $G_{\mu\nu}$ is the field tensor for the $\varrho$-meson and $g$ its renormalized coupling. (One conventionally defines the unrenormalized coupling as $g_0 = g \sqrt{Z} Z_0^{-1}$.) The important observation is that for $m_\varrho \to \infty$, the e.m. current (identical as we have said to the field) becomes identical to the unrenormalized current $J^0_\nu$. This situation, current-current identity, had been studied by Gell-Mann and Zachariasen [18]. Always limiting to the above Abelian case one verifies how the spectral function sum rules now follow from the canonical commutators and the field equations. For instance, the sum rule in eq. (22) follows directly from the spectral representation

\begin{align}
\langle [G_{\mu\lambda}(x), \varrho_\nu(0)] \rangle_0 = i \int d\sigma^2 \varrho(\sigma^2) (\delta_{2\nu} \varrho_\lambda - \delta_{\nu\lambda} \varrho_2) A(x, \sigma^2)
\end{align}

and the canonical commutator

\begin{align}
\delta(x_0) [G_{\mu\lambda}(x), \varrho_\nu(0)] = \frac{1}{Z} \delta_{ij} \delta(x).
\end{align}

The non-Abelian situation ($SU_2, SU_2 \times SU_2, SU_3, SU_3 \times SU_3$) [11] presents a formal difficulty connected to the occurrence of ambiguous terms proportional to bilinear expressions in the gauge fields taken at the same space-time point in commutators of fields and their time derivatives. Such products are not well defined, nevertheless they presumably contribute a vanishing vacuum expectation value. [A rough argument is: $\langle \varphi_\mu(x) \varphi_\lambda(x) \rangle_0$ should be proportional to $\delta_{\mu\lambda}$ on the basis of covariance, but $\langle \varphi_\mu(x) \varphi_\lambda(x) \rangle_0$ and $\langle \varphi_\mu(x) \varphi_\lambda(x) \rangle_0$]
have opposite sign. Furthermore in the non-Abelian cases (excluding $SU_2$) some currents are not conserved.

7. – In spite of the above remarks for the non-Abelian case we think it is useful to take the following attitude. We consider a general Yang-Mills theory and include in the discussion its limiting cases. This provides for a classification in terms of convergence or lack of convergence of the sum rules or, if one prefers, in terms of asymptotic behaviours of the $e^+ e^-$ annihilation cross-section. Alternatively one can present the classification in terms of limits on the bare quantities $m_0$ and $g_0$. (We have already discussed one realization of the situation $m_0 \to \infty$.) We shall briefly distinguish three cases:

1) finite field algebra (FFA);
2) divergent field algebra (DFA);
3) compound field algebra (CFA).

The three cases, 1), 2) and 3), are illustrated in Table I, II and III.

From the point of view of the cross-sections we thus have:

1) FFA: $\sigma(s)$ decreases faster than $s^{-6}$;
2) DFA: $\sigma(s)$ decreases faster than $s^{-4}$;
3) CFA: in the standard realization $\sigma(s)$ decreases as $s^{-2}$.

Note that only in FFA and in DFA one has a finite $\Delta'_{\mu\nu}$. In DFA and CFA $m_0^2 \to \infty$, i.e., the bare mass is infinite. The finiteness or lack of finiteness of the spectral integrals is a direct reflection of the respective finiteness or lack of finiteness of the two $c$-number terms appearing in the commutators

$$\delta(x_0)[j_{\alpha\beta}(x), j_{\beta\ell}(0)] \quad \text{and} \quad \delta(x_0)[\bar{\psi}_0 j_{\alpha\beta}(x) - \bar{\psi}_0 j_{\alpha\beta}(x), j_{\beta\ell}(0)].$$

In the latter commutator, besides a $q$-number Schwinger term transforming as the component of a four-vector and thus irrelevant here, there also appears a $q$-number $\delta$-function contribution which transforms as a reducible tensor. This last term is responsible for the e.m. mass differences [19]. Finally we note that the standard realization of CFA gives for the spectral integrals a behaviour identical to that obtained from currents bilinear in spin $\frac{1}{2}$ fermion fields. (If one wants one can call them « quarks » considering the current inflation in the use of such a word.) That the limit of vanishing renormalization constants is relevant to a composite particle picture emerges also from a number of field-theoretic investigations [20, 21]. We note that CFA came of interest to us to discuss the higher weak orders in our theory of the Cabibbo angle [9]. An extension of these concepts along the lines of Wilson’s approach to field theory [22] and employing approximate scale invariance has recently been developed [23].
REFERENCES


[21] The possible limits for $m_\pi \to 0$ of the Yang-Mills theory are also of great interest: see R. Brandt and J. D. Bjorken: *Phys. Rev.*, 177, 2331 (1968) where the connection with the Sugawara model is also discussed.


New Frontiers of High-Energy Physics.

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1. – Introduction.

One of the « frontiers » of physics has been and will be the study of phenomena at always decreasing small distances. Progress in this field has been swift: The frontier passed from the study of molecules to that of atoms, then to nuclei and to elementary particles. In terms of distances, one passed from distances of the order of $10^{-8}$ to $10^{-14}$ cm. The present frontier is the study of the so-called elementary particles; in particular we want to know if they are really elementary or composite.

The progress has always been connected with the development of new accelerators of ever increasing energies. They allowed the production of secondary beams of higher energies and of beams of altogether new particles. Atomic phenomena could be studied with photon and electron beams of eV energies. The study of nuclear phenomena required beams in the MeV range and new types of accelerators: Cockroft-Walton's, Van der Graaf's, cyclotrons, and betatrons produced the appropriate proton, electron, and photon beams. Another energy step and different types of machines were required before the new field of elementary particles could be entered; new types of objects, unstable ones, were produced, which did not exist in nature. Synchro-cyclotrons and electron-synchrotrons produced $\pi$ mesons; the 3 GeV Brookhaven cosmotron produced K-meson beams and allowed the study of various hyperons. At the 6 GeV Berkeley bevatron the antinucleons were discovered. The 28 GeV CERN Proton synchrotron (PS) and the 33 GeV Brookhaven AGS, together with some of the lower energy machines, brought into full swing the study of the resonances between various types of particles, and allowed the production of new objects, such as the muon neutrino. The high-energy frontier is at the moment represented by the 76 GeV Serpukhov
IHEP accelerator, while colliding beam machines are opening up a completely new field.

The increase in energy between 1 eV to 33 GeV has brought about the discovery of three types of spectroscopies—the atomic, nuclear, and particle spectroscopies—whose study is best done with beams of appropriate energies: (1 ÷ 1000) eV for atoms, (0.1 ÷ 10) MeV for nuclei, (0.1 ÷ 10) GeV for particles. The energy regions in between may be considered as transition energies or asymptotic regions for the preceding spectroscopy. The 76 GeV machine seems to cover such a type of asymptotic region for particle physics.

The race toward higher energies will have as the next steps the 400 GeV proton accelerator of Batavia, and the European 300 GeV. A glimpse of what is going to happen at much higher energies will be offered by the CERN 25 GeV pp ISR colliding beam machine and the 20 GeV pp Novosibirsk machine.

Professor Edoardo Amaldi works on the frontier, in particular looking for new phenomena [1], and he has been since a long time the Chairman of the European Committee for Future Accelerators (ECFA). It is also because of his efforts that European laboratories can work right on the frontier of high-energy physics.

In this note we shall discuss the recent beginning of the exploitation of the 76 GeV IHEP accelerator, comparing layouts and the first experimental results with those from CERN and BNL. Finally, we shall make some considerations and extrapolations for the future accelerators.

2. - Accelerator beams.

The experiments that can be performed at an accelerator depend critically on the quality and quantity of the available secondary beams. In a PS, the beams may originate either from an internal target or from a target placed in an extracted proton beam. While the first system was used extensively in the past, the trend for the future seems to be that of switching from internal to external targets. The main reasons for this trend are connected with problems of radiation damage, which become serious when accelerators reach $10^{18}$ protons per pulse (ppp); moreover, external beams offer greater flexibility. On the other hand, beams originating in an internal target may have better optical qualities, because of the smaller transverse dimensions of the source.

Table I offers a comparison of the CERN and IHEP accelerators.

The absence of long straight sections in the IHEP accelerator means that most of the secondary particle beams pass through the magnetic field of the
Table I. — Some characteristics of the 28 GeV CERN Proton Synchrotron and of the 76 GeV IHEP Proton Synchrotron. 1972 projected values represent rough estimates. Of the two numbers indicating the number of beams, the first gives the actual number of beams, irrespective of branches, while the effective number of beams is computed using the Brookhaven criteria, keeping in mind compatibility of operation and number of branches: i) a beam with two or more branches is worth 1.5 beams; ii) compatible beams are worth one each, while two non-compatible ones are worth only 1.

<table>
<thead>
<tr>
<th>Machine</th>
<th>CERN-PS</th>
<th>IHEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>1969</td>
<td>1969</td>
</tr>
<tr>
<td></td>
<td>1972 (projected)</td>
<td>1972 (projected)</td>
</tr>
<tr>
<td>Maximum energy</td>
<td>28 GeV</td>
<td>76 GeV</td>
</tr>
<tr>
<td>Normal energy for counter experiments</td>
<td>19 GeV</td>
<td>70 GeV</td>
</tr>
<tr>
<td>Average intensity per pulse</td>
<td>$10^{13}$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>2.4 sec</td>
<td>7 sec</td>
</tr>
<tr>
<td>Maximum burst length</td>
<td>0.5 sec</td>
<td>1.0 sec</td>
</tr>
<tr>
<td>Total number of counter beams</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>Effective number of counter beams</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Total number of bubble chambers</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Number of fast extracted proton beams</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>Number of slow extracted proton beams</td>
<td>1</td>
<td>—</td>
</tr>
</tbody>
</table>
Fig. 1. – Pressure curves (or mass spectra) at different beam momenta obtained with: a), b) differential Čerenkov counters, c) a combination of three threshold Čerenkov counters, d) a counter in the differential-threshold mode of operation.
3. – Particle separation at high energies.

Typical particle ratios in a negative, high-energy (*) unseparated beam are: \( \pi^-(1), \mu^- (10^{-2}), K^- (10^{-2}), e^- (10^{-3}), \bar{p}(10^{-3}) \); and in a positive beam: \( p(10), \pi^+(10^6), \mu^+(10^{-2}), K^+(10^{-2}), d(10^{-2}), e^+(10^{-3}) \). Since one is usually

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(*) In the following high energy will usually mean more than 5-10 GeV.
interested in only one of them, the problem arises of removing the others. Unwanted particles may either be physically removed by means of separators or simply not counted by electronics means.

The only types of separators available for high-energy beams are radio-frequency (RF) separators, which at the moment can be used only for bubble chamber beams, that is for beams of short time-duration.

The electronics separation of different types of particles is usually performed by time-of-flight techniques or by means of Čerenkov counters. The time-of-flight technique is limited by the present capability of resolving about 0.2 nsec. Many types of Čerenkov counters have been used for particle separation at high-energy [2]. The best resolutions achieved so far are of the order of $\Delta \beta < 10^{-5}$. Great care has to be exercised to achieve these resolutions: Threshold Čerenkov counters have to be many metres long; in differential counters, the Čerenkov light has to be chromatically corrected and the beam has to be quite parallel, so that a good performance of the Čerenkov counter also indicates an optically good beam. In a counter built at CERN, the index of refraction of the gas was measured by means of a laser-refractometer, providing an absolute measurement of the particle velocity and hence of the mass of the particle [2].

Figure 1 shows pressure curves at different momenta obtained by means of differential $a$, $b$, threshold $c$, differential-threshold $d$, and a combination of threshold and differential Čerenkov counter $e$). These curves show that present Čerenkov counter techniques allow the separation of kaons, when in 0.1% abundance, from pions up to 60 GeV/c (Fig. 1a-1d), and that a combination of Čerenkov counters is capable of rejection ratios of better than $10^5$ (Fig. 1e).

4. - Particle production.

One of the first experiments to be performed at a new accelerator is a measurement of the yield of different particles. These measurements are important both for practical purposes, as for instance the need for information for beam design, and on the techniques necessary to identify the particles, as well as to understand the physics of particle production at high energies.

Photon production is easily measured by means of a glass total absorption Čerenkov counter [3] or a sandwich-counter, where scintillator plates are alternated with lead plates. These systems are really total absorption calorimeters, where the total energy lost is simply given by a pulse-height measurement.

A study of the production of charged particles requires a system of Čerenkov counters as described in the previous chapter. Figure 2 shows a
Fig. 2. – Particle ratios $R$ vs. secondary beam laboratory momentum divided by the kinematically allowed maximum momentum of the heavier particle ($K^-$ and $\bar{p}$, respectively). The points represent the results of measurements performed at the IHEP accelerator operated at various energies [2]. Points corresponding to the same incident energy and secondary momentum correspond to different angles of production. The broken line represents the 19.2 GeV CERN data [4] for p-p collisions, which coincide with the dependence found by the same group for p-Al collisions. $E_0 = 70$ GeV (○), 52 GeV (△), 43 GeV (◇), 35 GeV (□), 20 GeV (＋), 19.2 GeV (——).
Fig. 3. – Laboratory spectra for $\pi^\pm$, $K^\pm$, $p$, and $\bar{p}$ produced in 19.2 GeV/c proton-proton collisions at 12.5 mrad laboratory angle [4]. The horizontal scale gives the secondary laboratory momentum, while the vertical scale gives the double differential cross-section in the laboratory frame.
compilation of negative particle ratios produced when the internal beam of energy between 20 and 70 GeV strikes an aluminium target [2]; the variable on the abscissa is \( \eta = p/p_{\text{max}} \), the secondary beam momentum divided by the maximum momentum the heavier particle (K\(^-\), or p) may carry, assuming the usual conservation laws of charge, baryon number, and strangeness. When plotted vs. \( \eta \), the particle ratios change very little from 20 to 70 GeV for large values of \( \eta \), while they increase slightly for smaller values of \( \eta \). It is interesting to notice that the ratio of K to \( \pi \) production remains small, even at energies so much larger than kaon threshold [2].

"Pionization", that is the production of a large number of pions, seems to be the dominant result of high-energy proton-proton collisions.

The measurements of the absolute fluxes from an internal target cannot be very precise; moreover the theoretical analysis is complicated by the presence of a complex nucleus. Refined measurements require an external proton beam and a liquid hydrogen target [4, 5]. Figure 3 shows the results of a recent CERN experiment performed along these lines.

Several production models have been proposed: They range from the statistical model with collective motion corrections [6], fireball mechanisms [5], etc., to semiempirical formulae of the types of Cocconi and Perkins [7].

These models give, for particle spectra, equations with a number of parameters to be determined experimentally. All the models are able to predict the pion data reasonably well, while the predictions of the kaon and antinucleon spectra are poorer. These models may at present be used as guiding lines, but are not expected to be very reliable at higher energies.

5. - Cross-section measurements.

The simplest cross-section measurement is the total cross-section; then follow, in order of complication, the elastic cross-section in the diffraction region, several two-body processes, etc.

5'1. Total cross-sections. - On the basis of the behaviour with energy of the total cross-sections, one may speak of two energy regions, the resonance and the high-energy regions, respectively [8]. In the first region (below 5 GeV), the total cross-sections are characterized by the presence of structures, most of which may be interpreted as resonances. In the high-energy region the total cross-sections are slowly varying functions of the energy and do not exhibit any appreciable structure. We shall be concerned with cross-section measurements in the second region. What is of interest here is the energy behaviour of the cross-sections and the relations among them. It was expected that in the limit of very high energies, hadron collisions do not depend
on the nature of the interacting particles, nor on the specific mechanism of strong interactions. In particular:

a) the ratios between the various total cross-sections should be governed by some internal symmetry;

b) the total cross-sections of the particles belonging to the same isospin multiplet; and

c) the total cross-section of particle and antiparticle should become equal (Pomeranchuk theorems [9]).

More specific models predict the behaviour of the cross-sections as functions of energy. At present their predictions are contradictory: some models predict that the cross-sections decrease toward constant non-zero values [10]; others predict that the cross-sections go to zero as the energy goes to infinity [11]; finally other models predict that the cross-sections reach a minimum and then rise to finite asymptotic values [12].

Figure 4 shows a compilation of the high-energy total cross-sections [8], including the recent CERN-IHEP results at the 76 GeV synchrotron [13].

![Figure 4](image)

Fig. 4. – A compilation of high-energy total cross-sections [8]. Data points are shown only for pp, pp, \(\pi^-p\), K\(^+\)p, and K\(^+\)p. The lines represent the results of the least squares fits of the total cross-sections above 5 GeV/c to the equation

\[
\sigma(p_{\text{lab}}) = \sigma(\infty) + c/p_{\text{lab}}.
\]
The total cross-sections for $\pi^- p$, $\pi^- n$ ($= \pi^+ p$), $K^- p$ and $K^- n$ seem to have become energy independent in the region above 30 GeV/c; the $K^+ p$ and $K^+ n$ cross-sections are already constant between 10 and 20 GeV/c, while the $\bar{p} p$ and $\bar{p} n$ cross-sections are still decreasing at 50 GeV/c.

The total cross-sections on protons and neutrons have become almost identical, suggesting that at these energies the strong interaction cross-sections are almost independent of isospin. The $K^- p$ and $K^+ p$ total cross-sections do not seem to come together as the energy increases.

The available high-energy data may be fitted to simple empirical formulae of the type:

\begin{align}
\sigma(p_{\text{lab}}) &= \sigma(\infty) + \frac{c}{p_{\text{lab}}^d} , \\
\sigma(p_{\text{lab}}) &= a p_{\text{lab}}^b .
\end{align}

The fittings to eq. (2) give worse $\chi^2$ than the fittings to eq. (1), suggesting that the data are in better agreement with finite asymptotic cross-sections. Table II gives the results of fitting the available data above 5 GeV/c to eq. (1) for $d = 1$. Both statistical and systematic errors have been taken into account by combining them quadratically. It is not clear how the differences between $K^- p - K^+ p$ and may be between $\bar{p} p$-pp can be reconciled within the framework of the existing theories. Also: the available data do not allow a definite conclusion about which model predicts correctly the energy behaviour. More experimental data are clearly needed.

**Table II.** - Least squares fits of total cross-sections to the formula $\sigma = c_0 + c_1/p_{\text{lab}}$ for $p_{\text{lab}} > 5$ GeV/c [8]. In order to obtain more satisfactory $\chi^2$ the systematic errors have been compounded quadratically with the statistical ones. No $\chi^2$'s are given for the $K^- n$ and $\bar{p} n$, since the fits come from interpolated data.

<table>
<thead>
<tr>
<th>Total cross-section</th>
<th>$c^0$</th>
<th>$c_1$</th>
<th>$\chi^2$</th>
<th>Number of points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^- p$</td>
<td>23.89 ± 0.04</td>
<td>27.56 ± 0.50</td>
<td>57.6</td>
<td>84</td>
</tr>
<tr>
<td>$\pi^+ p$ and $\pi^- n$</td>
<td>22.78 ± 0.06</td>
<td>19.40 ± 0.60</td>
<td>35.8</td>
<td>66</td>
</tr>
<tr>
<td>$K^- p$</td>
<td>20.18 ± 0.18</td>
<td>24.63 ± 2.22</td>
<td>22.1</td>
<td>30</td>
</tr>
<tr>
<td>$K^- n$</td>
<td>19.49 ± 0.31</td>
<td>6.13 ± 3.08</td>
<td>—</td>
<td>18</td>
</tr>
<tr>
<td>$K^+ p$</td>
<td>17.44 ± 0.24</td>
<td>-0.22 ± 2.42</td>
<td>31.6</td>
<td>20</td>
</tr>
<tr>
<td>$K^+ n$</td>
<td>17.63 ± 0.41</td>
<td>0.55 ± 3.54</td>
<td>—</td>
<td>11</td>
</tr>
<tr>
<td>$\bar{p} p$</td>
<td>42.44 ± 0.42</td>
<td>108.3 ± 6.1</td>
<td>24.8</td>
<td>24</td>
</tr>
<tr>
<td>$\bar{p} n$</td>
<td>42.48 ± 0.74</td>
<td>87.6 ± 8.2</td>
<td>—</td>
<td>17</td>
</tr>
<tr>
<td>$p p$</td>
<td>38.29 ± 0.08</td>
<td>15.24 ± 0.87</td>
<td>27.1</td>
<td>52</td>
</tr>
<tr>
<td>$p n$ and $n p$</td>
<td>36.48 ± 0.74</td>
<td>27.3 ± 5.9</td>
<td>21.9</td>
<td>16</td>
</tr>
</tbody>
</table>
The quark model prediction that the asymptotic \( \pi N \) total cross-section is \( \frac{3}{8} \) of the asymptotic \( pN \) cross-section, can be compared with 0.62, as obtained from Table II.

5.2. Elastic cross-sections. - The elastic cross-sections are either slowly decreasing or remain constant as functions of energy. This means that the opacities, defined as the ratios of elastic to total cross-sections, are either constant or slowly decrease with energy.

The elastic angular distributions may be subdivided into three regions:

a) the very small angle region where Coulomb and nuclear scattering interfere;

b) the diffraction region proper; and

c) the large-angle region, characterized by very small cross-sections.

Large counter hodoscopes or large spark chamber arrays are usually employed for measuring the elastic differential cross-section. A simpler experiment was recently performed at the IHEP accelerator by bombarding, with the internal proton beam, a polyethylene target [14] (*). The recoiling proton was detected by means of solid-state detectors, which measured angle and range. It was thus possible to separate elastic from inelastic events in the \(|t|\) range (0.01 \( \div \) 0.11) \((\text{GeV}/c)^2\) for proton incident momenta from 12 to 70 GeV/c. In this angular region the diffraction pattern is well represented by an exponential (Fig. 5):

\[
\frac{d\sigma}{dt} = ate^{bt},
\]

where the slope \( b \) changes monotonously from \( b \approx 10 \) \((\text{GeV}/c)^{-2}\) at 12 GeV/c to \( b = 11.5\) at 70 GeV/c (Fig. 6). Thus the shrinking of the diffraction peak initiated at energies around 5 GeV, continues when the energy is increased. Only higher energies will be able to tell if the shrinking goes on or eventually saturates. Within the framework of the Regge-pole theory, the elastic scattering data may yield the slope of the vacuum (Pomeranchuk) trajectory, which is found to be \( \alpha_p = 0.40 \pm 0.09 \) [14].

In another perspective the small angle elastic scattering is considered to arise from a diffraction mechanism, as the shadow of all the inelastic processes. In this framework, the simplest classical non-relativistic optical model

(*) In the future the polyethylene target will be substituted with a gaseous jet of hydrogen at supersonic speed, and the experiment will cover the Coulomb-nuclear interference region.
Fig. 5. – The differential proton-proton elastic cross-section at a laboratory kinetic energy of 58.1 GeV in the $0.01 < |t| < 0.11$ (GeV/c)$^2$ range [14].

(opaque dis) [8], predicts that the interaction radius, given by

$$r = 2\sqrt{b},$$

(4)

grows from 1.23 to 1.34 fm.

In more sophisticated optical models the size of the proton remains constant; the shrinking of the p-p diffraction pattern may then arise from the Lorentz contraction of the colliding particles in their direction of motion. The antishrinking of the $\bar{p}p$ elastic peak requires other hypotheses, such as that the total cross-sections are still decreasing and therefore one is nowhere near an asymptotic behaviour.

The opinion on the large angle scattering is even more divided: some authors suggest that it is purely diffractive, possibly arising from different spatial structures inside the nucleon; other authors suggest that a statistical mechanism may play some role.
Fig. 6. – The coefficient $b$ of eq. (3) for proton-proton elastic scattering in the diffraction region $[0.01 < |r| < 0.1 \text{ (GeV/c)}^2]$ vs. laboratory kinetic energy [14].

5.3. Other cross-sections. – The study of charge exchange cross-sections is particularly important because their theoretical analysis is simple, at least in the context of Regge-pole theory, where only one exchanged trajectory explains the main features of the data. Present experimental information stops at 18 GeV, but experiments will soon be done at higher energies.

Most of the available experimental results on the more sophisticated measurements are still not systematic, though a wealth of information is available.

It is clear that one is nowhere close to an understanding of strong interactions and that more and more experiments, particularly at higher energies, are required.

It may come as a surprise to learn that at the very high energies considered in this report, one may still obtain information on nuclear properties. The absorption cross-sections $\sigma_\alpha$ measured at 20 to 50 GeV/c with incident $K^-$, $\pi^-$, $\bar{p}$, and $\bar{d}$ on a variety of nuclei have revealed that their dependence on the atomic number $A$ is of the form [8, 13, 15] (Fig. 7):

$$\sigma = \sigma_0 A^\alpha,$$
where \( \alpha = 0.76, 0.75, 0.67, \) and 0.67, respectively for \( K^- , \pi^- , \bar{p} , \) and \( \bar{d} . \) The large value of \( \alpha \) for particles with small elementary cross-sections (\( K^- , \pi^- \)) may be explained qualitatively as being due to the fact that light nuclei are not completely black for \( K^- \) and \( \pi^- \). The antiprotons instead behave as if hitting a completely black nucleus (for which \( \alpha \) should equal \( \frac{3}{2} \)). The \( \bar{d} \) cross-sections, though poorly known, are very large, indicating both the large size of the antideuteron as well as a sensitivity to the nuclear matter density at the periphery of the nucleus.

Fig. 7. - Nuclear absorption cross-section for \( K^- , \pi^- , \bar{p} \) at 40 GeV/c [13] and \( \bar{d} \) at 25 GeV/c [15]. The lines represent the results of the least squares fits according to eq. (5); \( p = 40 \text{ GeV/c} \).
6. – Limits on new phenomena.

The discovery of new phenomena is one of the most exciting results of the race towards higher and higher energies. When a new energy region becomes available, a number of crude upper limits on new phenomena can be easily obtained as by-products of standard measurements. For instance, from the pressure curves of Fig. 1 one has upper limits for the production of negatively charged objects, in a rather ample mass region. Specific experiments are then required to refine these limits.

Physicists have invented a number of particles which could exist but have not yet been found: the quarks, the intermediate vector boson, antiparticles, tachions, magnetic monopoles, etc. Maybe these objects do not exist; maybe they have large masses, so that present accelerators are not capable of producing them. We shall now discuss some of the recent limits at the highest energies. Measurements concern only differential cross-sections over specific ranges of energies and angles; therefore estimates of the upper limits for the total cross-sections are necessarily model dependent.

6'1. The quarks. – The quarks were invented to explain the grouping of particles and resonances in unitary singlets, octets, and decuplets. Many searches were performed, employing accelerators, cosmic rays, and bulk matter methods. The quark detection is based on the fact that the charge of the quarks is fractional, for instance \( \pm \frac{1}{3} \) or \( \pm \frac{1}{6} \); therefore they ionize less than minimum ionizing particles (\( \frac{1}{3} \) and \( \frac{1}{6} \), respectively) and their apparent momentum is larger than the momentum of normal particles. In particular, it is possible to have quarks with an apparent momentum larger than the momentum of the accelerator. Using these last properties, the two most recent experiments at the CERN and IHEP accelerators have yielded the limits quoted in Fig. 8 [16-18]. Although the limits have been computed for charges \( \pm \frac{1}{3} \), \( \pm \frac{1}{6} \) the experiments are usually sensitive to charges in the range \( 0.3 \div 0.8 \) times the electron charge.

The interpretation of the limits quoted in Fig. 8 are ambiguous among several possibilities:

a) quarks do not exist as physical entities;

b) the conservation of some quantum number prohibits their existence as free entities;

c) they are so massive as to be beyond the energies available at the present accelerators.

Ambiguities of this type will be with us in any unsuccessful search for new objects.
Fig. 8. – Summary of quark production data from accelerators [18]. The total cross-sections are expressed per nucleon and have been calculated assuming isotropic c.m. angular distributions and four-body phase space according to $N^0N^0 \rightarrow N^0N^0 + QQ$. The «diagonal» curves represent the statistical model predictions. The curves A-F come from earlier experiments; curves H from [16] and curves G and I from [17].

A recent cosmic ray experiment reported indications for the existence of quarks of charge $\frac{2}{3}$ [19]. A Wilson cloud chamber was triggered by a system of scintillation counters sensitive to large air-showers, initiated by extremely high-energy particles, estimated at about $10^8$ GeV. A few low-ionizing particles were found in the core of the showers. This experiment seems to have been now contradicted [20].

6.2. Heavy objects with unit charge. – Limits on these objects are obtained from Čerenkov pressure curves such as those in Fig. 1 [2], or from time-of-flight measurements [2, 21]. The last method is particularly useful for heavy mass objects, especially when triggered by Čerenkov counters which veto light particles. A rough summary of the upper limits obtained in negative beams with momenta between 25 and 40 GeV/c at 70 GeV primary energy is the following: The production cross-section $\sigma_m$ for masses $m$ in the range $m_\pi < m < m_\Delta$ is $\sigma_m < 10^{-7} \sigma_\pi$ while, for $m_\Delta < m < (5 \div 6)$ GeV, it is $\sigma_m < 10^{-9} \sigma_\pi$, where $\sigma_\pi$ is the pion production cross-sections.

The antideuteron yield increased by almost an order of magnitude when the energy was raised from 30 to 70 GeV, and one may now work with about one d per minute. It is likely that much will be learned about antinuclei
in general at future accelerators, as indicated by the first absorption cross-section measurements for $\bar{d}$ [15] and the successful finding of few events of $^3\text{He}$ [22].

6.3. Intermediate vector boson. – It is attractive to consider that weak interactions are mediated by a vector boson, the $w$. The smallness of the $K^0_L - K^0_S$ mass difference suggests that the $w$ mass cannot be very large, while neutrino experiments indicate that it must be greater than about 2 GeV [23].

At Brookhaven, a number of experiments have investigated, without success, the mass region $(2 \div 5)$ GeV, by looking for muons produced at large angles by the decaying boson [24] or by determining the intensity and the polarization of muons originating very near to the point of interaction of 28 GeV protons with uranium nuclei [25]. The last experiment quotes an upper limit of $B \sigma_w < 6 \times 10^{-36}$ cm$^2$, where $B$ is the branching ratio of the $w$ into $\mu + \nu$.

At very high energies, the relativistic time dilatation makes the $\pi$ and $K$ mesons less liable to decay. Therefore, the muon contaminations in particle beams will become smaller, and one can obtain a direct limit of the number of $\mu$ mesons produced directly at the target. At the Serpukhov accelerator such simple limits are at the level of $10^{-3} \div 10^{-4}$ of the pion flux at the same energy and angle. These limits are adequate only for excluding a strong production of the intermediate boson.

6.4. Magnetic monopoles. – The possible existence of a magnetic pole would have some appealing aesthetical implications:

$a)$ it would re-establish the symmetry between electric and magnetic charges in Maxwell equations, in a formal way, not in a numerical way, since the magnetic pole strength is probably so much greater;

$b)$ it would provide some understanding of why the electric charge is quantized; and

$c)$ of why the photon mass is zero [1].

Also the experimental implications would be quite interesting:

$a)$ the monopole would ionize thousands of times more than a minimum ionizing particle;

$b)$ it would be easily accelerated to thousands of GeV;

$c)$ it could be «stored» in some materials, and so on.

A very complete review of monopole properties as well as of the present situation about its existence can be found in the review article by Amaldi [1]. Here we shall discuss further possibilities.

The methods for detecting magnetic monopoles, at accelerators are usually based on the fact that they curve toward the poles of a magnetic field, that
their ionizing power is larger than that of fission fragments, and that the ratio of Čerenkov to ionization loss is different from that of ordinary particles [1]. A simple detector is a plastic material, placed a few centimetres downstream from a target in a magnetic field: only heavy ionizing objects which get bent towards the poles can be detected. The fission fragments, already present in natural plastic materials, provide a calibration of the ionization, though they have to be removed (by heating) in order to reduce the background. A crude test on these lines performed at IHEP gave an upper limit of $10^{-38}$ cm$^2$ for monopole production.

6'5. Tachions. – In nature one finds massive particles that travel at speeds smaller than light and massless particles that travel at the speed of light. The question arises: is it possible to have particles which travel faster than light? Theoretically it is possible [26]: These objects would have an unobservable imaginary mass, they would gain velocity while losing energy and, since they travel faster than light, they should produce Čerenkov radiation also in vacuum. Finally, they would end up as transcendent tachions, with infinite velocity, zero energy, and a constant momentum [26]. Their theoretical interpretation requires that they travel backward in time, like antiparticles; present theories may have problems with unitarity.

Tachions could be produced at accelerators, very likely with velocities not too different from light. Actually the situation should be symmetric about $c$, the ordinary particles having speeds close to $c$ on the low side, with tachions on the other side.

Again some limits can be obtained from pressure curves such as those in Fig. 1, when they are continued to the left. A simple limit is given by the threshold counters when they are set to count electrons: There are no tachions to a limit of $10^{-3}$ of the pions. In at least one case (25 GeV/c secondary momentum), the differential counter was set to count «on the other side» up to velocities equivalent to «protons»: there were no counts to a limit of $10^{-6}$ of the pions.

6'6. Long-lived particles. – It would be rather difficult to detect a high-energy neutral particle with a lifetime longer than $10^{-6}$ sec and a relatively small production cross-section [27]. The neutron is of such type, but is abundantly produced. In analogy with the neutron detection method we may speak of various detection methods for such objects:

a) by missing-mass at production;

b) by interaction in various target materials; and

c) by activation and subsequent radioactivity.
Let us consider, in particular, the last method. The delayed radioactivity would be of the normal type if an excited nucleus is formed; it would consist of high-energy particles if a type of hypernucleus would have been formed. Such a delayed radioactivity, resulting in high-energy $\gamma$-rays and electrons was searched for at CERN with negative results [28].

7. — Future perspectives.

As already stated, the present frontier of high-energy physics is represented by the 76 GeV machine. Broadly speaking one can anticipate that the accelerator will be used for:

- $a)$ the study of resonances in production experiments;
- $b)$ asymptotic behaviour, or better energy behaviour of cross-sections;
- $c)$ systematic study of two body, quasi-two-body, and many-body processes;
- $d)$ neutrino physics;
- $e)$ searches for new phenomena, etc.

The 400 GeV Batavia accelerator and the 300 GeV European one will probably follow the same lines.

Altogether new possibilities were opened up by the colliding beam machines, though their small luminosities have until now precluded all but the simplest measurements. The next generation of this type of accelerators is starting now with the successful operation of the 1.5 GeV $e^+e^-$ ADONE colliding beams of Frascati. Electron-positron colliding beams should, in principle, allow a detailed study of many electromagnetic phenomena at small distances, a precise investigation of the $J^{P_L} = 1^{-+}$ boson resonances and so on, while the CERN 25 GeV pp and the Novosibirsk 20 GeV $\bar{p}p$ colliding beams should allow a first study of what is happening at much higher energies. The program for the «first generation experiments» at the CERN-ISR is now taking shape. It is not too different from the program at a new high-energy proton-synchrotron:

- $a)$ particle production;
- $b)$ total and elastic $p$-$p$ cross-sections;
- $c)$ search for new phenomena, etc.

Increasing energies mean physically larger accelerators, bigger laboratories, and much higher costs. Therefore it is clear that the number of super-high-energy laboratories will be small, and that they will have an international
character. Fortunately, high-energy physics has at present no strategic implication; so it is the field of science best suited for supranational co-operation.

Higher-energy conventional machines are becoming physically too large. Several solutions to this problem may be envisaged:

a) the use of large superconducting magnetic fields;

b) colliding beam machines;

c) the development of completely new principles of particles acceleration.

At the same time, one has to worry about particle beam optics at these very high energies. Here, superconductors should be of great help in reducing the dimensions of the beam elements; superconducting RF cavities may allow the separation of particles at much higher energies than at present.

As far as the possibility of doing experiments at such energies is concerned, the results of the CERN-IHEP collaboration have shown that up to 60 GeV/c the electronic separation of particles required only established, though somewhat more refined, Čerenkov techniques, and did not pose any major technical problem. The same techniques will probably be used for the beams of higher energy machines. The counter beams of the future may incorporate many long Čerenkov counters of different types. The time-of-flight technique will instead remain useful when searching for heavy mass objects. The bubble chamber technique will still play an important role in first surveys of what is happening, in particular for seeing new phenomena; it will probably be used extensively to study neutrino interactions. Large magnetic spark chamber spectrometers, and eventually total absorption spectrometers of large dimensions, show prospects of becoming very important types of detectors for the future accelerators. Cosmic rays may still allow a glimpse at what is happening at much higher energies: do quarks really exist? Will everything tend to energy-independent asymptotic limits? Will the Pomeranchuck theorem be violated or will something new happen, such as a new spectroscopy? Only accurate experiments at higher and higher energies will give an answer to the questions.

***

I would like to express my thanks to all my colleagues [2, 13] of the first collaborative experiments between the European Organization for Nuclear Research and the Institute of High-Energy Physics, Serpukhov. Most of this note comes from their work and from discussions with them. I would also like to thank all the people who made the collaboration possible, and the members of the Directorate of IHEP for their hospitality.
REFERENCES


The $K^0-\bar{K}^0$ System in p-$\bar{p}$ Annihilation at Rest.

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In the annihilation of stopped $\bar{p}$ by p, neutral K meson pairs are produced in $\sim 10^{-3}$ of the events. This $K^0-K^0$ system is of a very special nature [1] and it seems worthwhile in the light of the discovery of $CP$ nonconservation to discuss [2] what one can learn from a more detailed study of this system. The discussion is perhaps especially relevant since in the not too distant future it should become possible to stop a sufficient number of antiprotons in hydrogen in a well-defined geometry [3] to pursue statistically significant experiments concerning the system.

In the reaction

$$p + \bar{p} \rightarrow K^0 + \bar{K}^0 + \text{neutrals},$$

for fixed momenta $P_1$ and $P_2$ of the two K's, there are four states: $|SS\rangle$, $|SL\rangle$, $|LS\rangle$, and $|LL\rangle$ where the first (or second) letter denotes the short or long lived nature of the K with momentum $P_1$ (or $P_2$). Assuming $CPT$ invariance, but not separate $C$, $P$, or $T$ invariance, one has [4], with suitable normalization,

$$|K_S\rangle = p|K\rangle + q|\bar{K}\rangle,$$

$$|K_L\rangle = p|K\rangle - q|\bar{K}\rangle.$$  

It follows that

$$(3a) \quad |SS\rangle + |LL\rangle = 2p^2|KK\rangle + 2q^2|\bar{K}\bar{K}\rangle$$ 

(forbidden)

$$(3b) \quad |SL\rangle + |LS\rangle = 2p^2|KK\rangle - 2q^2|\bar{K}\bar{K}\rangle$$ 

(forbidden)
and

\[(4a) \quad |SS\rangle - |LL\rangle = 2pq|KK\rangle + 2pq|KK\rangle \quad \text{(allowed, parity = + 1)}\]

\[(4b) \quad |LS\rangle - |SL\rangle = 2pq|KK\rangle - 2pq|KK\rangle \quad \text{(allowed, parity = - 1)}\]

The two states \((3a, b)\) are forbidden by strangeness conservation. The indicated parity of the two states \((4a, b)\) is the intrinsic parity of the two \(K\)'s taken as a single system in the final state of (1).

1. – Experiments requiring the detection of one \(K^0\).

It follows from (4) that in \(p\bar{p}\) annihilation one has an incoherent source of \(K_S\) and \(K_L\) with an intensity ratio of 1 to 1, if one confines oneself to the observation of only one neutral \(K\) decay. (It seems to us difficult to arrange for another source of this type.) One has thus a rather direct method for measuring the decay rates of \(K_S\) into \(\pi^+\pi^-\pi^0\), \(\pi^0\pi^0\pi^0\), \(\pi^-\mu^+\nu\), \(\pi^+\mu^-\bar{\nu}\), \(\pi^-e^+\bar{\nu}\) or \(\pi^+e^-\bar{\nu}\). (These rates have so far not been experimentally measured. Comparison of the rates of \(K_S \rightarrow \pi^+\pi^-\) and \(K_L \rightarrow \pi^+\pi^-\) yields, of course, direct information on the \(\Delta Q \neq \Delta S\) question.) The sum total rate of these decays is expected to be \(\sim 10^{-3}\) that of \(K_S \rightarrow \pi^+\pi^-\). The spatial distribution of the point of decay of \(K_S \rightarrow \pi^+\pi^-\), which is readily measurable, should be the same as that of any other \(K_S\) decay mode. This allows for a simple method to separate out the \(K_L\) decay modes (which form the main background events) from the \(K_S\) decay modes under study. Scattering of the \(K_S\) and \(K_L\) before decay can introduce uncertainties in the experiment. The amount of matter between the target and the detector is therefore best minimized.

In \(p\bar{p}\) annihilation, detection of a charge asymmetry in the decay \(K_L \rightarrow \pi^+\pi^-\) would be a direct proof of \(CP\) violation, independent of the usual theoretical analysis of the time dependence of the \(K^0\bar{K}^0\) complex, since the initial particles producing the \(K_L\) (i.e. \(p\bar{p}\)) are their own \(CP\) conjugates. (In contrast, the usual charge asymmetry experiments [5] examine \(K_L\) produced in hadronic collisions not involving antinuclei. They therefore demonstrate \(CP\) violation only if one accepts the analysis that the \(K_L\) observed are the same whether they are produced in \(CP\) self-conjugate collisions or not. While this analysis is in all probability correct, there is an explicit advantage [6] in a direct experimental demonstration of the important phenomena of \(CP\) violation without having to use such an analysis.) The experiment can only be successful, however, if one observes \(\sim 10^7\) or more \(K_L\) decays.
2. Experiments requiring the detection of two $K^0$'s.

The decay amplitudes of the two states (4a) and (4b) are tabulated in Table I, for the case when the $\Delta Q = \Delta S$ rule holds. The Table requires minor modifications if $\Delta Q = -\Delta S$ is also allowed. The notations used in Table I are as follows:

$$\mathcal{S}_i = \exp[-\frac{1}{2} \lambda_s t_i + i \Delta m t_i], \quad \mathcal{L}_i = \exp[-\frac{1}{2} \lambda_L t_i],$$

$$\Delta m = m_{\pi} - m_S = 0.469 \lambda_S,$$

where $\lambda_S = \text{decay rate of } K_S,$

$$\eta = \eta_{++}, \quad |\eta_{++}|^2 = 3.7 \times 10^{-6},$$

$$|q|^2 = 6.6 \times 10^{-4}; \quad |p|^2 \propto |q|^2 = \frac{1}{3},$$

**Table I. Decay rate into various modes at $t_1$ and $t_2$.** It is assumed that the $\Delta Q = \Delta S$ rule holds (*).

<table>
<thead>
<tr>
<th>Decay</th>
<th>Amplitude of $LS-LS$</th>
<th>Amplitude of $SS-LL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi \pi^{+}\pi^{+}$</td>
<td>$pa(\mathcal{L}_2^1 \mathcal{S}_2 - \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$pa(\mathcal{L}_2^1 \mathcal{S}_2 - \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi \mu^{-}\pi^{-}$</td>
<td>$-qa^*(\mathcal{L}_2 \mathcal{S}_2 + \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$qa^*(\mathcal{L}_2 \mathcal{S}_2 + \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi^{+}\pi^{+}$</td>
<td>$\eta(\mathcal{L}_2 \mathcal{S}_2 - \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$(\mathcal{L}_2 \mathcal{S}_2 - \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi \mu^{+}\pi^{+}$</td>
<td>$p^2a^2(\mathcal{L}_2 \mathcal{S}_2 - \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$p^2a^2(\mathcal{L}_2 \mathcal{S}_2 - \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi \mu^{-}\pi^{-}$</td>
<td>$-q^2a^2(\mathcal{L}_2 \mathcal{S}_2 + \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$-q^2a^2(\mathcal{L}_2 \mathcal{S}_2 + \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi \mu^{+}\pi^{-}$</td>
<td>$paqa^2(\mathcal{L}_2 \mathcal{S}_2 + \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$paqa^2(\mathcal{L}_2 \mathcal{S}_2 + \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
<tr>
<td>$\pi \pi^{\pm}\pi^{\mp}$</td>
<td>$pa(\eta \mathcal{L}_2 \mathcal{S}_2 - \mathcal{L}_2 \mathcal{S}_2)$</td>
<td>$pa(\mathcal{L}_2 \mathcal{S}_2 - \eta \mathcal{L}_2 \mathcal{S}_2)$</td>
</tr>
</tbody>
</table>

(*) For explanation of notation used in Table, see text.

if $\pi \pi \nu$ is considered instead of $\pi \mu \nu$, $|q|^2$ increases by a factor of 1.34. For those cases where the decay products at $t_1$ and $t_2$ are the same, such as $(\pi^{+}\pi^{-})_t (\pi^{+}\pi^{-})_t$, each observed event should count as $\frac{1}{2}$ event at $t_1$, $t_2$ and $\frac{1}{2}$ event at $t_2$, $t_1$. The amplitudes are normalized such that, e.g., the probability of the state $2^{-1}[|SS\rangle - |LL\rangle]$ to decay into $(\pi \mu^{+}\nu)$ at $t_1$ and $(\pi^{+}\pi^{-})$ at $t_2$ is $\frac{1}{2}|pa(\mathcal{L}_2 \mathcal{S}_2 - \eta \mathcal{L}_2 \mathcal{S}_2)|^2(\lambda_3^2 dt_1 dt_2)(\frac{2}{3})$. The factor $\lambda_3^2 dt_1 dt_2(\frac{2}{3})$ are common to all decay modes. Notice that decay into $(\pi \mu^{+}\nu)$ at $t_2$ and $(\pi^{+}\pi^{-})$ at $t_1$ is a separate entry in the Table.

A traditional type of experiment in which one concentrates on events with no final pions:

$$p\bar{p} \rightarrow SL \text{ or } SS$$
has led to the conclusion that there are few, if any, events of type \((4a)\). (This result is to be expected [7] if the pp annihilation occurs in the \(S\) state, resulting in total parity equal to \(-1\).) Improved statistics for this type of experiment would be useful.

Observation of the interference of \(|LS\rangle\) states with \(|SL\rangle\) states, or of \(|SS\rangle\) with \(|LL\rangle\), in the mode \((\pi\pi)_{t_1}(\pi^+\pi^-)_{t_2}\) would lead to a measurement [8] of both the absolute value and the phase of \(\eta_{\pm}\), as is obvious from Table I. For this experiment, the two \(K\)'s could be produced together with pions. Even so, intensity is a severe problem, and it is doubtful that the experiment could be successful with less than \(10^{11}\) \(K\) pairs, or \(10^{13}\) stopped \(\bar{p}\).

The \(K-\bar{K}\) system should exhibit especially interesting properties if \(CPT\) invariance should be violated. Such a violation cannot be discussed at present at any fundamental level, since no model violating \(CPT\) invariance can be constructed that satisfies the requirements of Lorentz invariance and minimum analyticity. However, one could discuss the possibility that the usual decay formalism still holds phenomenologically, with the two eigenmodes \(|K_S\rangle\) and \(|K_L\rangle\) in eq. (2) given instead by

\[
\begin{align*}
|K_S\rangle &= p'|K\rangle + q'|\bar{K}\rangle, \\
|K_L\rangle &= p|K\rangle - q|\bar{K}\rangle.
\end{align*}
\]

In such a case the two states with zero strangeness are, instead of \((4a)\) and \((4b)\):

\[
\begin{align*}
|SS\rangle - \frac{p'q'}{pq}|LL\rangle + \frac{1}{2} \left( \frac{q'}{q} - \frac{p'}{p} \right) (|LS\rangle + |SL\rangle) &= \frac{1}{2} \frac{(qp' + q'p)^2}{pq} (|KK\rangle + |\bar{K}\bar{K}\rangle), \\
|LS\rangle - |SL\rangle &= (pq' + qp') (|KK\rangle - |\bar{K}\bar{K}\rangle).
\end{align*}
\]

Equation \((6a)\) exhibits the possibility of observing an interference between \(|SS\rangle\) and \(|LS\rangle\) term, which is present only when

\[
\frac{q'}{q} - \frac{p'}{p} = 2\beta \neq 0,
\]

implying violation of \(CPT\) invariance. To search for such an interference it is probably best to look for the decay \((\pi\pi)\) at \(t_1\) together with the decay \((\pi^+\pi^-)\) at \(t_2\) from a single \(\bar{p}p\) annihilation. The probability of such events depends on \(t_1\) and \(t_2\) as follows:

\[
\exp \left[ -\lambda_S t_2 \right] \exp \left[ -\frac{1}{2} (\lambda_S - 2i\Delta m) t_1 \right] + \beta \exp \left[ -\frac{1}{2} \lambda_L t_1 \right]^2.
\]
The $K^0 - \bar{K}^0$ system in p-$\bar{p}$ annihilation at rest

The interference term would yield a measurement of $\beta$. To appreciate the difficulties of the experiment let us assume that $|\beta| \sim 2 \times 10^{-3}$. The most bothersome background is then due to the decay of (6b) into the same state with time dependence:

$$\sim \exp \left[ -\lambda_S t_2 \right] \exp \left[ -\lambda_L t_1 \right].$$

While the time dependence on $t_1$ can serve to separate (9) from (8), the small magnitude of $\beta$ makes the separation possible only with enormous statistics. It is doubtful that the experiment could be done with fewer than $10^{11}$ $K$ pairs or $10^{13}$ stopped $\bar{p}$. On the other hand, if CPT invariance should be violated, the essential parameter $\beta$ seems measurable only in an experiment of this type.

REFERENCES

[1] Previous discussions of this topic can be found in M. Goldhaber, T. D. Lee and C. N. Yang: Phys. Rev., 112, 1796 (1958); and ref. [7].


[3] From a practical point of view it may be desirable, in order to enrich the sample with $K^0 - \bar{K}^0$ events, to surround the hydrogen target with anticoincidence counters. To observe $K_\pi$ decays high precision is needed, such as is already obtainable with streamer chambers. To observe $K_\mu$ decays, very large volume detectors would be needed.


[8] Unlike a number of experiments under progress on the phase angle of $\eta_{+-}$ which suffer somewhat from a lack of knowledge of the precise $K^0 - \bar{K}^0$ composition, this approach is free of such problems.
Symmetry Principles in Physics.

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1. Introduction.

Symmetry considerations have played an important role since the beginning of physics. A great deal of our understanding of nature can be formulated through symmetry principles. However, especially over the past decade, we have learned that many of these symmetry considerations turn out to be not strictly correct. Why do natural laws have a connection with symmetries; but often with a slight amount of asymmetry rather than with perfect symmetry? For example, physical laws are almost symmetrical with respect to left and right. However, in the weak interactions where

\[
\frac{\text{weak forces}}{\text{strong forces}} \sim 10^{-6},
\]

the right-left symmetry is broken. Similar symmetry breaking effects have been observed for many other transformations; these are summarized in Table I.

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Violation amplitude (relative magnitude)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P) (space inversion)</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>(C) (particle anti-particle conjugation)</td>
<td>(10^{-3} - 10^{-6})</td>
</tr>
<tr>
<td>(T) (time reversal)</td>
<td>(10^{-3} - 10^{-15})</td>
</tr>
<tr>
<td>(CP)</td>
<td>(10^{-3} - 10^{-16})</td>
</tr>
<tr>
<td>(SU_2) (iso-spin)</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>(SU_3)</td>
<td>(10^{-1})</td>
</tr>
</tbody>
</table>
This situation may appear to some people as aesthetically disturbing. Why should nature be slightly asymmetrical?

Before starting the technical discussion of these symmetries and asymmetries in physics, let us recall that the word « symmetry » has two different meanings even in our daily language. According to the 1949 Webster’s Dictionary (*), these two definitions are:

\[ \text{sym\'me\-try (sīm\'ə-trē), n. [F. or L.; F. \textit{symmétrie} (now symétrie), fr. L., fr. Gr. \textit{symmetria}, fr. \textit{syn-} + \textit{metron} a measure.] 1. \textit{Now Rare.} Due or balanced proportions; beauty of form arising from such harmony. 2. Correspondence in size, shape, and relative position, of parts that are on opposite sides of a dividing line or median plane.} \]

It is of interest to note the italic \textit{Now Rare} for the first definition. Can it be anticipating the broken symmetry principles that physicists have now discovered about nature? Indeed, as we shall see, perhaps beauty should be associated with a slight asymmetry, rather than with total symmetry.

The concept of beauty is, of course, quite subjective. Which is a more beautiful object, one with total symmetry, or one with a slight asymmetry? The answer is clearly open to debate. However, we may take a look at some of the well-known art pieces. For example, both the Greek statue and the mosaic from S. Apollinare in Classe near Ravenna, shown in Figs. 1 and 2, emphasize bilateral symmetry. The painting of Poplars by Monet in Fig. 3 suggests a discrete space-translational symmetry. While the beauty of symmetry is clearly demonstrated in each case, this beauty is greatly enhanced by the presence of slight asymmetries.

In physics, our concern is with the laws of nature. The concept « beauty » comes in only because of our belief that nature is beautiful. The concepts « symmetry » and « asymmetry » are applied to the various transformations which are connected with both space-time co-ordinates and the interactions between elementary particles. Both symmetry and asymmetry should be formulated in terms of precise mathematical language, so that their implications can lead to predictions which can be tested experimentally.

There are four main groups of symmetries, or broken symmetries, that are found to be of importance in physics.

1) Permutation symmetry: Bose-Einstein and Fermi-Dirac statistics,

2) continuous space-time transformations: translations, rotations, accelerations, etc.,

Fig. 1. – Praying Boy, Greek sculpture. Reprinted from H. Weyl: *Symmetry* (Princeton University Press, Princeton, New Jersey, 1952).
Fig. 2. – Mosaic from S. Apollinare in Classe.

Fig. 3. – Poplars by Claude Monet (on display at the Metropolitan Museum of Art).
3) discrete transformations: space inversion $P$, time reversal $T$, particle-antiparticle conjugation $C$, $G$-parity, etc.,

4) unitary transformations:

$U_1$-symmetries: conservation laws of charge $Q$, baryon number $N$, and lepton numbers $L_e$ and $L_{\mu}$, $SU_2$ (isospin) symmetry, and $SU_3$ symmetry.

Among these, the symmetries connected with the first two groups of transformations are, at present, believed to be exact. In the third group only the product $CPT$ is perhaps exact, but each individual discrete symmetry operation is not. In the fourth group only the $U_1$-symmetries are exact.

2. – Symmetries, nonmeasurables and conservation laws.

The root of all symmetry principles in physics lies in the assumption that it is impossible to measure certain basic quantities; these will be called nonmeasurables in the following discussion. For example, we may consider the interaction energy $V$ between two particles at positions $r_1$ and $r_2$. The physical assumption that it is not possible to measure an absolute position leads to the mathematical conclusion that the interaction energy $V$ should be unchanged under a space translation

$$r_1 \rightarrow r_1 + \Delta$$

and

$$r_2 \rightarrow r_2 + \Delta.$$ 

Therefore, the interaction energy $V$ is a function only of the relative distance $(r_1 - r_2)$, i.e.,

$$V = V(r_1 - r_2).$$

(1)

From this, we deduce that the total momentum of this system of two particles must be conserved, since its rate of change is equal to

$$-(\nabla_1 + \nabla_2)V$$

which, due to (1), is zero.

This simple example illustrates the close connection between three aspects of a symmetry principle: the assumption of a nonmeasurable, the implied invariance under the connected mathematical transformation and the physical consequence of a conservation law. In an entirely similar way, we assume
absolute time to be a nonmeasurable; the physical laws must then be invariant under a time translation

\[ t \to t + \tau \]

which results in the conservation law of energy. By assuming absolute direction to be a nonmeasurable, we derive rotation invariance and obtain the conservation law of angular momentum.

Similar reasoning extends to all other symmetry considerations. The special theory of relativity assumes that absolute velocity is a nonmeasurable, and the general theory of relativity assumes that the difference between an acceleration and a gravitational field is not a measurable quantity. The permutation symmetry rests on the impossibility of observing any difference between identical particles.

In order to derive the conservation law of electric charge, we assume that it is not possible to measure the relative phase between states of different charges; therefore, one must have invariance under an arbitrary multiplication of phase factor \( e^{i\theta} \) between states of different charges. Since \( e^{i\theta} \) is a simple \( 1 \times 1 \) unitary matrix, this invariance is called « \( U_1 \)-symmetry ». It implies that the transition matrix elements between states of different charges must be zero, for otherwise there could be interference between two states of different charges, and their relative phase would be measurable. This \( U_1 \)-symmetry then leads to the well-known conservation law of electric charge. Similarly, the impossibility of measuring relative phases between states of different baryon numbers implies the conservation of baryon number; the assumption that relative phases between states of different lepton numbers are nonmeasurables results in the conservation of lepton number.

3. – Symmetry violations.

Violations of symmetries arise when what were thought to be nonmeasurables turn out to be actually measurable. Let us take as a first example the question of right-left symmetry.

The concept that nature (i.e., physical law) is symmetrical with respect to right and left dates back to the early history of physics. Of course, in our daily life left and right are quite distinct from each other. Our hearts, for example, are usually on our left sides. The word « right » means also correct, while the word « sinister » in its Latin root means left. In English, one says right-left, but in Chinese, 左 (left) always precedes 右 (right). However,
such asymmetry in daily life is attributed to either the accidental asymmetry of our environment or the initial condition in organic life.

The principle of the symmetry between right and left has been found to be true in classical physics, in atomic physics, and in many areas of nuclear physics. Yet, in 1957 it was discovered that the laws of nature are in fact not symmetrical with respect to right and left. The apparent symmetry previously found in macroscopic physics, atomic physics, and nuclear physics is only an approximate one. For example, the neutrino emitted in a $\pi^+$ decay has its spin always antiparallel to its momentum. Although the initial $\pi^+$, being of zero spin, is in a totally spherically symmetrical state, nevertheless we may use its final decay product to give an absolute definition of left versus right.

The same applies to other violations of symmetries. Previously, in electromagnetic theory, the sign of electric charge could only be defined in terms of that of a test charge. Now, because of $C$ and $CP$ asymmetries, one may give an absolute definition of the sign of electric charge. For example, in the decay of the long-lived neutral K meson, $K^0_L$, the decay rates $[1, 2]$ to $e^+$ and $\mu^+$ are different from those to $e^-$ and $\mu^-$:}

\[
\frac{\text{rate}(K^0_L \rightarrow e^+\pi^-\nu_e)}{\text{rate}(K^0_L \rightarrow e^-\pi^+\bar{\nu}_e)} = 1.00315 \pm 0.0003
\]

and

\[
\frac{\text{rate}(K^0_L \rightarrow \mu^+\pi^-\nu_\mu)}{\text{rate}(K^0_L \rightarrow \mu^-\pi^+\bar{\nu}_\mu)} = 1.00405 \pm 0.00135.
\]

Such slight differences in decay rates enable one to give an absolute definition of the sign of electric charge, without the use of a test charge.

As discussed in the previous Section, the validity of all symmetry principles rests on the theoretical hypotheses of nonmeasurables. Some of these hypotheses may indeed be correct in a fundamental sense, some may simply be due to the limitations in our present abilities to measure things. As we improve our experimental techniques, our domain of observations naturally becomes enlarged. It should not be too surprising that we may even succeed in observing some of those supposedly nonmeasurables, and therein lies the root of symmetry breaking.

In this sense, we should be prepared for the eventual possibility that we might be able to measure absolute space-time positions, absolute direction, and absolute velocity, and even relative phases between states of different charges, different baryon numbers, and different lepton numbers. Even if these were possible, it should be expected that such discoveries could lead only to small symmetry breakings in all of the presently known physical
phenomena, because, otherwise, these supposedly nonmeasurables would have been measured long ago.

Just as in most of our artistic creations, the harmony and beauty of symmetry is always enhanced by the presence of a small degree of asymmetry. From an aesthetic point of view, it is rather satisfying to find nature also has a similar preference in small symmetry violations.

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The Design and Use of Large Electron Synchrotrons.

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1. – Introduction.

The large proton synchrotrons built in the last 20 years have been hugely successful in their contributions to knowledge in elementary particle physics. This has been due primarily to the fact that they serve as intense sources of secondary particles in the giga electronvolt region.

The contribution of the large electron accelerators has not, until recently, been anything like as striking as that of the proton accelerators, but now that many electron accelerators are working in the giga electronvolt range this is a situation which has changed, and is changing, rapidly. It is very clear that the large electron accelerators can be used to investigate phenomena in a way quite complementary to the proton accelerators.

There is a fundamental peak energy limitation for large cyclic accelerators, first pointed out by Iwanenko and Pomeranchuk which, while quite unimportant for proton accelerators, is critical to the building of electron accelerators. This is the classical electromagnetic radiation emitted by a charge undergoing acceleration—in the case of an accelerator, the centripetal acceleration caused by the charged particle following a curved orbit. It can easily be shown that the energy loss $\Delta E$ (in MeV) of a charged particle of energy $E$ (in MeV) in one complete orbit of a circle of radius $R$ (in meters) is given by

$$\Delta E = \frac{4\pi}{3} \frac{e^2}{R} \left( \frac{E}{mc^2} \right)^4,$$

where $e$ is its charge and $m$ its rest mass. For electrons this reduces to

$$\Delta E = 8.84 \times 10^{-14} \frac{E^4}{R}.$$  

(*) Present address: University of Bristol, Bristol, U. K.
As a practical example, the 5 GeV electron synchrotron NINA has a bending radius of 21 m which leads to an energy loss at the maximum energy of 2.6 MeV/revolution. But since this radiation loss occurs inversely as the fourth power of the rest mass it is completely negligible for all practicable proton accelerators.

It can be shown also that if the magnetic guide field varies sinusoidally with time, which is usually the case with electron synchrotrons, then the mean energy loss (in watts) $W$ through synchrotron radiation, if the mean accelerated current of electrons is $i$ (in amperes), is given by

$$W = 5.78 \times 10^{-7} \frac{E^4 i}{f R^2},$$

where $f$ is the repetition frequency in cycles/sec. This shows clearly that where the synchrotron radiation is a serious consideration it can be reduced by increasing the repetition frequency of the synchrotron and its radius.

Apart from the difficulty of providing sufficient radio-frequency power to overcome this loss and then to accelerate the electrons, a more fundamental limitation is created by this so-called synchrotron radiation. Since the electron loses energy not continuously but by the discrete emission of photons, synchrotron oscillations are created which can lead to serious difficulties with the available magnet aperture for practical accelerators. However, for the accelerators discussed in this paper this is not likely to be a serious limitation, and schemes to reduce it have been put forward by, for example, Robinson.

Of course, none of these considerations applies to linear accelerators and this is why the largest electron accelerator built so far, the 20 GeV accelerator at Stanford, is a linear machine. Such accelerators, since they are not cyclic, suffer the serious disadvantage that their duty cycle is very short; in the case of SLAC, for example, it is 0.05%. This limits the class of experiment which can be carried out with this kind of accelerator, though it is entirely possible that this difficulty will be overcome by the current development of superconducting radio-frequency cavities. This limitation in turn has concentrated a great deal of study on the possibility of building large electron synchrotrons, which have the virtue of a good duty cycle.

2. -- What is the maximum practicable energy of an electron synchrotron

This question has been discussed in some detail by Crowley-Milling [1] and some of his simple arguments will be presented.

We can get an absolute limit to the maximum energy in a synchrotron
of given configuration by setting the radiated energy loss per revolution equal to the energy gain per revolution.

In a conventional synchrotron the perimeter is made up of guide fields, where the radius of curvature of the electrons is \( R \), and straight sections which contain other ancillary equipment—such as, for example, the accelerating cavities—leading to a «mean radius» \( r \) for the synchrotron. The maximum energy gain per revolution is given by

\[
\Delta E_0 = 2\pi\varepsilon\tau\epsilon r \left( 1 - \frac{R}{r} \right) k \cos\varphi,
\]

where \( \varepsilon \) is the maximum electric accelerating field, \( \tau \) is the transit time factor, \( k \) is the fraction of total straight length available for acceleration, and \( \varphi \) is the synchronous phase angle. By setting this equal to the synchrotron radiation loss at peak energy and using practicable values for the parameters in (1) of \( \tau = 1 \), \( R/r = 0.6 \), \( k = 0.8 \), \( \cos\varphi = 0.7 \) and \( \varepsilon = 1 \text{ MV/m} \) we arrive at the peak energy \( E \)

\[
E = 1.78 \times 10^9 r^4.
\]

Again as a practical example for \( r = 1.2 \text{ km} \) (which is the radius of the proposed European 300 GeV proton synchrotron) we have \( E = 62 \text{ GeV} \).

Another limitation discussed by Crowley-Milling is that given by the fact that the radio-frequency losses in the normally conducting structures usually used as radio-frequency accelerators increase rapidly as the energy increases. If we say that the electric field required to maintain the particles in acceleration varies as \( E^4 \) then clearly the radio-frequency power required to set-up these fields will vary as \( E^8 \). Again by making reasonable assumptions about the overall accelerator structure the dependence of the maximum energy on available radio-frequency power can be deduced.

If one assumes that it would not be unreasonable to use 10 MW of radio-frequency power in such an accelerator the peak energy from an electron synchrotron with parameters similar to those used above (including a mean radius of 1.2 km) will be about 45 GeV. Since most of this power would still be absorbed in the accelerating structure, beam loading would not be a serious problem and it looks feasible to accelerate mean currents of up to 10 \( \mu \text{A} \).

3. – The Daresbury « Booster ».

For some time now a certain amount of study has been given to the possibility of using the 5 GeV electron synchrotron NINA as the injector to a very much larger synchrotron in the (15–20) GeV range [2]. This was
prompted by a number of considerations: the clear need in the future for a long duty-cycle accelerator in this energy range; the high cost of the injector for such a machine; and the fact that the topology of the Daresbury site is well suited to the construction of the main accelerating ring of the large radius required if the radio-frequency power is to be kept within acceptable limits. By the time the electrons in NINA have been accelerated to 2 to 3 GeV, the emittance and energy spread of the beam are very small and this means

Fig. 1. - Layout of Daresbury site for the NINA Booster. \(\ldots\) represent tunneling.
that the magnet aperture of the main accelerator (the «Booster») can be made correspondingly small. Again even with a large radius, the high injection energy means that the magnetic field in the main ring at injection can be very high. Since in this case much larger remanent and eddy current fields can be tolerated at injection this in turn means that one can contemplate quite simple forms of construction for the vacuum chamber.

One possible layout which has been studied for the NINA Booster is that shown in Fig. 1. This particular layout has the advantage of making the maximum possible use of the experimental facilities, including the existing NINA experimental hall, on the Daresbury site. It can be seen too from this layout how much of the main ring would lie in a tunnel through the hillside on which NINA has been built.

The general form of the booster would be that of a superperiod machine with 4 «quadrants» each on a mean radius of about 150 m with long straight sections, each of 100 m, separating them. Two of these straight sections would be sufficient to house the radio-frequency accelerating sections required for acceleration to 20 GeV. A list of the leading parameters is given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak energy</td>
<td>15/20 GeV</td>
</tr>
<tr>
<td>Mean current</td>
<td>1/3 μA</td>
</tr>
<tr>
<td>Mean radius</td>
<td>146.9 m</td>
</tr>
<tr>
<td>Bending radius</td>
<td>120.0 m</td>
</tr>
<tr>
<td>Magnetic field on equilibrium orbit (20 GeV)</td>
<td>0.56 T</td>
</tr>
<tr>
<td>Length of full magnet</td>
<td>7.25 m</td>
</tr>
<tr>
<td>Total number of full magnets</td>
<td>104</td>
</tr>
<tr>
<td>Betatron oscillations per turn</td>
<td>17.75</td>
</tr>
<tr>
<td>Length of straight section</td>
<td>100.0 m</td>
</tr>
<tr>
<td>Magnet excitation frequency</td>
<td>53 Hz</td>
</tr>
<tr>
<td>Radiation loss/turn 20 GeV</td>
<td>118 MeV</td>
</tr>
<tr>
<td>15 GeV</td>
<td>38 MeV</td>
</tr>
<tr>
<td>Radio frequency power (mean) at 15 GeV, 1 μA</td>
<td>95 kW</td>
</tr>
</tbody>
</table>

It seems quite clear that there are no real problems in building a quite conventional synchrotron to accelerate the NINA beam to 15 GeV with perhaps 1 μA (mean) of accelerated electrons. Before embarking on a second stage which would take 3 μA, say, to 20 GeV it is certain that the possibility
of superconducting radio-frequency structures must be investigated thoroughly. The preliminary cost estimates of the first stage are £ 3.82 million, with an additional £ 0.7 million required for the second stage.

4. – Experimental use of a 70 GeV electron synchrotron.

It is important to bear in mind in thinking about possible experiments for a high energy electron synchrotron that for the most part they will be experiments which it will be impossible to do with a proton synchrotron of comparable or higher energy. There are certain experiments which would be done on either kind of accelerator and for which the electron machine will offer certain advantages, but these form rather a special class.

As an illustration of this one can look at the secondary particle yields worked out by P. G. Murphy [3] for the NINA Booster at 20 GeV with 3 μA of accelerated electrons and compare them with the CERN PS. In all cases the Booster yields are a factor of between 2 and 5 down on those from the CERN PS. But there are situations where it would be more advantageous to use the Booster secondary particle beams. Long-lived neutral kaons, for example, are much more free (by a factor of 10 or more) from neutron contamination, and in addition because of the high radio-frequency used in the booster (816 or 1224 MHz) timing information on the kaons can be obtained. It must be emphasised however that the principal experiments to be done with high energy electron synchrotrons will certainly be those using electrons or photons as the primary particle.

A number of specific experiments which can be done with (15–20) GeV electrons are investigated in some detail in the report referred to by Murphy and Clegg [3]. They have confined their attention to experiments where a good duty cycle is essential or, at the least, very desirable. Since the photon has a unique combination of quantum numbers (spin, helicity, C) no rest mass and a relatively weak interaction there are many experimental situations where the photon is an ideal tool to use as the primary particle. This is particularly the case when one studies the vector mesons which, leaving aside the mass, are rather « photon-like » in their properties. The success of the vector dominance model in the strong interaction does not need underlining. In the same sense, in inelastic electron scattering experiments, the electron can be looked on as a source of « massive » virtual photons. But it is quite clear that all such investigations will call for complicated coincidence experiments and will depend on a long duty cycle accelerator for exploration in depth. The field however is extraordinarily rich and can be approached only with an accelerator of the kind discussed in this paper.
REFERENCES

EL/TM/49 (1967); and Daresbury Report DNPL/R2 Preliminary design study for a
15-20 GeV electron synchrotron: NINA Booster.
Breaking of the $SU_3 \times SU_3$ Symmetry in Hadronic Physics.

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1. - Introduction.

In this paper we analyze the properties of the three fundamental interactions (strong, electromagnetic, and weak) from the point of view of the $SU_3 \times SU_3$ group. For this analysis we will use an extension of the geometrical approach which we have introduced before [1, 2] for $SU_3$. In that case the three charges conserved by each interaction namely the hypercharge $Y$, the hadronic electric charge $Q_H$, and the weak hypercharge $Z$, are generators of the unitary linear representation of $SU_3$ on the Hilbert space of hadronic states. That is, in the representation $a \mapsto Q(a) \in \mathcal{L}(H)$ of the $SU_3$ Lie algebra on $H$, $Q_H$, $Y$, $Z$ are the images of three vectors $-q$, $y$, $z$ of $\mathbb{R}^3$, the octet space, _i.e._, the eight-dimensional real vector space of the Lie algebra of $SU_3$. As we have shown in ref. [1] the isotropy groups of these vectors are maximal subgroups of $SU_3$ and the vectors themselves are solutions of a nonlinear equation.

It is however clear that for a full understanding of the properties of the interactions and of their relations we need to consider the group $SU_3 \times SU_3$. Indeed the different behavior under space reflections of the three interactions, cannot be described in terms of the diagonal $SU_3$ subgroup alone.

We will see that some of the interesting geometrical properties of the vectors $y$, $q$, $z$ can be carried over to $SU_3 \times SU_3$. We will show that the directions along which the symmetry group is broken are, also in this case, solutions of nonlinear equations of the type postulated by the bootstrap approach to symmetry breaking.

Two subgroups of $SU_3 \times SU_3$ are of special significance for hadron physics: $SU_3$ and $SU_2 \times SU_2$. Both represent approximate invariances of the strong
interactions which are valid when one neglects either the difference between the K- and π-meson mass (for \(SU_3\)) or the pion mass (for \(SU_2 \times SU_2\)).

Recently Gell-Mann, Oakes and Renner [3] have suggested that the strong Hamiltonian which breaks the \(SU_3 \times SU_3\) symmetry transforms approximately like an element of the \((3, \overline{3}) \oplus (\overline{3}, 3)\) representation which is left invariant by \(SU_2 \times SU_2\). We will show that in the space of the \((3, \overline{3}) \oplus (\overline{3}, 3)\) representation we can define two directions which are solutions of nonlinear equations and whose isotropy groups are precisely \(SU_3\) and \(SU_2 \times SU_2\).

In Sect. 2 after a brief resumé of the relevant results of refs. [1] and [2] we will discuss the unique symmetrical algebra, on the space of the \((1, 8) \oplus (8, 1)\) and of the \((3, \overline{3}) \oplus (\overline{3}, 3)\) representations, which have \(SU_3 \times SU_2\) as a group of automorphism. The existence on these spaces of symmetrical algebras insures the possibility of having nonlinear equations whose solutions define the directions along which \(SU_3 \times SU_3\) is broken.


2'1. Geometry of the octet. We begin by briefly reviewing a co-ordinate-free formulation [1] of the \(SU_3\) invariant algebras on the octet space \(\mathbb{R}^8\).

We can realize \(\mathbb{R}^8\) as the real vector space of all \(3 \times 3\) Hermitian traceless matrices \(a, b, c, \ldots\). Any element \(u\) of the group \(SU_3\) is the form \(u = \exp[-i\varphi a/2], \ a \in \mathbb{R}^8\). The action of \(SU_3\) on \(\mathbb{R}^8\) (which is the space of its adjoint representation) is

\[
a \rightarrow uau^* = uau^{-1}.
\]

(1)

We can define on \(\mathbb{R}^8\) an \(SU_3\)-invariant scalar product and two algebras which have \(SU_3\) as automorphism group:

Scalar product:

\[
(a, b) = \frac{1}{2} \text{tr} \ ab
\]

(2)

\(SU_3\) Lie algebra:

\[
a \wedge b = -\frac{i}{2} [a, b].
\]

(3)

Symmetrical algebra:

\[
a \vee b = \frac{1}{2} (ab + ba) - \frac{3}{8} (a, b) = \frac{1}{2} [a, b] - \frac{3}{8} (a, b).
\]

(4)

If \(a\) and \(a \vee a\) are linearly independent they generate a two-plane \(\mathcal{C}_a\) (i.e. a two-dimensional subspace of \(\mathbb{R}^8\)) which is a Cartan subalgebra (i.e. a maximal Abelian subalgebra) of the \(SU_3\) Lie algebra. Thus \(\mathcal{C}_a\) which is isomorphic
to $U_1 \times U_1$ is the Lie algebra of the isotropy group (or little group) of $a$. If on the contrary
\begin{equation}
q \sqrt{q} + \eta(q)q = 0,
\end{equation}

the isotropy group is a $U_2$ group which we denote by $U_2(q)$. Any vector whose isotropy group is a $U_2$ will be called a «q-vector». From now on we will consider only normalized «positive» q-vectors, i.e. such that: $(q, q) = 1$, $\eta(q) > 0$. This implies, $\eta(q) = 1/\sqrt{3}$.

The Cartan subalgebras of the $SU_3$ Lie algebra are all conjugate (i.e. transformed into each other) by the $SU_3$ group. One of them is of course made with the diagonal matrices $u \in SU_3$. It can be proved that any $C$ contains three positive normalized q-vectors at $120^\circ$ from each other. Conversely if $x, y \in \mathbb{R}^8$ commute, $\alpha x + \beta y$ and $\alpha' x + \beta' y$ commute, and generate a $C$ (denoted $C_{x,y}$). For positive normalized q-vectors we thus have
\begin{equation}
(q_i, q_j) = -\frac{1}{2} \iff q_i \wedge q_j = 0 \quad \text{and} \quad q_i \neq q_j.
\end{equation}

Given a q-vector $y$, the vectors $t_y$ of $U_2(y)$ which are orthogonal to $y$ form the $SU_2(y)$ subalgebra of $U_2(y)$. They satisfy the following relations:
\begin{equation}
y \wedge t_y = y \wedge t'_y = 0; \quad (y, t) = (y, t') = 0; \quad \sqrt{3} t_y \wedge t'_y = (t_y, t'_y)y.
\end{equation}

The normalized $t_q$ of the three q-vectors of a Cartan subalgebra $C$ form the hexagon of the «roots» different from zero.

2.2. The $SU_3 \times SU_3$ algebra. – To extend this formalism to $SU_3 \times SU_3$ we consider the space $\mathbb{R}^{16} = \mathbb{R}^8 \oplus \mathbb{R}^8$. We call $a_+$ and $a_-$ the elements of the first and the second $\mathbb{R}^8$, respectively (the index $\pm$ corresponds in physics to the chirality) and denote by $\bar{a} = a_+ \oplus a_-$ an element of $\mathbb{R}^{16}$. The Lie algebra of $SU_3 \times SU_3$ is then defined by
\begin{equation}
\bar{a} \wedge \bar{b} = (a_+ \oplus a_-) \wedge (b_+ \oplus b_-) = (a_+ \wedge b_+) \oplus (a_- \wedge b_-),
\end{equation}

where $\wedge$ in the right-hand side has been defined on $\mathbb{R}^8$ in eq. (3). The scalar product invariant under $SU_3 \times SU_3$ is the Cartan-Killing form which we write
\begin{equation}
(a_+ \oplus a_-, b_+ \oplus b_-) = \frac{1}{2} (a_+, b_+) + \frac{1}{2} (a_-, b_-).
\end{equation}

It is also convenient to use another decomposition of $\mathbb{R}^{16}$ into a direct sum $\mathbb{R}^8 \oplus \mathbb{R}^8$. In this decomposition, which is symbolically illustrated in Fig. 1, we denote the element $\bar{a} = a_+ \oplus a_-$ by $(a|a')$ with
\begin{equation}
a_+ = a + a' \quad \text{and} \quad a_- = a - a'.
\end{equation}
In this notation the Lie algebra law (6) becomes

\[(11) \quad \tilde{a} \wedge \tilde{b} = (a|a') \wedge (b|b') = (a \wedge b + a' \wedge b'|a \wedge b' + a' \wedge b)\]

and the scalar product

\[(12) \quad (\tilde{a}, \tilde{b}) = (a, b) + (a', b').\]

In a similar way we can extend to $\mathbb{R}^{16}$ the symmetrical algebra on $\mathbb{R}^8$:

\[(13) \quad \tilde{a} \vee \tilde{b} = (a|a') \vee (b|b') = (a \vee b + a' \vee b'|a \vee b' + a' \vee b).\]

One verifies that the equation

\[(14) \quad \tilde{a} \vee \tilde{a} = \lambda \tilde{a},\]

has only two types of solutions:

\[(15) \quad \tilde{a} = (q|0)\]

and

\[(16) \quad \tilde{a} = (q|\pm q),\]

where $q$ is a $q$-vector.

The subalgebra of $SU_3 \times SU_3$ which leaves invariant (i.e. commutes with) a $q$-vector $(y|0)$ of the diagonal $SU_3$ subalgebra is the set of all $(a|a')$ such that $y \wedge a = 0$, $y \wedge a' = 0$; it will be denoted $(U_3|U_3)_y$. With the notation of (8) it is the direct sum $U_2^{(+)}(y) \oplus U_2^{(-)}(y)$.
23. The \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation. A special role in the physical applications is played by the \((3, \bar{3})\) representation of the group or of the Lie algebra \(SU_3 \times SU_3\). We can realize the 9-dimensional space of this representation as the complex vector space of the \(3 \times 3\) matrices \(m\). Under the transformation \(u_+ \times u_- = \exp[-i\varphi a_+ / 2] \times \exp[-i\varphi a_- / 2]\), \(m\) goes over into:

\[
m \mapsto u_+ m u_-^*.
\]

The representation of the Lie algebra is thus

\[
D(\bar{a}) m = D(a_+ \oplus a_-) m = -\frac{1}{2} (a_+ m - ma_-),
\]

or

\[
D(\bar{a}) m = D(a | a') m = -\frac{i}{2} ([a, m] + \{a', m\}).
\]

(Equation (18) is obtained from (17) by differentiation with respect to \(\varphi\), at \(\varphi = 0\).)

The representation is unitary for the group, i.e. it leaves invariant the Hermitian scalar product

\[
\langle m_1, m_2 \rangle = \frac{1}{2} \text{tr} (m_1^* m_2).
\]

The 9-dimensional complex vector space \(C_{3, \bar{3}}\) can be considered as an 18-dimensional real vector space \(R^{18}\). The 18-dimensional representation of the group \(SU_3 \times SU_3\) on this space is the direct sum of the \((3, \bar{3})\) and its complex conjugate \((\bar{3}, 3)\). It is real and unitary, hence orthogonal. It leaves invariant a Euclidean (i.e. real orthogonal) scalar product which is the real part of (20), while the imaginary part becomes an antisymmetrical real (i.e. sympletic) scalar product. Explicitly we have:

\[
(m_1, m_2) = \text{Re} \langle m_1, m_2 \rangle = \frac{1}{4} \text{tr} (m_1^* m_2 + m_2^* m_1),
\]

\[
\text{Im} \langle m_1, m_2 \rangle = \frac{1}{4i} \text{tr} (m_1^* m_2 - m_2^* m_1).
\]

Any \(3 \times 3\) complex matrix can be written in the form

\[
m = \sqrt{3} \mu 1 + m + i \sqrt{3} \mu' 1 + im' = (\mu | m||\mu' | m'),
\]

where \(\mu\) and \(\mu'\) are real members and \(m\) and \(m'\) are vectors of the octet space.

In this notation (21), (22), and (23) read

\[
(m_1, m_2) = \mu_1 \mu_2 + \mu'_1 \mu'_2 + (m_1, m_2) + (m'_1, m'_2),
\]

\[
\text{Im} \langle m_1, m_2 \rangle = \mu_1 \mu'_2 - \mu'_1 \mu_2 + (m_1, m_2) - (m'_1, m'_2),
\]
and eq. (19) reads

(26) \[ D(a|a')(\mu|m||\mu'|m') = \]
\[ = (\sqrt{\frac{3}{2}}(a', m')|a\wedge m + a'\wedge m' + \sqrt{\frac{3}{2}}\mu'|a'| - \sqrt{\frac{3}{2}}(a', m)|a\wedge m' - a'\wedge m - \sqrt{\frac{3}{2}}\mu a') \]

Tensor operators which represent physical observable must be Hermitian on \( \mathcal{H} \). It is therefore necessary that they belong to a real representation of the invariance group. This is the case of the \( (3, \bar{3}) \oplus (\bar{3}, 3) \) representation which, we want to emphasize, is irreducible as a real representation.

The tensor product of \( (3, \bar{3}) \oplus (\bar{3}, 3) \) by itself when decomposed into real irreducible representations contains the \( (3, 3) \oplus (\bar{3}, \bar{3}) \) once.

Hence from two vectors \( r, s \in (3, \bar{3}) \oplus (\bar{3}, 3) \) it is possible to form a new vector of the same representation which we denote \( r_T s \). The symbol \( T \) is the law of a symmetrical algebra on \( \mathbb{R}^{18} \) which has \( SU_3 \times SU_3 \) as automorphism group. By standard methods we find

(27) \[ r_T s = \frac{1}{6} \mathbf{1}(\text{tr} r^* \text{tr} s^* - \text{tr}(r^* s^*)) - \frac{1}{3} r^* \text{tr} s^* - \frac{1}{2} s^* \text{tr} r^* + \frac{1}{2} \{r^*, s^*\} \]

We leave to the reader to check that

(28) \[ D(a|a') r_T s = (D(a|a') r)_T s + r_T (D(a|a') s) \]

which means that \( SU_3 \times SU_3 \) is a derivation algebra of the \( T \)-product.

With the notation (23) we can write eq. (27) in the form

(29) \[ r_T s = (\tau|\tau||\tau'|\tau') \]

where

\[ r = (\sigma|\sigma||\sigma'|\sigma') \quad \text{and} \quad s = (\sigma|\sigma||\sigma'|\sigma') \]

and

\[ \tau = \frac{1}{\sqrt{6}} \left( 2\sigma\sigma' - 2\sigma'\sigma' - (r, s) + (r', s') \right) , \]

\[ t = \frac{1}{\sqrt{6}} \left( -\sigma s - \sigma r + \sigma's' + \sigma' r' \right) + r\vee s - r'\vee s' , \]

(30) \[ \tau' = \frac{1}{\sqrt{6}} \left( -2\sigma\sigma' - 2\sigma'\sigma + (r, s') + (r', s) \right) , \]

\[ t' = \frac{1}{\sqrt{6}} \left( \sigma s' + \sigma' s + \sigma' r + \sigma r' \right) - r\vee s' - r'\vee s . \]
We add two more properties of this product

\begin{align}
\langle x_T y, z \rangle &= \langle x, y_T z \rangle , \\
\langle x, x_T x \rangle &= \frac{2}{3} \det x = (x_T x, x) + i x_T x, x .
\end{align}

The \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation of \(SU_3 \times SU_3\) has no invariant for the subgroup \((U_2, U_2)_y\), i.e. if for all \((a|a') \in (U_2 \times U_2)_y\), \(D(a|a')m = 0\) eq. (26) shows that \(m = 0\). However if \(a'\) is restricted to be in \(SU_3(y)\), the same equation shows the existence of a two-dimensional invariant subspace spanned by the vectors

\begin{equation}
(\eta| - \sqrt{2} \eta y| |\eta'| - \sqrt{2} \eta' y) ,
\end{equation}

where \(y\) is a \(g\)-vector and \(\eta, \eta'\) are real numbers. We will denote the isotropy group (or its Lie algebra) of the vectors (33) by \((U_2(y)|SU_2(y))\). This Lie algebra is the following direct sum

\begin{equation}
(U_2(y)|SU_2(y)) = SU_2^+(y) \oplus SU_2^-(y) \oplus U_1^d(y) ,
\end{equation}

where \(U_1^d(y) = (U_1(y)|0)\) is the Lie algebra generated by \((y|0)\) (see Fig. 1).

The vectors (33) have an interesting property under the \(T\)-product. Let \(\mathcal{Y}(\varphi)\) be the vector (33) with

\begin{equation}
\eta = \sqrt{\frac{3}{2}} \cos \varphi , \quad \eta' = \sqrt{\frac{3}{2}} \sin \varphi .
\end{equation}

These vectors are normalized

\begin{equation}
\langle \mathcal{Y}(\varphi), \mathcal{Y}(\varphi) \rangle = 1 .
\end{equation}

They belong to the \(SU_3 \times SU_3\) orbit of \(y(0)\) and satisfy the quadratic equation

\begin{equation}
\mathcal{Y}(\varphi)_T \mathcal{Y}(\varphi') = 0 .
\end{equation}

Moreover one shows that all unit vectors of \(R^{18}\) satisfying such an equation are on the \(SU_3 \times SU_3\) orbit of \(y(0)\).

Equation (37) is a particular case of the equation

\begin{equation}
m_T m = \lambda m .
\end{equation}

If \(\lambda \neq 0\) (and \(m \neq 0\) one also shows that the only unit vectors which are solutions of (38) are on the two orbits of \(\pm \mathbf{n} = \pm \sqrt{2/3} \mathbf{1}\) which are \(SU_3^g\) invariants. The unit vectors which have \(SU_3^g\) as isotropy group form a circle

\begin{equation}
\mathbf{n}(\varphi) = \sqrt{\frac{3}{2}} \exp[i \varphi] \mathbf{1} = (\cos \varphi|0)\langle |\sin \varphi|0 \rangle
\end{equation}

\begin{equation}
\mathbf{n}(\varphi \pm \pi) = - \mathbf{n}(\varphi)
\end{equation}
and they generate the \( T \)-subalgebra

\[
\mathfrak{n}(q)\mathfrak{n}(q') = \sqrt{\frac{2}{3}} \mathfrak{n}(-q - q').
\]

3. Geometrical properties of the three interactions.

3'1. The \( SU_3 \) symmetry. – We begin by recalling the basic properties of the interactions under the \( SU_3 \) group, \textit{i.e.}, the diagonal subgroup \( SU_3^d \) of \( SU_3^{(+) \times SU_3^{(-)} }\).

\( a \) The hypercharge \( Y \) and the three isospin operators \( T_1, T_2, T_3 = Q_H + \frac{1}{2} Y \) generate the invariance group \( U_2(y) \) of the strong interactions. The extension of this invariance to \( SU_3 \) implies considering \( U_2(y) \) as a subgroup of \( SU_3 \). This means that \( y \) is a \( q \)-vector of which \( Y \) is the image in the representation of the \( SU_3 \) algebra in the Hilbert space \( \mathcal{H} \) of hadron physics.

The electric hadronic charge \( Q_H \) is the corresponding image of \(-q\) and the relation \( Q_H = T_3 + \frac{1}{2} Y \) implies that \( q \) is a \( q \)-vector. The \( SU_2(q) \) group is called the «\( U \)-spin group».

\( b \) According to Cabibbo’s hypothesis the two charged components of the vector current \( v^\pm_\mu \) coupled to the leptons and the electromagnetic current \( j_{\mu}^{em} \) belong to the same \( SU_3^d \) octet. We denote by \( c_1 \pm ic_2 \) the directions of \( v^\pm_\mu \). Using the additional property that the electric charges of \( v^\pm_\mu \) are \( \pm 1 \) we can deduce

\[
\sqrt{3}c_1 \vee c_1 = \sqrt{3}c_2 \vee c_2 = z,
\]

where \( z \) is a \( q \)-vector. The operator \( Z \), which is the image of \( z \), is the weak hypercharge conserved in weak interactions.

The vector \( z \) commutes with \( q \) but not with \( y \). We thus have in \( \mathbb{R}^8 \) two distinct algebras \( \mathfrak{c}_{qy} \) and \( \mathfrak{c}_{qz} \) which have \( q \) in common. The noncommunicativity of \( y \) and \( z \) reflects the existence of strangeness violating weak interactions. As one can see from (6) the difference from \( 0^\circ \) or \( 120^\circ \) of the angle between \( y \) and \( z \) gives a measure of the noncommunicativity of \( Y \) and \( Z \) and is therefore related to the Cabibbo angle \( \theta \). Explicitely we have

\[
(y, z) = 1 - \frac{3}{2} \sin^2 \theta.
\]

It can be proved that two noncommuting \( q \)-vectors \( y \) and \( z \) uniquely define another \( q \)-vector which commutes with both of them. This vector is given by the relation

\[
q = ((y, z) - 1)^{-1}(\sqrt{3}y \vee z + \frac{1}{2}(y + z)).
\]
Thus the strong and weak interactions determine uniquely the direction of the electromagnetic interactions.

Cabibbo has also postulated that the axial currents \( a_{\mu}^x \) belong to another \( SU_3^g \) octet in the same directions \( e_1 \pm ie_2 \) as \( v_{\mu}^\pm \). The two assumptions about the vector and the axial vector currents are in good agreement with experiment.

3'2. \( SU_3 \times SU_3 \) symmetry. – Since the weak interactions have a definite (negative) chirality whereas the electromagnetic and strong interactions have a defined parity, their relations can only be fully understood by considering the enlarged group \( SU_3 \times SU_3 \). It has indeed been suggested [3, 4] that this group and its subgroups provide a reasonably approximate frame for the study of hadron physics. Cabibbo's hypothesis can be generalized to \( SU_3 \times SU_3 \) by assuming that \( j_{\mu}^{em}, v_{\mu}^\pm, a_{\mu}^\pm \) belong to the same representation of this group namely the adjoint representation \((8, 1) \oplus (1, 8)\). We can thus write for the currents:

\[
(44) \quad j_{\mu}^{em} = h_\mu(q|0); \quad v_{\mu}^\pm = h_\mu(c^\pm|0); \quad a_{\mu}^\pm = h_\mu(0|c^\pm).
\]

The weak currents are thus

\[
(45) \quad h_{\mu}^\pm = h_\mu(c^\pm - c^\pm).
\]

As \( Q_H \) is the integral over space of the time component of \( j_{\mu}^{em} \), the integrals

\[
(46) \quad Q(a|a') = \int d^3x h_0(a|a'),
\]

are at a given time the generators of \( SU_3 \times SU_3 \). We shall now list the covariance properties of the three interactions under \( SU_3 \times SU_3 \):

a) The isotropy group of the electromagnetic current \( h_\mu(q|0) \) and therefore of the electromagnetic interactions (see Sect. 2'2) is \((U_2^+ U_2^-)q = \cup_2^+ (q) \oplus \cup_2^- (q)\).

b) The pair of weak currents \( h_\mu(c^\pm - c^\pm) \) and therefore the semi-leptonic weak interactions have for isotropy group \( SU_3^{(+)} \oplus U_1(\pm)(z) \).

c) The covariance of the CP conserving Hamiltonian \( \mathcal{H}_{NL} \) for non-leptonic weak interactions is not yet established. If it involves only charged currents as many physicists would prefer [5] then it would have components outside the \((1, 8) \oplus (8, 1)\) representation. It is however compatible with the present evidence to assume that \( \mathcal{H}_{NL} \) belongs entirely to the representations \((1, 8) \oplus (8, 1)\) in the direction \((z|z)\) [6]. If this were the case the isotropy group of \( \mathcal{H}_{NL} \) would be \( SU_3^{(+)} \oplus U_2^- (z) \) which is a maximal subgroup of \( SU_3 \times SU_3 \). Nothing is known for the CP violating part.
d) We have said that \( U_2(y) \) and \( SU_3^g \) are approximate invariances of the strong interactions. Another interesting approximate invariance has been recently proposed by Gell-Mann, Oakes, and Renner [3]. According to them, in the limit where the pion mass can be neglected the strong Hamiltonian is of the form

\[
\mathcal{H}_s = \mathcal{H}_0 + \mathcal{H}(m),
\]

where \( \mathcal{H}_0 \) is invariant under \( SU_3 \times SU_3 \) and \( \mathcal{H}(m) \) transforms like the \((3, \bar{3}) \oplus (3, 3)\) representations. They also suggested that to a good approximation \( m \) coincides with the vector \( y(0) \) of eq. (37).

In this model the approximate isotropy group of the strong interactions would be \((U_2|SU_3)_y\) which is a maximal isotropy group for the nonzero vector of the \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation.

Even though the mass difference \( m_K - m_\pi \) is larger than \( m_\pi \), \( SU_3 \) remains an interesting approximation for the strong interactions. The \( SU_3 \times SU_3 \) breaking part in eq. (47) is in the \( SU_3 \) invariant direction denoted by \( n(0) \) in eq. (39). It is remarkable that its isotropy group \( (SU_3^g) \) is the other maximal isotropy group of the nonzero vectors of the \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation.

3'3. \( SU_3 \times SU_3 \) and space reflections. – In the limit where they are exact the \( U_2(y) \) and \( SU_3^g \) symmetries of the strong interactions commute with the Poincaré group (without time reflection).

For the exact \( SU_3 \times SU_3 \) symmetry the invariance group is no longer a direct product of the internal symmetry group by the Poincaré group but the following semidirect product

\[
(\mathcal{P}_0 \times SU_{3}^{(+)} \times SU_{3}^{(-)}) \rtimes Z_2,
\]

where \( \mathcal{P}_0 \) is the connected Poincaré group and the nontrivial element \( r \) of \( Z_2 \) acts on \( \mathcal{P}_0 \) like the space inversion and interchanges \( SU_{3}^{(-)} \) with \( SU_{3}^{(+)} \). The action of \( r \) on the \((8, 1) \oplus (8, 1)\) representation is

\[
(a|a') \mapsto (a| - a').
\]

This allows to assign a parity to the elements of the \((8, 1) \oplus (1, 8)\) representation; the primed vectors have odd parity, the unprimed ones have even parity.

For the \((3, \bar{3}) \oplus (\bar{3}, 3)\) representation, eq. (26) shows that the primed and unprimed quantities which appear in (23) have opposite parity. For example \( y(0) \) and \( y(\pi/2) \) (see eq. (33)) are eigenvectors of \( r \) with opposite parity. As we have seen, \( SU_3 \times SU_3 \) implies the existence for this represen-
tation of the $T$-algebra and this fixes in the $SU_3$ limit the assignment of the parity. Indeed, as we have shown, the direction along which $SU_3 \times SU_3$ is broken in an $SU_3$-invariant way satisfies the nonlinear equation

\begin{equation}
(49) \quad n(0) = \sqrt{\frac{2}{3}} n(0) = n \left( \frac{\pi}{2} \right) T \cdot n \left( \frac{\pi}{2} \right).
\end{equation}

Thus under $r$

\begin{equation}
(50) \quad m = (\mu | m | \mu' | m') \mapsto (\mu | m || \mu' | m').
\end{equation}


It has been suggested by several authors [7] that the $SU_3$ or the $SU_3 \times SU_3$ symmetries are spontaneously broken. Such a symmetry breaking occurs when the invariance group $K$ of a stable state of a physical theory is only a subgroup of the invariance group $G$ of the theory itself. In this case all states of the same orbit $G/K$ of solutions are all stable states.

We have shown [1, 8] that in a theory based on a variational principle spontaneous symmetry breaking can occur and one expects the subgroup $K$ to be a maximal isotropy group among those of all possible orbits. As we have seen the breaking of $SU_3 \times SU_3$ by the strong interactions has the above property both in the $SU_3$ or in the $SU_3 \times SU_2$ approximation [9]. The same is true of $\mathcal{N}_L$ if its invariance group is $SU_3^{(+)} \oplus U_2^{(-)}(z)$ (see Sect. 3'2). This may therefore suggest that the $SU_3 \times SU_3$ symmetry of the hadronic world is spontaneously broken by the strong and perhaps also the weak interactions.

The intersection between the two isotropy groups of the weak nonleptonic interactions and of the strong interactions in the Gell-Mann, Oakes, and iRenner model is:

\begin{equation}
(51) \quad (SU_3^{(+)} \oplus U_2^{(-)}(z)) \cap (SU_3^{(+)}(y) \oplus SU_3(y) \oplus U_1^d(y)) = SU_3^{(+)}(y) \oplus U_1^d(q),
\end{equation}

where $q$ is a $q$-vector commuting with $y$ and $z$ which, as we have seen, is uniquely defined once $y$ and $z$ are fixed. The intersection of the two groups in the left-hand side of (51) and $SU_3^d$ is $U_1^d(q)$ which is thus the only invariance group for the interactions between hadrons (when the hadron-lepton interactions are disregarded) and corresponds to the conservation of the electromagnetic charge. We have thus the following scheme of decreasing invariance inside the hadronic world

$$SU_3 \times SU_3 \supset (U_2 \downarrow SU_2)_y \supset SU_3^d \supset U_3(y) \supset U_1(q).$$
Let us remark that the isotropy group of the electromagnetic Hamiltonian is \((U_3)^g\). This group is not maximal in \(SU_3 \times SU_3\) for the \((1, 8) \oplus (8, 1)\) representation.

However the direction \((q|0)\) of the electromagnetic interactions shares with the directions of the two other interactions, \(i.e. (z|z)\) and \(y(0)\) or \(n(0)\) the following properties: they are the different types of solutions of \(SU_3 \times SU_3\) invariant nonlinear equations:

\[
(a|a') \lor (a|a') = \lambda (a|a'),
\]

for \((q|0)\) and \((z|z)\);

\[
m \times m = \lambda m,
\]

for the two directions along which \(SU_3 \times SU_3\) is broken with approximate \(SU_3\) or \(SU_2\) invariance. Bootstrap approaches to symmetry breaking lead to this quadratic type of nonlinear equations.

It is interesting to note that for an \(SU_3 \times SU_3\) invariant theory, the space inversion operator \(r\) can only be defined modulo an inner \(SU_3 \times SU_3\) automorphism. However, as we have discussed in Sect. 3, the existence of the \(T\)-algebra fixes naturally the parity of the vectors of the \((3, 3) \oplus (\bar{3}, 3)\) representation of \(SU_3 \times SU_3\) and the vector \(n(0)\) has even parity. Thus the requirement that the breaking due to strong interactions satisfies eq. (53) fixes the parity of the hadronic states.

It is also worthwhile to point out that one of the solutions of eq. (52), \((z|z)\) has a pure chirality corresponding to maximal violation of the parity fixed by the strong interaction. The other solution \((q|0)\) has a definite parity and its direction \(q\) is fixed when \(y(0)\) and \((z|z)\) are known. From the point of view of \(SU_3 \times SU_3\) the three directions according to which the symmetry is broken have thus fairly simple properties and correspond to all three types of solutions of the nonlinear equations (52) and (53).

There is nothing however to tell us why the directions \(y\) and \(z\) should make precisely the angle that is experimentally observed.

We do not want to discuss here the attempts [10, 11] to calculate \(\theta\). We only remark that in the \(SU_3 \times SU_3\) scheme, \(m\) and \((z|z)\) are not in the same representation space. Thus an \(SU_3 \times SU_3\) invariant depending on these two vectors has to be at least quadratic in \(m\). For example if we define the vector (see eq. (18))

\[
v = D(z|z)m = \frac{1}{2} mz,
\]

we can form an \(SU_3 \times SU_3\) invariant \(\langle v, v \rangle\) which is a function of \((y, z)\).
However the length of the vectors has not been given here a physical meaning as we did not take into consideration the strength of the coupling.

On the other hand a projective invariant such as $\mathcal{I}_3 = \langle \mathbf{v}_T, \mathbf{v} \rangle \langle \mathbf{v}, \mathbf{v} \rangle^{-\frac{1}{2}}$ depends upon both $\theta$ and the matrix elements of $\mathbf{m}$ which are functions of the physical masses. We note that in the limit where $\mathbf{m} = \mathbf{y}(0)$ (invariant under $(U_3|SU_3)_\gamma$, $\mathcal{I}_3$ vanishes.

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Structure of Matter Investigations by Thermal Neutrons in Rome.

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1. - Introduction.

As it is well known, the slowing down of neutrons and the interacting properties of thermal neutrons with matter were discovered in Rome by Amaldi, Fermi, Pontecorvo, Rasetti and Segrè [1] thirty-five years ago.

Since then a tremendous amount of work has been done in all the world in the field of nuclear, atomic and molecular physics by means of thermal neutrons which at the present time are still successfully employed in many first class investigations.

Until 1955 there was very little in nuclear energy researches going on in Italy; particularly in the field of the so-called «pile neutron research» every thing was to be started «ex-novo».

E. Amaldi had always followed with great interest this kind of research and was convinced of the necessity of promoting it in Italy also. Among the several lines of research with neutrons he suggested neutron diffraction for solid state investigations warmly promoting also the use of thermal neutrons in nuclear physics.

This point of view was completely accepted by the Comitato Nazionale per le Ricerche Nucleari [2] leading to the constitution of the Laboratorio di Fisica Nucleare Applicata in Rome and of similar units in other Italian nuclear Centers [3].

In this paper we would like to briefly describe some recent research both in nuclear and solid state physics carried out at the Casaccia Center of CNEN, Rome, using thermal neutrons.

The thermal neutrons are provided by the 1 MW, RC-1 reactor [4]. Two radial beam tubes are employed for solid state physics work, while two tangential beam tubes, one completely crossing the biological shield, are used for nuclear physics experiments.
2. – Nuclear physics work.

The \((n, \gamma)\) reactions are intensively used at the Casaccia Center to study nuclear levels in different classes of nuclei. Besides these experiments we would like to describe one which makes use of a very fruitful technique developed in our laboratory [5]. High energy gamma-lines \((5\text{–}8\text{ MeV})\) produced from \((n, \gamma)\) reactions in a target located very close to the reactor core, come from a beam tube completely opaque to neutrons and impinge on a scatterer which is viewed by Ge(Li) and NaI(Tl) large detectors. When a gamma-line resonantly excites a high energy level of the target (scatterer) nucleus, a gamma-ray cascade takes place originating an inelastic scattering process. In this way the physical characteristics of the high energy level starting the cascade, together with the lower ones populated by the cascade can be efficiently studied analysing the high resolution single spectra, the angular distributions, the coincidence spectra, etc.

In Fig. 1 a sketch of the experimental set-up [6] is shown. In general this technique allows one to study several nuclear parameters such as resonant cross-sections, ground state and total radiation widths [7], and finally to determine the level scheme of nuclei restricted to the transitions starting

![Diagram](image-url)

Fig. 1. – Nuclear spectroscopy work at the RC-1 Reactor: The experimental set-up concerning resonant scattering.

from the resonant level. This kind of spectroscopy gives information similar to those obtainable with \((n, \gamma)\) reactions with the advantage that level spins can be determined by measuring the directional correlations of the elastic and inelastic gamma-rays; further \((\gamma, \gamma')\) reactions are the most suited for
the study of stable isotopes which may not be reached by \((n, \gamma)\) reactions. The choice of the target is considerably restricted by the requirement that a random overlap should exist between the incident gamma-line and a level in the particular nuclide to be studied.

Nevertheless about 50 cases of nuclear resonance fluorescence, almost near the closed-shell regions, have been since observed [8].

We have systematically studied ten nuclei by the nuclear resonant technique [7]. Of these the level scheme of \(^{62}\text{Ni}\), \(^{112}\text{Cd}\), \(^{118}\text{Sn}\) [6], \(^{205}\text{Tl}\) and \(^{65}\text{Cu}\) [9] have been deduced.

An example of the power of the method in spectroscopy work is given in Fig. 2. Monocromatic photons (7646 keV) produced by \(\text{Fe}(n, \gamma)\) reaction, resonantly excite a \(^{205}\text{Tl}\) level. The analysis of all the spectrum shows the existence of more than 20 levels with energies between ground state and \(\sim 3.5\) MeV. In the figure a portion of inelastically scattered gamma-rays, detected by a 30 cc Ge(Li) counter, is reported.

![](image)

**Fig. 2.** – Nuclear spectroscopy work at the RC-1 Reactor: A portion of the (inelastic) spectrum of \(^{205}\text{Tl}\) resonantly excited by \(\text{Fe}(n, \gamma)\) 7646 keV \(\gamma\)-line.

In conclusion we would like to point out that nuclear spectroscopy by gamma-rays resonant scattering excitation leads to results very similar to that achieved by Coulomb excitation and therefore it may be considered a very important part of a research program to be performed with reactors.
3. – Solid state physics work.

Neutron diffraction is a powerful tool for studying some microscopic properties of solids such as magnetic structures, magnetization densities, lattice dynamics, magnetic excitations, phase transitions, etc.

At the Laboratorio di Fisica Nucleare Applicata research has been performed in ferromagnetism by use of the polarized neutrons. The static and dynamic behaviour of unpaired electrons in metals and alloys has been investigated mostly by determining magnetization density maps and dispersion relations of spin waves. So far neutron diffraction has been used at Casaccia mostly for 3d electrons [10]; however recently experiments have been carried out also on 4d and 5d electrons.

Transition metals of 4d or 5d group do not exhibit a magnetically ordered state as elements, but some of their alloys with 3d metals show bulk magnetization properties which were interpreted as indicating a contribution of 4d and 5d electrons to magnetization. Polarized neutrons provide an unique tool for obtaining direct microscopic information and have been successfully used both for elastic and inelastic scattering from some Pt alloys, namely CoPt$_3$, CoPt, and MnPt$_3$ [11]. It has been possible to ascertain that a fairly well-localized magnetic moment of about 0.2 Bohr magnetons is generally present on Pt atoms, and magnetization density maps have indicated the spatial distribution of 5d electrons and the symmetry of their wave functions. These results are appreciably affected by the interaction with neighbour atoms as it is proved by their dependence on the order parameter. A typical magnetization density around a Pt site in a MnPt$_3$ alloy with order parameter $S = 0.22$, is given in Fig. 3. One notices that:

1) The magnetization changes from negative to positive values going from the center of the site outward.

2) The distribution of unpaired electrons is highly aspherical.

The first result can be consistently interpreted by assuming that the moment of Mn atoms occupying the Pt site, because of the incomplete order are directed opposite to bulk magnetization. Their negative magnetization due to 3d electrons which are fairly concentrated prevails around the nucleus while the positive magnetization of 5d electrons which are more spread out is prevalent away from the nucleus. (One must remember that neutron diffraction provides information on the single sites of the unit cell but averaged over all the unit cells of the sample.)

MnPt$_3$ has been also investigated from the dynamical point of view. The acoustical branch of the spin waves dispersion curve has been determined
with polarized neutrons by the «diffraction method», which provides such information with a simple two crystals experiment. The results have been interpreted according to calculations performed at Casaccia [12] on alloys of Cu₉Au type and provided the values for the Mn-Pt and Mn-Mn exchange integral in fair agreement with estimations based on the Ising model.
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Search for New Stable Particles.

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1. Introduction.

Everybody knows the extraordinary contribution which Edoardo Amaldi has given to the development of modern physics, from X-ray to molecular physics, from his classical neutron investigations to problems of nuclear techniques etc., not to speak of his great merits in forming young scientists and, last but not least, in organizing modern physics in Italy (and not only in Italy!).

As a rule the research work of Amaldi is fundamental and quantitative in character, a fact which is evident also in all his first class numerous books and review articles, and yet he occasionally likes to perform brave, qualitative experiments whose significance is going together with a very small \textit{a priori} probability of finding a positive result (for example, the search for magnetic monopoles, or several unpublished old time experiments «a porte chiuse» performed in Rome). For this reason I would like to dedicate the present paper to my friend Edoardo, with whom my first steps in science are closely connected, in the hope that he will not form a too severe judgement of this extremely naive piece of fantasy.

In modern accelerators, to the development of which Amaldi is devoting much of his time, the available collision energy is steadily increasing, so that the question naturally arises among physicists as to whether there might not exist unknown and entirely unpredictable «stable particles» which are produced in such accelerators.

Here stable particles are defined as objects with a mean life \( > 10^{-8} \) s; as it will be explained below, the figure \( 10^{-8} \) s is arbitrary and corresponds simply to the shortest available pulse of accelerated protons in modern machines.

There has already been performed a number of experiments in order to search for new stable particles [1]. All the relevant investigations and proposals
made up till now are characterized by the following circumstances:

a) the search is made for electrically charged particles,

b) for the identification of such particles a beam well resolved in momentum is analyzed and various quantities (momentum, ionization, time of flight,...) are measured without the decay properties of the new particles being investigated.

Below, a method is proposed for the search of both neutral and charged «stable» particles. The advantage of a method which may be operative for neutral as well as for charged particles is immediately evident if one keeps in mind that among the known particles the number of neutral objects is about equal to the number of charged ones.

In order to discover the new particles it is proposed to study their radioactivity properties with the help of a special method.

2. - The idea of the experiment.

There are reasons to assume as a working hypothesis that new particles with mean lives \( \geq 10^{-8} \) s might exist, that is that the transformation of such particles into lighter particles is strongly forbidden in some way. As an illustration we could think, for example, that the decay of the new particle is due to the second order weak interaction in \( G \), the Fermi constant being \( G = 10^{-5}/\text{M}^2 \). Then the probability of decay will be \( 1/\tau \approx G^2 E^0 \), where \( E \) is a certain energy characteristic of the process. If, for example, the \( \Xi \)-hyperon had a mass \( \leq 1115 \) MeV, instead of 1315 MeV, its mean life could be longer than hours! Besides, the existence of a hyperon with strangeness \( -2 \) and mass \( \leq 1115 \) MeV might lead to the appearance of long living quasi-nuclei (a sort of hypernuclei) with special properties and, in particular, to a new form of radioactivity of matter, in which the decay energy is not measured in million electronvolts but is of the order of 100 MeV. However, I wish to stress again that this example is only an illustration and the possibility that the metastability of the new particles, if they really exist, is to be found outside the boundaries of the known physics seems to me much more plausible. Such metastability might be related to the existence of yet unknown quantum numbers, or to something else, for example, to an unusual combination of known quantum numbers [2].

Generally speaking, the body of information accumulated in the region of atomic and nuclear physics tells us that metastability is a property appearing in the most various phenomena, from phosphorescence to nuclear isomery, from the existence of strange particles to the decay \( K^+ \rightarrow \pi^+ + \pi^0 \), etc.
I am just proposing to use electronic methods for the search of a new type of high energy radioactivity, related to the existence of particles which, due to a forbidity of unknown nature, decay with a very long mean life ($\geq 10^{-8}$ s). Below, the assumption will be made that these new particles are strongly scattered by nucleons. As to the production mechanism of such particles, there will not be made any hypothesis.

3. – How to detect the new particles?

I shall illustrate here the case when the new particles are electrically neutral. Then the discovery of the neutron and of its properties tells us how it is possible, in principle, to detect new neutral particles. As it is well known, neutrons may be detected in many ways:

1) There are detected nuclear recoils in elastic collisions of fast neutrons with nuclei (especially protons). Such a method is not adequate for the discovery of new particles, because their flux is expected to be very small, so that the number of nuclear recoils due to the new particles is negligible in comparison with the number of recoils produced by neutrons.

2) Nuclear reactions produced by fast neutron bombardment with the emission, for example, of protons, alpha-particles, etc., are looked for. Such a method is also inadequate for detecting new particles, because their flux is very small.

3) There are observed nuclear reactions (n, γ), (n, p), (n, α), fission, etc. produced by neutrons after they have been slowed down. The possibility of slowing down new particles is not to be excluded, but since such particles are expected to be generated with an energy of several $10^{10}$ eV, the slowing down process requires very large dimensions of moderator (a fact which greatly complicates the detection of the new particles, whose intensity is very small at best). Under certain circumstances, however, (see below) slowing down of new particles could be used.

4) There are observed radioactive properties of the neutron (generally speaking, of the nucleon). Today the observation of the free neutron decay is not a difficult problem; however, it is necessary to have a very intense neutron beam to observe the decay of free neutrons. The detection of the decay of the new particles in their free state is a very unpractical proposition, especially if their mean life exceeds $10^{-7}$ s. But the detection of neutrons turns out to be quite effective if the decay of bound neutrons (that is if the beta radioactivity induced by neutron bombardment) is looked for. The
analogy for new particles would be the search for a special type of radioactivity of pseudonuclei, that is of quasi-nuclei within which the new particle is found together with ordinary nucleons (I do not call these quasi-nuclei « hypernuclei », because by definition hypernuclei are Λ quasi-nuclei: hypernuclei cannot have a mean life much longer than $10^{-10}$ s).

It is natural to expect that the new particle (probably produced together with other particles) in high energy collisions of protons or γ quanta with nuclei, as a rule will leave the original nucleus and then will be « stopped », either suddenly (after a few collisions) or gradually after slowing down by many collisions. For such « stopping » of the new particles a large amount of condensed matter is required; I will not discuss here the corresponding experiments and I shall note that only radiochemistry, which permits the separation of « pure » source of quasi-nuclei from a large amount of irradiated material may give positive results (if the lifetime is long enough).

Below, however, I shall consider the relatively rare but experimentally favourable possibility that in a proton or photon collision with a nucleus a new particle is produced, which is trapped « at the place of birth » (that is, which is found eventually inside the nucleus product of spallation); in such a circumstance, a radioactive quasi-nucleus, analogous to a hypernucleus, will be produced. Of course this requires that the new particle is being strongly scattered (and attracted) by nucleons. Thus the experiment, which will be discussed below, consists in the search for a new type of « radioactivity » (with mean life $>10^{-8}$ s) in a target, irradiated in a very high energy accelerator, the radioactivity being notable for the high energy of its decay products (hundreds of million electronvolts instead of million electronvolts as in the ordinary radioactivity).

Immediately there arises the question: What limits on the production cross-section of such particles can be obtained from experiments already performed? If the mean life of the new particles is less than a few days, there are no limits for the cross-section, because to the best of my knowledge no relevant experiments have been performed. Some limits on production cross-sections, for mean lives greater than, say a few days, can be obtained from the underground experiments of Reines et al. [3] on the degree of accuracy with which the baryon conservation law is known. In these experiments it was found that the carbon nucleus has a mean life longer than $10^{27}$ y (for high energy decays). If we take into account that the carbon compound, of which the detector was made, had been irradiated at the earth surface by a cosmic ray nucleon flux of $10^{-4}$ to $10^{-5}$ cm$^{-2}$ s$^{-1}$, the upper limit for the production cross-section by nucleons of a radioactive quasi-nucleus turns out to be quite large — $10^{-30}$ cm$^2$/nucleus.
3'1. Possibilities of the method proposed. – Let us discuss now what possibilities are given by the method just proposed. An estimate will be made for the case of the Serpukhov accelerator, although it is clear that such experiments could be performed on an accelerator of the CERN, Brookhaven, or SLAC types. Let us consider for example a mean life of the new type of radioactivity of the order of days; in such a case the radioactivity can be investigated far away from the accelerator, in conditions of low cosmic ray background. In spite of the fact that radiochemical separations will not be considered here, still a detection efficiency of about 0.2 or more can be achieved. With an average intensity of $10^{12}$ protons/s, at saturation it is possible to detect the production of radioactive quasi-nuclei with a cross-section of the order of $10^{-40}$ cm$^2$/nucleus, which corresponds to about one decay event per day. If the production cross-section of quasi-nuclei by protons colliding with nuclei is known, one may then obtain the cross-section for the production of new particles in nucleon-nucleon collisions after the introduction of a small coefficient. It is just the requirement that the new particle is found inside the spallation product which leads to the necessity of introducing this small coefficient, the value of which, of course, cannot be estimated a priori. However, if we fantasticate on the analogy between the process considered here and the well-known process of hypernucleus production, we may give a rough estimate, starting from the corresponding experimental data on hypernuclei. It is known that the probabilities of hypernucleus production in photoplates by K mesons of energy 3, 5 and 10 GeV are $(3\pm0.1)$%, $(2.2\pm0.1)$% and $(1.2\pm0.1)$% [5] of the total nuclear collision probability, respectively. Unfortunately at present there are no available data for higher energy kaons, but from the quoted information, and also from the fact [6] that for 25 GeV protons the fraction of nuclear interactions in emulsions which results in hypernucleus formation is $0.5\%$, we may guess a value of 0.005 for the indicated small coefficient.

Thus the proposed method is capable of revealing cross-sections for the formation of new particles in nucleon-nucleon collisions which are ten orders of magnitude smaller than the total nucleon-nucleon cross-section (of course, if the assumptions made are true).

3'2. Remarks on the proposed method. – If possible, the irradiation of the target should require a time comparable with the mean life of the activity which is looked for. For short mean lives one should use the extracted particle beam (at Serpukhov such a beam will consist of 30 proton pulses the length of each pulse lasting $1.5 \times 10^{-8}$ s); this permits us also to take the measurements in the immediate proximity of the target. By means of the classical delayed coincidence method (when the radioactivity is looked for in the time interval between accelerated proton pulses) one may search for mean lives of
the order of $10^{-8}$ s with effective beam intensities of a few percent of the full beam intensity and of the order of $10^{-6}$ s or more at full beam intensity. When investigating mean lives from $10^{-8}$ s to a few microseconds one must pay attention to the pion and muon background.

By the way, when searching for the new type of radioactivity with a mean life in the microsecond region, the most adequate beam time structure is to be found in electron linear accelerators (SLAC and Kharkow), where the beam time length is of the order of microseconds with a repetition rate of 100 Hz.

An extracted proton beam is convenient also when looking for mean lives less than a few hours, although in such a case the internal target may be used.

The shortest mean life which can be looked for in the internal target of the Serpukhov accelerator is of the order of milliseconds (as such is the time required to put the target into the beam). If the internal target is used, it is highly desirable to take measurements in one of the straight sections, because this allows a larger solid angle to be seen by the detector at the target.

In the search for activities with mean lives greater than a few hours, the internal target can be removed and investigated in conditions of very low cosmic ray background and a high solid angle detector. One can consider the possibility of using a liquid internal target, which can be easily removed from the vacuum chamber.

In the search for «radioactivities» with long mean lives there are two difficulties which are present also, to a less degree, in the search for shorter mean lives.

1) The main source of background is due to cosmic ray muons, the integrated flux of which at the earth surface is about 0.01 cm$^{-2}$ s$^{-1}$, and also to nuclear «stars» produced by cosmic ray neutrons. It is evident that investigating the target «radioactivity» underground has great advantages in the search for long mean lives. In the most deep existing underground laboratories the cosmic muon intensity decreases by a factor of $10^8$. In such conditions there is no background even in the absence of an anticoincidence system. Such system, which can easily decrease the muon background by a factor of 1000, should be used if the measurements are made near the earth surface.

2) The irradiated target is strongly active due to the presence of spallation products. This has the effect that no full advantage for decreasing the cosmic ray muon background can be made of the fact that a target of very small dimensions (say < 1 cm$^2$) can be used; as a matter of fact, there will be many accidental coincidences between the counters through which pass cosmic muons and the small area counter, placed in the immediate proximity
of the small target. It may be necessary to place a filter between the target and the detector to decrease strongly the beta radioactivity.

One of the detector elements must be an energy spectrometer, let us say a NaI chrystral (or a lead glass spectrometer, etc. if high energy gamma's are looked for).

If the measurements are made at the earth surface it may turn out to be necessary to use some kind of track chamber to reject the events in which the particles are not coming out of the (small) target.

Here I would like to mention another possible registration arrangement. When a heavy ($Z \gg 80$) quasi-nucleus decays, the decay products may induce with reasonable probability the fission of the nucleus. Consequently there raises the probability of searching for a «radioactivity» with emission of fission fragments in a thin heavy target (made of an element not undergoing spontaneous fission, let us say Th) irradiated by high-energy particles. The interest in this arrangement is due to the possibility of detecting (even at the earth surface) very rare fission events of a substance having an extremely high beta activity.

One might also consider the search, deep underground, for a delayed emission of a few neutrons from a heavy material irradiated by high-energy particles, because it is well known that a heavy nucleus excited at a few hundred million electronvolts emits many evaporation neutrons.

4. — Conclusion.

The well-known methods of observing neutral particles (decay in flight, missing mass spectrometer) are adequate only if the mean life is short enough or if the corresponding production cross-section is relatively large.

It is evident that the present proposal (a search for a «radioactivity» of a special type) is quite naïve, a fact which I fully recognize. However the proposal is relatively simple and, independent of the ideas expressed in this paper, the suggested experiment has a definite phenomenological interest.

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Note added in proof. - After this paper was written, Dr. Giacomelli has kindly informed me about an interesting investigation [7], which is relevant to the question discussed above from an experimental point of view, although it originated from a completely different «phylosophy». A search was made for magnetic monopoles, which might have been
produced in collisions of high-energy protons with nuclei. In order to detect the products of a possible monopole-antimonopole annihilation, the authors looked for a high energy radiation from a target irradiated by 27.5 GeV protons. No effect was found, the detector being sensible to electrons and photons in the time interval from 0.1 s to 1 day after the production of the monopole-antimonopole pair in targets of Al, polyethylene, and Cu. According to this investigation the upper limit for the production cross-section in light elements of a radioactive quasi-nucleus of the type discussed in this paper turns out to be several orders of magnitude smaller than that from ref. [3].

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The Isobaric Analog Resonances in Phenomenological Nuclear Spectroscopy.

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1. - Introduction.

The remarkable achievements of nuclear spectroscopy in the last ten years have been made possible by the advent of new types of high resolution accelerators, such as Tandem Van de Graaff, to produce particle beams in a large interval of masses and energies, and by the solid state detectors upon which the so-called «in beam nuclear spectroscopy» is based. These technical advances have allowed measurements that only a few years ago were impossible and a considerable volume of new information is now available for theoretical investigations. A large number of nuclear states have been identified and classified by means of precise determinations of energy, angular momentum, parity, and transition probability. A systematic knowledge of such attributes of nuclear levels is essential for any attempt to relate them theoretically and provide the experimental ground for testing the validity of the current microscopic theories the goal of which is to disclose the fundamental features of the nuclear structure.

The aim of this paper is to give a comprehensive outline of the so-called isobaric analog resonances, a promising and relatively new topic in nuclear physics. The outstanding importance of the isobaric analog resonances (IAR) is brought about by the fact that in many cases they give the clue for the correct interpretation of the observed structures in nuclear excitation spectra.

It is well known that the low excitation levels (bound states) are characteristic of the collective motion of the nucleus, namely rotation and/or vibration modes or motion of mutually interacting particles. This situation corresponds to an average width \( \Gamma \) (decay probability) very small as com-
pared to the average distance $D$ between levels, i.e., $\Gamma \ll D$. By gradually increasing the excitation energy, the level density also increases and, above the nucleon threshold, the nucleus « enters » the continuum region. In the neighborhood of this region the nuclear states, obtained through capture reactions (formation of the compound nucleus), decay to the levels of the residual nucleus and the yield relative to the different exit channels exhibits a resonant behavior (compound nucleus resonance): Since $\Gamma$ is of the order of $D$ ($\Gamma \sim D$) the resonances are observed separated provided the measurements are carried out with adequate energy resolution ($\delta E \ll \Gamma$).

At higher excitation energies the situation becomes more complicated due to the overlap of the nuclear levels. The existence of statistical fluctuations, namely a sort of background noise due to the behavior of the reaction amplitudes, is nowadays a well-established fact. The reaction partial amplitudes, randomly distributed over a large number of levels, sum up coherently with a width $\Gamma_c$ (coherence width): this implies that when $\delta E < \Gamma$, namely when the measurements are performed with high resolution, the fluctuations can be observed as superimposed to an average behavior generally nonresonant-like [1]. Characteristic resonances, however, can distort the nonresonant behavior: evidence has been obtained in the last few years of such an anomalous behaviour in experiments with not very high energy resolution. This kind of resonances has been called « intermediate structures »: They correspond to peculiar nuclear states, completely determined by the entrance channel, which give rise to the excitation of resonances of the compound nucleus through particular correlations (doorway states) [2]. The improvement of the resolution brings to view a fine structure composed by several narrow resonances which should be properly correlated in order to give the single gross resonance observed in experiments which de facto average the reaction cross-section. It is evident that a clear-cut identification of an intermediate structure is bound to the fact that the observed effect on the cross-section be unambiguously independent of any averaging procedure.

It is conceivable that by virtue of some specific interaction the doorway state mixes coherently in the resonant region with the compound nucleus resonances nearby localized. The stronger such a mixing is, the more correlated are the nuclear states with the entrance channel, contributing to the intermediate resonance and more crucial is the effect of the presumptive doorway state on the decay behavior of the compound nucleus. So far these predictions have not yet been corroborated by a clear-cut experimental support because of the difficulty of performing measurements satisfying the following conditions [3]:

$$D_I > \Gamma_I, \quad D_I \gg D, 2\Gamma,$$
where \( D_I \) is the average spacing of intermediate states and \( \Gamma_I \) is their average width, while \( D \) and \( \Gamma \) refer to the compound nucleus. There are two cases where the above conditions are fulfilled: the giant resonance and the isobaric analog resonance.

2. – Isobaric analog states as resonances.

The essential standpoint was the discovery of the possibility of producing isobaric analog states as compound-nucleus resonances in reactions initiated by low-energy protons (below the Coulomb barrier) on medium and heavy nuclei. The pioneer experiment was performed by the Florida State group [4] who found strong anomalies in the excitation curve of the \(^{89}\text{Y}(p, n)\) reaction clearly arising from the \(^{89}\text{Y} + p\) compound system (\(^{90}\text{Zr}\)) and reproducing with good accuracy the intervals of the low-lying bound states of the \(^{89}\text{Y} + n\) (\(^{90}\text{Y}\)) isobar.

The standard analysis of the elastic proton-scattering data revealed the exact correspondence of spin and parities; moreover the « analog spectrum » was found to be displaced relatively to the isobaric « parent spectrum » by the expected Coulomb energy difference. This was the second important step in the modern history of the isobaric-spin nuclear spectroscopy. The first one was the identification of new modes of excitation in charge-exchange reactions as isobaric analog states. The anomalous sharp peak found in the neutron evaporation spectrum following the \(^{51}\text{V}(p, n)^{51}\text{Cr}\) reaction by Anderson and Wong [5] was interpreted as the isobaric analog of the target ground state. Since then this situation was found to hold for other heavier nuclei and not to be restricted to light nuclei as it was believed for a long time.

These experiments and their interpretation in terms of isospin dependence of the optical potential, as given by Lane [6], opened the current interest in isobaric analog states (IAS) and buried the old superstition that the classification of nuclear states in terms of isobaric spin was useless for medium and heavy nuclei (\( i.e., Z > 10 \)) because of the strong Coulomb interaction. In this connection it is worthwhile to mention that since 1961 French and Mac Farlane [7] have introduced the concept of isobaric splitting in describing the distribution of spectroscopic strengths in particle-transfer reactions even in medium-weight nuclei.

In fact, the use of isobaric spin in nuclear spectroscopy since the theory developed by Wigner in 1937 [8], was strictly connected with the occurrence of isobaric-spin multiplets in the assumed charge-independent nuclear world. In this world the knowledge of the properties of one member of a given multiplet is sufficient to describe completely the other members (the « isobaric
Isobaric analog resonances

analogs» of the former). However, the nuclear Hamiltonian is not exactly charge-independent; in the limits that charge-dependent effects such as the Coulomb interaction between protons are not so strong to cancel the regularities predicted by the charge-independent part, the isobaric spin is still a good quantum number. In light nuclei the Coulomb mixing is only a very slight perturbation because of the quite large average spacing between states of different isospin; consequently, isobaric multiplets were found to hold in this case with analog states obtained by simply applying the charge-exchange operator $T^-$ to the parent states.

The situation is different for medium and heavy nuclei, where the symmetry and pairing energies can put the analog states in the continuum region and the strong Coulomb interaction can mix them with nearby states of different isospin; moreover they can become proton unstable, while the parent states (low-lying) are stable against particle emission (bound). The connection between such states cannot be simply assured by the $T^-$ operator and the isobaric-spin correspondence may be completely destroyed.

The discovery of isobaric analog resonances (IAR) has been taken as an indication of the survival of the isobaric-spin (charge-independent) description in the continuum region, where the IAS can be interpreted as some kind of «special states» embedded in a dense spectrum of complicated compound-nucleus states [9].

A schematic picture of the correspondence between such IAS and parent states is shown in Fig. 1.

The $(Z-1, N+1)$ and $(Z, N)$ nuclei are connected through the charge-exchange operation $(n \rightarrow p)$; neglecting the Coulomb interaction, it may happen that the levels of the former ones are located at exactly the same energy in the continuum region of the latter ones. Due to symmetry effects in the nuclear interaction, the displacement of the states $T_>$ and $T_<$ is expressed by the term $E_{sym}$ the value of which for medium and heavy nuclei may be higher than the threshold for neutron emission. Without Coulomb interaction the states $T_>$ in the $(Z, N)$ nuclei would have exactly the same configuration of the parent states in the $(Z-1, N+1)$ nuclei; because of the isospin difference, these states do not mix with those of the continuum and therefore are bound states.

The Coulomb interaction gives rise to a displacement of the spectrum of the $(Z, N)$ nucleus which amounts to the difference $\Delta E_C$ (*) between the Coulomb energy of the two systems; furthermore, the proton emission

(*) The term $\Delta E_C$ considered here accounts also for the neutron-proton mass difference which should be included in the atomic mass scale. Only relative shifts are considered here.
threshold is lowered and it may occur that the analog states become unstable against proton emission. In this case it is possible to excite an analog state as a resonance by means of elastic scattering of protons. It has to be remarked that the Coulomb interaction favors the mixing of the $T_>$ state with the neighboring ones correlated with it by the same set of spin and parity.
attributes. In this way, exit channels responsible for the decay of the $T_<$ states are open; it follows that, by virtue of charge-dependent effects brought about by the Coulomb mixing, an analog state becomes indeed observable through an isospin-forbidden decay (for example, neutron emission). In the first experiment carried out by the research group of the Florida State University [4], which led to the discovery of the isobaric analog resonances, anomalous nuclear structures were observed in the excitation curve of a $(p, n)$ reaction.

In conclusion, an analog state in the continuum appears as an intermediate structure induced by an elementary excitation, namely a charge-exchange interaction, which stimulates the excitation of the neighboring states of the compound nucleus through the Coulomb mixing and acquires their decay properties.

3. – Production and decay modes of IAR.

Figure 2 shows a schematic diagram of a proton-induced reaction with excitation of IAR and the isospin-allowed and forbidden decay of these latter.

Fig. 2. – Schematic diagram of a proton-induced reaction with production of IAR; the neutron or $\alpha$-decay of IAS is made possible by the coupling with the normal $T_<$ compound nucleus states.
The bombarding proton energy (in the center-of-mass) needed for exciting IAS via a resonance reaction is:

$$E_{p}^{cm} = \Delta E_{C} - S_{n},$$

where $\Delta E_{C}$ is the Coulomb shift and $S_{n}$ the neutron separation energy.

In light nuclei, in spite of the low Coulomb barrier, states which are the analog of low-lying bound states cannot be excited in resonance reactions, since $\Delta E_{C} < S_{n}$. Now, $\Delta E_{C}$ increases with $Z$ while $S_{n}$ decreases slowly with increasing $A$; consequently IAR become available for medium and heavy nuclei. On the other hand the required proton bombarding energy becomes higher and higher; this explains the fact that such experiments are generally done with Tandem accelerators which cover the energy interval required for a large amount of nuclei and whose energy resolution is far superior to the energy resolution of either the incident proton or the outgoing particle detection devices.

Nevertheless, in recent years, some interesting investigations on IAR have been performed even at subtandem energies (up to 5.5 MeV) with Van der Graaff accelerators in connection with properly used proton and gamma high-resolution spectrometry. In fact, the available experimental data on the fine-structure and gamma-decay properties of IAR have been obtained mostly at subtandem energies and in the region of medium-weight nuclei (2s-1d and 1f_{5} shell).

These properties are of fundamental importance in the analysis of IAR and the related spectroscopic information. Furthermore, the standard elastic scattering analysis in terms of interference pattern between resonance and potential scattering allows the extraction of spectroscopic parameters (reduced proton widths) related to the properties of the parent bound states.

3'1. Elastic scattering and spectroscopic information. – An IAS in the continuum is a resonance. A fundamental problem is to explain how it can be putted in the same isobaric multiplet of a bound state. More precisely, the fundamental task is to understand the connection existing between the appearance of an isobaric analog resonance and the strict isobaric correspondence with a bound state and what role is played by correlations, different from the simple Coulomb mixing, between the entrance channel and the normal compound nucleus state in generating the observed resonance. Several theoretical approaches attempt to find an answer for all these open problems and to develop reliable methods for the analysis of excitation curves with the final goal of extracting the spectroscopic parameters characterizing the IAR [9, 10, 11].
From a phenomenological point of view, which is of interest here, the IAR observed in proton elastic scattering can be related directly with the corresponding parent states via their spectroscopic properties. A typical curve for a \((p, p)\) elastic excitation function is shown in Fig. 3.

The curve is analyzed in terms of a resonance plus potential scattering yielding to the resonance energy \(E_R\), the total width \(\Gamma\), and the partial proton width \(\Gamma_p\); moreover, the interference pattern observed at different scat-

![Graph showing excitation function for elastic proton scattering on \(^{92}\text{Zr}\) in the region of the \(^{92}\text{Nb}\) ground-state isobaric analog (schematic, see ref. [12]). Also shown is the neutron excitation curve for the \(^{92}\text{Zr}(p, n)\) reaction in the same energy range.](image-url)
tering angles yields to the proton angular momentum \( l_p \) and consequently to the parity, and, in some cases, to the spin of IAR. The spin is uniquely determined when also the polarization of the scattered protons could be measured [13]. The correspondence with the isobaric parent state is first given by energy and quantum numbers. Furthermore, a clear correlation exists between the elastic width and the stripping spectroscopic factors of these states.

From Fig. 2 it is seen that the low excitation states of the parent isobar \( (T = T_z = T_\sigma) \) may be produced by means of a stripping reaction through the transfer of one neutron to the same target nucleus. The spectroscopic information thus obtained concerns not only the energy, spin, and parity but also the spectroscopic factor \( S_{d,p} \), which, in single-particle reduced width \( \gamma_{s,p}^2 \) units, expresses the reduced width \( \gamma_n^2 \) for neutron capture into a bound parent state: \( S_{d,p} = \gamma_d^2/\gamma_{s,p}^2 \).

Starting from the same target state the elastic scattering cross-section for the analogue resonance yields to the reduced width \( \gamma_p^2 \) for proton capture into the analog state, through the relation \( \gamma_p^2 = \Gamma_p^{obs}/2P_t \), where \( \Gamma_p^{obs} \) is the observed partial width and \( P_t \) is the transmission coefficient which allows the proton to traverse the momentum discontinuity at nuclear surface and to penetrate the angular momentum and Coulomb barrier.

If \( T_0 \) is the isobaric spin of the target the isobaric analog state spends only a \( 1/(2T_0 + 1) \) fraction of the time as a proton state; consequently:

\[
\frac{\gamma_p^2}{\gamma_n^2} = \frac{1}{2T_0 + 1},
\]

i.e.,

\[
\frac{\gamma_p^2}{\gamma_{s,p}^2} = \frac{1}{2T_0 + 1} S_{d,p}.
\]

This result gives the correspondence between the nuclear structure of the analogue resonance and that of the parent state through the correlation with the same initial state (target nucleus). Unfortunately, current methods used for extracting the spectroscopic factors by means of distorted wave analysis give results affected by large uncertainties (of the order of (20 ± 30)\%), whereas the determination of the reduced width from elastic scattering experiments is generally limited because of difficulties in evaluating correctly the transmission coefficient which may be modified in a complicated way by the presence of the \( T_\sigma \) states. The study of the IAR will become an independent spectroscopic tool of outstanding importance as soon as appropriate methods will be developed for the determination of accurate spectroscopic factor from the data [9]. So far the study of the inelastic scattering through the
IAR seems to be more promising. In this case, the excited states of the target nucleus are correlated with the isobaric analog resonance. The comparison of the inelastic and elastic excitation curves gives information on the configurations existing in those states.

3'2. Fine structure. – The « gross anomalies » observed in proton elastic scattering have generally total widths of the order of 10 times the widths expected for the normal compound-nucleus resonances. This is what is found in poor-resolution experiments and is generally interpreted in terms of a « gross structure » averaging over individual resonances just as an intermediate state. The main effect of the \( T_\rangle \) resonance is associated with the interference between Coulomb and resonance scattering. However, the shape of the gross anomaly shows some departures from the pure Breit-Wigner form, with a typical asymmetric behavior (see Fig. 3).

This was interpreted by Robson [9] in the framework of the \( R \)-matrix theory as a consequence of a coherent interference between two contributions arising from two different regions of the configuration space: one is internal to the nucleon-charge distribution \( (r < r_c) \), where \( T_\rangle \) is a good quantum number (no Coulomb mixing); the other is the external region allowing Coulomb mixing. When the two solutions of the Schrödinger equation are smoothly joined in order to calculate the collision matrix, the presence of the \( T_\rangle \) IAS contributes to the widths of the normal compound-nucleus states with the same attributes defined in the internal region, giving rise to an enhancement of such states. This enhancement is asymmetric as a function of the energy and vanishes just at the energy of the IAS, which is found to be higher than the energy at which the elastic resonance occurs.

This prediction was confirmed in the high-resolution experiments where the fine structure of IAR resolved. The typical example is shown in Fig. 4, where the fine structure covered by the IAR found in the \( ^{92}\text{Mo}(p, p) \) experiment is seen to quench more rapidly on the high-energy side of the excitation curve.

We are dealing, in this case, with a typical heavy compound nucleus \( (^{99}\text{Tc}) \) with high normal level density in the analog state region; then the fine structure is interpreted as being due to the fluctuations of many overlapping levels with the same spin and parity as the analog one \( (T_\rangle) \) but with lower isospin \( (T_\langle) \). This is related to the fact that the best fit to the experimental cross-section, in the poor resolution measurement, yields a partial proton width \( \Gamma_p \) which accounts for only \( \sim \frac{1}{3} \) of the total width \( \Gamma_R \); \( \Gamma_p = \Gamma_R \) would have been expected in this case, since we are below the Coulomb barrier and the sole proton channel is open [15]. More insight in this matter is gained when the experiment can resolve all the \( T_\langle \) individual components in the region of the
Fig. 4. – Gross and fine structure of the IAR corresponding to the $^{92}$Mo ground state ($T = T_z = 9/2$) in the $^{92}$Mo(p, p) reaction (schematic, see ref. [15]). On the left-hand side the schematic diagram of the reaction is shown.

analog state. Few experiments of this type have been performed so far, all confined to medium-weight nuclei such as $^{41}$K [16], $^{48}$Sc [17-20], $^{23}$Na [21], $^{43,45}$Sc [22]. It is interesting to note that, due to the rather low neutron separation energy in all these cases such experiments have been performed at substand energies (i.e., with the Van de Graaff accelerators of the Duke, Padua and Utrecht Universities). The high overall resolution obtained in these experiments (from 0.2 to 0.8 keV) could separate the individual compound-nucleus resonances averaged over by the IAR. In fact we are dealing here with nuclei where the level density at the IAS energy is not so high as in heavier nuclei, so that the best energy resolution actually available from accelerating machines is enough for such measurements.

All these experiments clearly show that the sum of the partial widths $\Gamma_{p\lambda}$ is less than the total width $\Gamma_R$ of the intermediate resonance, i.e., the tails of the individual resonances add coherently to produce one large resonance with a great amount of nonresonant range inbetween. The difference $W = \Gamma_R - \sum \Gamma_{p\lambda}$ is the so-called «spreading width» (statistical contribution in the region of the IAS).

Figures 5 and 6 show schematically two typical examples of fine structure experiments, the first one being the $^{40}$Ar+p reaction with essentially the proton
channel open [16], the second being the $^{48}$Ca + p experiment where also the neutron and gamma-ray channels are open and indeed measured [17-19].

In the first case 17 individual resonances were found, every one with spin and parity $3/2^-$ and with a distribution corresponding exactly to the shape of the Lane-Wigner-Thomas giant resonance in $^{41}$K; this «micro-giant» resonant structure corresponding to the analog strength being shared among the surrounding normal states, was already predicted by Bloch and Schiffer [23], for the case where the spacing between such states is much less than the mixing strength. The enhancement of the compound-nucleus resonance appears clearly as due to the mixing with the nearby analog state, while the fact that, for each resonance, $I'_p < I$ shows the presence of a nonvanishing spreading (damping) width, arising from the coupling with other nonresonant degrees of freedom of the target plus proton system [11].

In the second case the IAR corresponding to the $^{49}$Ca ground-state isobaric analog in the $^{48}$Ca + p ($^{49}$Sc) system was resolved in individual components singled out in the isospin-forbidden (p, nγ) reaction [17] performed with the 5.5 MeV Van de Graaff of Padua indicating a more detailed structure than that found in a first (p, p) experiment with not very high energy resolution [19]; a successive high-resolution elastic scattering experiment performed at the Utrecht Van de Graaff accelerator [18] confirmed the presence of at

![Reaction diagram](image)

Fig. 5. – Reaction diagram (a) and fine structure (c) of the 1.87 MeV IAR found in $^{49}$A(p, p) experiment at Duke University [16] (schematic); the IAS ($T_z = 3/2$) corresponds to the fourth excited state ($T = 3/2^-$) of the parent $^{41}$A. The corresponding gross structure found earlier at Yowa [12] is also shown schematically (b).
least 7 individual resonances with the expected $3/2^-$ assignment and with $\sum I_{p,i} = (2.2 \pm 0.3)$ keV rather smaller than the single-particle value 4.4 keV obtained from the stripping spectroscopic factor $S_{d,p} = 1$ found in the $^{48}$Ca(d, p)$^{49}$Ca reaction [24]. A similar result has been obtained in the recent experiment performed at Duke [20], where an additional $3/2^-$ resonance has been found and a value of 1.85 keV for the sum of the proton widths has been reported; this value, compared with the estimated [19] width for a single-particle ($3/2^-$) resonance, gives a spectroscopic factor $S_{d,p} = 0.6$.

3'3. Shell model description of IAR. – The $^{48}$Ca+p system is interesting from several points of view. First, it corresponds to a particle outside a doubly-closed shell core ($^{48}$Ca has $Z = 20$ and $N = 28$); consequently it is a case suitable for a shell-model description of the isobaric analog resonances [10, 25].
The basis for a microscopic shell model analysis of IAR is the particle-hole picture of Bloch and Feshbach [2], where the nucleon-nucleon interaction allows the mixing between the virtual state of the proton plus target system with a 2 particle-1 hole state in the compound nucleus; such $2p-1h$ state is the doorway state with isobaric analog configuration ($T = T_>$), giving rise to the intermediate structure in the presence of the neighboring $T_<$ states; these latter correspond to much more complicated configurations ($3p-2h$, etc.). The particle-hole picture for the $^{49}\text{Ca}$ ground-state analog is schematically represented in Fig. 7.

![Diagram of particle-hole picture for the $^{49}\text{Ca}$ ground-state analog]

The $2p-1h$ state serves as an intermediary between the incident proton and the complicated $3p-1h$ states, since it can mix with them while the proton entrance channel cannot; the doorway can decay into the $T_<$ states forming an intermediate structure, whose total width is given by

$$\Gamma_B = \Gamma^i + \Gamma^1,$$

where $\Gamma^i$ is the «escape width» for the single-particle decay in the continuum and $\Gamma^1$ is the «damping width» (corresponding to the spreading width already mentioned) i.e., the decay width into more complicated modes. Now $\Gamma^1 = \sum_i \Gamma_i$, where the index $i$ refers to all the open exit channels (i.e., p, n
and $\gamma$ in our case), consequently

$$\Gamma_R = (\Gamma_p + \Gamma_n + \Gamma_\gamma) + \Gamma^1.$$  

Following the results of refs. [17-19] and the analysis of the intermediate structure performed in ref. [19], one has

$$\Gamma_p + \Gamma_n = (2.4 \pm 0.5) \text{ keV (refs. [17, 18])} = (3.4 \pm 0.8) \text{ keV (ref. [19])},$$

$$\Gamma_\gamma = 0.24 \text{ eV (ref. [18])},$$

$$\Gamma_R = (4.7 \pm 0.4) \text{ keV (ref. [19])},$$

$$\Gamma_{\text{exp}} = (2.3 \pm 0.7) \text{ or } (1.3 \pm 0.8) \text{ keV},$$

$$\Gamma_{\text{theory}} = 0.6 \text{ keV (ref. [19])}.$$

A better evaluation of the widths for the isospin-forbidden neutron decay seems to be necessary in order to ascertain if there is a real discrepancy between the experimental and calculated damping width; this is of some importance since a large spreading width corresponds to the interference of other possible degrees of freedom to be considered in a suitable classification [11].

3'4. Neutron decay. – A second interesting aspect of the $^{48}\text{Ca}+\text{p}$ experiment is the presence of the neutron exit channel. Since it is generally assumed that the low-lying levels of the residual nucleus arising from the (p, n) reaction ($^{48}\text{Sc}$ in the present case) are normal $T_\prec = T_z = T_\circ - 1$ states, with reasonably pure isobaric spin, the neutron decay from the $T_z = T_\circ + \frac{1}{2}$ IAS should be forbidden ($\Delta T = \frac{3}{2}$) (see Figs. 2 and 6). As already mentioned, its occurrence is taken as an indication of the mixing between the IAS and the surrounding $T_\prec$ states; on the other hand this makes it possible to characterise the IAR through the sharing of its strength among the normal compound-nucleus states. The case in point here is a typical example.

Indeed it was found in a first poor resolution experiment (see ref. [17]) that the neutron decay at a proton bombarding energy corresponding to the gross resonance was selective for the residual states of $^{48}\text{Sc}$. In other words, there exists a preferential decay of the analog state to one of the levels of the residual nucleus (due to a particular configuration of the latter one), namely to the level at 1.4 MeV which undergoes a strong gamma transition of 0.78 MeV. The yield of this transition, determined by means of gamma spectroscopy, provides an excellent measurement of the excitation of the analog resonance and of its fine structure. This is clearly seen in Fig. 8, where the yields of 0.78 MeV and 0.37 MeV $\gamma$-rays are compared, the latter
one being originated in transitions from a lower level which collects most of the neutron decay from the compound nucleus states. The selection of the 0.78 MeV γ-ray allows, therefore, the suppression of the background due to resonances and decay modes different from those mixed with the analog state.

Another interesting fact is that the shape of the enveloping distribution is almost Gaussian, as is expected in as much as the entrance channel can only produce the higher isospin and the neutron emission can only proceed from the lower isospin; the asymmetry of such a distribution is less pronounced than in other typical cases, where a dip is found in the neutron excitation curve just at the energy of the IAS at which the isospin-violating processes should vanish following the Lane-Robson model (see Fig. 3).

3’5. Gamma (radiative)-decay. – The third point of interest in the present example is the determination of the gamma-decay properties of the IAR. Such properties are of considerable interest in nuclear spectroscopy, since the gamma-transition probabilities are strongly correlated with the intrinsic structure of the nuclear states. In fact it has been pointed out that the gamma-decay of IAS observed in (p, γ) reactions may be very simple [26]. Let us consider the case of quite pure IAS \( (T_\pi = T_\pi + 1) \) with single-particle character in the compound nucleus formed in a (p, γ) reaction; in the same
nucleus at least 1 bound state with exactly the same nuclear configuration, but with different isospin symmetry \((T_<, T_>)\) exists, due to the isospin splitting of single-particle states [7] (see Fig. 9). Such a state is called an «antianalog» [26] or «homolog» [27] state and should be connected with the IAS via a strong M1 transition [28] of the order of the Weisskopf unit. The work of the Utrecht [26] and Ohio [28] groups in the 2s-1d shell was successful in locating, through \((p, \gamma)\) reactions, several strong IAR with rather little isospin mixing and strong M1 analog to antianalog transitions. Such cases are rather simple, because only the channels for proton and gamma-ray emission are open, the analog and antianalog states are essentially not fragmented and the corresponding gamma-transition connecting them stands out clearly in the gamma-spectrum taken at the resonance energy.

This is due to the weak coupling between the single-particle state and the core which forms a \((T_<, T_>)\) doublet with spin \(J = j\) if the core is a \(J_0 = 0\) nucleus (see Fig. 9). Theoretical calculations performed by Maripuu [29] show that a strong enhancement of M1 transition rates between the two members of the isospin-doublet arises for the «parallel» case \(J = j = l + \frac{1}{2}\) (i.e., when the orbit and the spin of the added proton are parallel) as compared with the «antiparallel» case \(J = l - \frac{1}{2}\). The experimental data in the
2s-1d shell nuclei generally agree with these predictions. A rather different situation is found in the 1f\textsubscript{3/2} shell, where data concerning 49Sc (we are discussing here), 49V [30] and 51V [31] are now available.

In the 49Sc case, for instance, the coupling between the 1p\textsubscript{3/2} single-particle state and the 48Ca core seems not to be weak so that it becomes fragmented, especially for the lower (T\textsubscript{<}) member; moreover, as we have seen, the IAS is rather strongly mixed with the neighboring compound-nucleus states with complicated configurations. The gamma-branching from the IAR is then less simple, but could give important information for a microscopic description.

The result found in the 48Ca(p,\gamma) experiment [18] show the presence of an E2 ground-state transition from the major resonance present in the fine structure of the IAR and the absence of M1 transition (less than 6 \times 10\textsuperscript{-4} Weisskopf unities) to the expected strongest fragment of the antianalogue state (the 3.08 MeV level with stripping spectroscopic factor of about 0.6). This could be related to a much more fragmentation of the 1p\textsubscript{3/2} single-particle state and to a strong mixing of the IAS with more complicated (3p-1h) configuration at such a resonance energy.

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REFERENCES

The Crab Nebula.
Ancient History and Recent Discoveries. (*)

B. B. Rossi


1. – The Chinese and Japanese chronicles for the year 1054 of the Christian era registered the sudden appearance in the constellation of Taurus of a new star—a «guest star»—of extraordinary brightness, which gradually faded away until, some two years later, it was no longer visible.

Centuries went by, and hardly anyone was aware of this event when, in 1771, the French astronomer Messier compiled a catalogue of all known comet-like objects (nebulae and clusters) that appeared to occupy fixed positions in the sky. The first item on his list (M1) was a nebula in the constellation of Taurus, about 4 arc minutes across, whose existence had been known for about 40 years. During the following decades this nebula was observed repeatedly with improved telescopes. In 1848 the shape of the object suggested to the British astronomer Lord Ross the name of Crab Nebula, which has been since generally accepted.

The next event of crucial importance for the present story was the detection at a Baltic observatory, in 1885, of an exceedingly bright star in the Andromeda galaxy, that was the result of a sudden flare up. In the subsequent years, a number of similar stellar outbursts were observed in external galaxies. In some cases the brightness of the «new» star was comparable to or even greater than the total brightness of the galaxy before the outburst. By 1920, it had become generally accepted that these extraordinary outbursts were not limiting cases of ordinary novae, but were to be regarded as an entirely different class of astronomical events. Since the late thirties these events have been known as supernovae.

The discovery of supernovae in external galaxies stimulated the interest of astronomers in the historical records of events that might be interpreted

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as supernovae outbursts within our own galaxy, and prompted them to search for celestial objects that might be regarded as remnants of these outbursts. In the early twenties astronomers noticed the coincidence between the position of the Crab Nebula and the position of the «guest star» of 1054, as could be deduced from the descriptions contained in the oriental chronicles. They also discovered that the angular dimensions of the Crab Nebula were gradually increasing. Under the assumption that the nebula had originated from a point-like object and had undergone uniform expansion since its birth, it was possible to compute its age, which turned out to be close to the time elapsed since the appearance of the «guest star». On the basis of these results Hubble, in 1928, suggested that this event had been a supernova outburst, and that the Crab Nebula was its remnant.

In the following years the very powerful optical telescopes which by then had become available were applied to a systematic study of the Crab. It was

Fig. 1. – a) Picture of the Crab Nebula in «white light» (taken through a polaroid filter), showing the diffuse luminosity.
found that this object consisted of an «amorphous mass», in which long and thin «filaments» were embedded. The light from the «amorphous mass» (which accounted for over 90% of the whole optical emission from the Crab) had a continuous featureless spectrum. In the light from the filaments, on the other hand, the lines of the known elements (especially the Hα line of hydrogen) appeared prominently (see Fig. 1).

Fig. 1.  b) Picture of the Crab Nebula in Hα (taken through an interference filter), showing the filamentary structure (Mt. Wilson and Palomar Observatories).

The spectral lines of individual filaments were observed to exhibit Doppler shifts, which were interpreted as due to the expansion of the nebula. From this effect, the radial component of the velocity of expansion was found to be a little over 1000 km/s. This result together with the observed rate of increase of the angular radius (0.21 arc s/y) provided an estimate of 5000
light years for the distance of the Crab Nebula, under the assumption that the velocities of expansion along the line of sight and perpendicularly to it were identical. (However, it is now believed that the expansion may not be exactly isotropic, and consequently that the above estimate of the distance may be in error by some 20\%, probably on the low side.)

In the meantime, theoretical ideas pertinent to the supernova phenomenon began to emerge. Already in 1939 Oppenheimer and his collaborators addressed themselves to the problem of what happens when the nuclear fuel in the central part of a star is nearly exhausted, so that the pressure of the radiation generated by the nuclear reactions can no longer balance the forces of gravitational attraction. They found that, depending on its mass, the star will collapse either into a «white dwarf», or into a lump of nuclear matter, \textit{i.e.}, a «neutron star». According to present views, prior to the final collapse, part of the stellar mass is blown out into space, perhaps because of the sudden ignition of the remaining nuclear fuel. This outburst manifests itself as a supernova, and the ejected matter forms the cloud later found at the location of the outburst.

By then, astronomers had discovered two faint stars near the center of the Crab Nebula, and had suggested that either of them might be the residual condensed object of the supernova explosion of 1054. However, while one of these stars had an entirely «normal» spectrum, the other (known to the astronomers as the \textit{south preceding star}) was found to have a featureless spectrum, quite different from the spectra of ordinary stars. Furthermore, for some time astronomers had been observing certain peculiar «ripples», which traveled through the cloud at enormous speed (about \(\frac{1}{10}\) the speed of light). Careful measurements showed that the vector velocities of these ripples were directed away from the south preceding star. For both these reasons Baade and Minkowsky in 1942 concluded that this object rather than the other member of the doublet should be identified as the supernova remnant.

2. – Astronomical research in the years following the end of the second world war was dominated by the almost explosive development of radio astronomy. One of the first discrete radio sources to be identified with an optical object was the Crab Nebula [1].

The discovery of the radio emission of the Crab brought to a sharper focus the problem of the origin of the radiation from this object, which had puzzled astronomers for several years. Indeed, while it had been found very difficult to explain the shape and the intensity of the optical continuum in terms of thermal processes (the only celestial radiation processes well understood at the time), in no way could processes of this kind account for the strong radio signals.
The solution of the problem came in the early fifties when Shklovsky suggested that both the radio emission and the optical continuum were due to the same, nonthermal process, a process to be identified with the so-called synchrotron effect, i.e., the emission of electromagnetic radiation by highly relativistic electrons traveling in a magnetic field [2]. Unlike thermal radiation, synchrotron radiation is linearly polarized. Although it was difficult to predict whether or not a polarization might actually be observable (since in the case of a source of finite dimensions the net effect depends on the degree of randomness of the magnetic field), Shklovsky's suggestion prompted astronomers to search for a polarization of the optical continuum of the Crab. The positive results of these observations, and the detection, some time later, of a similar polarization in the radio band of the spectrum, have been generally accepted as a crucial test of the synchrotron hypothesis. Today, of course, the synchrotron process is known to play a major role in many astrophysical phenomena. But it is worth noting that it was in the Crab Nebula that the occurrence of this process on a cosmic scale was first established.

Synchrotron emission extending into the optical band implies that the Crab Nebula is permeated by a magnetic field (of an estimated strength between $10^{-3}$ and $10^{-4}$ G) and contains electrons with energies extending up to at least $10^{19}$ eV. Various suggestions about the origin of these electrons were put forward (although none was worked out quantitatively into a theory). High-energy electrons might have been left over from the original explosion; or they might be ejected continuously from the central star; or they might be accelerated while moving through the cloud by some sort of Fermi-type process. Whatever mechanism was responsible for the acceleration of electrons, it was thought that the same mechanism would also accelerate protons and heavier nuclei. While the electrons lost their energy (or most of it) within the cloud by synchrotron emission, protons and heavier nuclei (for which synchrotron losses are negligible) would escape into interstellar space without appreciable energy loss, and would thus contribute to the galactic cosmic-ray flux. In fact, it was argued that all galactic cosmic rays may originate from supernovae, being produced primarily at the time of the initial outburst.

3. – In 1962, the discovery of surprisingly strong celestial sources of X-rays—including both localized sources and a diffuse background [3]—opened up the new field of X-ray astronomy. X-rays, of course, can only be observed at very high altitudes, because of their strong absorption in the atmosphere. Most of the results available to this date have been obtained by means of rockets, although balloons have made important contributions to the study of the «hard» component of the X-ray flux. The second X-ray rocket, flown in October 1962 [4] already gave some tentative indication
of an X-ray source in the general direction of the Crab Nebula. The following spring a rocket equipped with a detector of improved angular resolution established the existence of an X-ray source within a few degrees of the Crab [5]. The crucial proof that this source was indeed coincident with the Crab came in the summer of 1964 when a rocket flown during an eclipse of the Crab by the moon showed the simultaneous disappearance of the X-ray and of the optical flux [6]. The identification was confirmed in 1967 by means of a collimator of very fine angular resolution, which measured both angular coordinates of the X-ray source with a precision of about 20 arcsec [7].

The results of the 1964 and 1967 observations are summarized in Fig. 2. They agree in showing that, within the observational uncertainties, the center of the X-ray source is coincident with the center of the visible nebula. Moreover both experiments indicate that the X-ray source is not point-like, but

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**Fig. 2.** Observational results on the location and size of the X-ray source in the Crab Nebula, superimposed on a photograph of the nebula in ordinary light [7]. The data were obtained by Bowyer *et al.* [6] who observed the occultation of the Crab by the moon, and by Oda *et al.* [7], using a modulation collimator. The arc marked «NRL 1964» shows the position of the moon's limb at the time when it crossed the center of the X-ray source, as given by Bowyer *et al.* The arc marked «NRL (Manley 1965)» shows the same data, corrected for the motion of the rocket during the experiment [31]. The intersection of the «preroll» and «postroll» lines is the most likely position of the center of the source, as determined by Oda *et al.*; the observational errors of this determination are also indicated. The dotted circle represents the approximate dimensions of the X-ray source.
The Crab nebula. Ancient history and recent discoveries

has an angular diameter of about 100 arc s (i.e., of the same order as that of the visible nebula, although perhaps somewhat smaller).

Since its discovery, the X-ray source in the Crab has been the object of many observations. In reporting the results of these observations, it may be instructive to compare them with those concerning another strong X-ray source, Sco X-1, which has also been extensively investigated. Unlike the Crab, Sco X-1 had not been recognized by the astronomers as a peculiar celestial object before its discovery as an X-ray emitter. Subsequently it was identified with a faint star of unusual spectral characteristics [8]. Again unlike the Crab, Sco X-1 appears point-like (to the limit of the resolution achieved so far) both in the optical and in the X-ray band.

The X-ray emission from the Crab, as well as its light emission, were found to be nearly constant in time, at least when averaged over periods of seconds (Sco X-1, on the contrary, was found to be highly variable both in the X-ray and in the optical bands).

In the X-ray band, the spectral function of the Crab (energy flux per unit interval of photon energy) was found to follow closely a power law with exponent close to unity from \( h\nu = 1 \) keV to \( h\nu = 100 \) keV. (The X-ray spectrum of Sco X-1 has a very different shape, being represented approximately by an exponential function, similar to that expected from a thermal, optically thin source at about \( 5 \times 10^7 \)°K. This implies that the spectrum of Sco X-1 is much « softer » than that of the Crab; indeed, while Sco X-1 is about 10 times brighter than the Crab at photon energies of the order of 5 keV, the Crab becomes brighter than Sco X-1 at photon energies above about 30 keV.)

A log-log plot of measurements in the radio, visible, ultraviolet, and X-ray bands suggests that the whole electromagnetic spectrum of the Crab may be described by a single smooth function. This has been taken as an argument in favor of a common origin (i.e., synchrotron radiation) for the entire spectrum. Although not yet definitely proven, the assumption of a synchrotron origin for X-ray spectrum of the Crab is accepted by most scientists, to a large extent because of the difficulty of finding a more likely alternative. The only other process that has been considered seriously is thermal radiation from a hot, optically thin plasma cloud. As already noted, if the cloud is at a uniform temperature, this process gives rise to an exponential spectrum, i.e., a spectrum more similar to that of Sco X-1 than to that of the Crab. Of course, if the plasma temperature varies from point to point, as it may well do in the Crab Nebula, the X-ray spectrum will be a sum of exponentials which might conceivably simulate a power law over a limited range of photon energies. However, beyond a photon energy corresponding to the temperature of the hottest region, the spectrum should drop sharply. Therefore
the possibility of a thermal radiation process became increasingly remote as spectral measurements were extended to higher and higher energies and failed to detect any cut-off.

With the magnetic fields that supposedly exist in the Crab, synchrotron emission in the X-ray band requires electron energies of the order of $10^{14}$ eV. It is worth noting that for these very energetic electrons the synchrotron process is exceedingly effective. Consequently the electrons lose energy at a very fast rate, which appears to rule out the possibility that they might have originated from the initial explosion.

At this point it may be useful to quote some figures. The X-ray flux from the Crab Nebula, in the spectral band from $h\nu = 1 \text{ keV}$ to $h\nu = 100 \text{ keV}$, amounts to about $7 \times 10^{-8} \text{ erg/cm}^2\text{s}$ at the earth. Taking the distance of the Crab as 5000 l.y., its X-ray emission turns out to be about $2 \times 10^{37} \text{ erg/s}$, i.e., about 5000 times the total emission of the Sun in all wavelengths. The emission in the optical band is about $\frac{1}{4}$ and the emission in the radio band ($\lambda > 3 \text{ cm}$) is about 1/1000 of the X-ray emission. (For Sco X-1, the corresponding figures are about 1/1000 and about $2 \times 10^{-8}$.)

4. - We now come to the very recent developments of astronomical research, and here again we find that the Crab Nebula occupies a central position in the new discoveries.

Early in 1968, Hewish and his co-workers announced the discovery of pulsating radio sources, or pulsars [9]. At the end of that year, some 25 pulsars were known, with periods ranging from about 2 to 1/30 s. Of these, only two had been identified with previously known celestial objects, both of them supernova remnants. One of them was Vela X [10] the other was the Crab Nebula [11]. The pulsar in Vela X had a period of about 89 ms, that in the Crab (known also as NP 0532) had a period of about 33 ms, the shortest among all known pulsars.

The periods of the «slow» pulsars were found to be remarkably constant (for some of them it was established that the rate of change was less than one part in $10^8$ per year). The periods of the «fast» pulsars in Vela X and the Crab, on the other hand, were found to increase very slowly. For the Crab, the rate of increase amounts to one part in 2400 per year (*).

In January 1969 another important discovery took place, with the detection, in the Crab Nebula, of the first and thus far the only optical pulsar [14]. The period of the optical pulsations was found to be exactly identical to that of the radio pulsations, which proved beyond any reasonable doubt that the

(* In the case of Vela X, the gradual increase of the period was interrupted, between February 4 and March 3, 1969, by a sudden decrease of two parts in one million [12, 13].
Fig. 3. – Stroboscopic pictures of the stars near the center of the Crab Nebula taken by J. S. Miller and E. J. Wampler at the Lick Observatory. The pulsar appears as the brightest object in the picture at the top; it is nearly invisible in the picture at the bottom. The change in the apparent brightness is due to the gradual phase change of the light pulses relative to the «open periods» of the stroboscopic disk [16] (Lick Observatory photograph).
radio and the optical pulsars were the same object (although, of course, the radiations belonging to the two spectral bands may come from different regions of this object). Precise determinations of its position showed that the pulsating star is the south preceding member of the doublet found near the center of the Crab [15], and thus confirmed unequivocally the previous tentative identification of this star as the condensed residue of the supernova explosion. A further dramatic verification of this result came from a series of photographs taken through the slots of a rotating disk, which showed that the brightness of the south preceding star changed periodically between a maximum and practically total extinction when the time between successive « open » intervals was nearly equal to the period of the pulsations (see Fig. 3).

Quite naturally, the discovery of the optical pulsar in the Crab suggested a search for a pulsating component in the X-ray emission of the same object. During the month of April 1969, two rockets provided with detectors sensitive to « soft » X-rays (photon energies of several keV) were launched for this purpose, the first by the NRL group [17], the second by the MIT group [18]. Both experiments did, in fact, detect the expected pulsations, with a period exactly equal to that of the radio and of the optical pulsations (33.099522 ms at the time of the MIT flight).

Finally, a recent analysis of balloon data obtained in 1967 revealed that also the « hard » X-ray flux of the Crab (photon energies greater than about 35 keV) contains a pulsating component [19]. A balloon flight carried out in May 1969 confirmed this result and provided quantitative information on the size and shape of the pulses [20].

Examples of the pulse shapes observed in different spectral bands appear in Figs. 4-7. Shown in each case is the time dependence of the radiation flux during one period, averaged over a large number of periods.

One sees that, at all wavelengths, each pulse contains two peaks, separated by a time interval slightly less than one half the period. In the optical and in the X-ray bands, the shape of the pulses appears to be quite constant. In the radio band, however, the pulse shape varies greatly from pulse to pulse, and even averaging over thousands of pulses does not result in a stable pattern. It has been pointed out that this instability may be due, at least in part, to refraction of radio waves, possibly in the ionized gases within the nebula itself [23, 24]. This interpretation is consistent with the observed stability of the optical and of the X-ray pulses because refraction effects decrease rapidly with decreasing wavelengths.

There is evidence that at all wavelengths the radiation level between the first and the second peak is somewhat higher than the radiation level after the second peak. We shall take the view that this lowest level of radiation represents the steady emission of the nebula. In other words we shall assume
that the emission of the pulsar actually drops to zero during each period. (Stroboscopic pictures such as those shown in Fig. 3 tend to support this assumption, but do not prove that it is rigorously correct.) By taking the lowest radiation level as the zero line, we can then separate the pulsating component of the radiation originating from the pulsar, from the steady component originating from the nebula.

Fig. 4. – Average pulse shapes of the pulsar in the Crab Nebula, as observed on three different days and at three different radio frequencies with the 1000 ft antenna at the Arecibo Ionospheric Observatory; a) Nov. 14, 1968; 196.5 MHz; 18000 pulses. b) Nov. 26, 1968; 198 MHz; 21153 pulses. c) Dec. 2, 1968; 430.0 MHz; 53427 pulses [21].
Fig. 5. – Light curves for the Crab pulsar in white light. a) sum of 100000 periods; b) sum of 30000 periods, taken 3½ h earlier. The abscissa is channel number, each channel being of 100 μs duration; the left-hand scale refers to curve a) and the right-hand scale to curve b) [22].

Observations show that the ratio between the power in the pulsating mode and the power in the steady mode varies by a very large factor over the spectrum. In the radio and in the optical bands this ratio amounts to only several parts in one thousand. In the « soft » X-ray band it reaches the value of about 9% and in the « hard » X-ray band it seems to be higher still. From these results and from the spectral data on the total emission of the Crab reported previously it follows that all but a minute fraction (perhaps less than 1%) of the radiation from the pulsar is in the form of X-ray. This object, then, may be properly described as an X-ray pulsar.

The pulses observed in the optical and the X-ray bands, while very different in their size relative to the steady component, have strikingly similar shapes. In both spectral bands, one of the two peaks observed during each period has a width of about 1.5 ms, and the other has a width of about 3.5 ms (*). Within the experimental errors, the separation of the two peaks

(*) However, one should note that the observed width of the narrow X-ray peak is not much greater than the time resolution of the instrument.
is the same (about 13.5 ms). In the experiment by Bradt and his co-workers (see Fig. 6) recording of time signals from the WWV radio station during the rocket flight made it possible to correlate the X-ray observations with optical observations carried out, within a few hours of the flight, at the McDonald Observatory and at the Palomar Observatory. It was thus shown that the narrow peaks in the X-ray and in the optical bands are simultaneous within 1 ms.
The great variability of the radio pulses denies the possibility of a detailed comparison of their shape with that of the optical and radio pulses. Furthermore, the wavelength-dependent delay of the radio pulses due to dispersion in the interstellar medium makes it difficult to establish an exact time correlation between the radio peaks and the optical peaks. All one can say on the basis of published reports is that the peaks in the radio and optical bands are simultaneous, with an uncertainty of about 6 ms, due almost entirely to the interstellar dispersion [25].

5. A reliable theoretical interpretation of the observational data that have been described above is still lacking. From these data, however, there begins to emerge a model which, although tentative and incomplete, may be worth discussing.

When pulsars were first discovered, two different kinds of models were suggested to account for their equally-spaced signals; i.e., a) vibrational models and b) rotational models. The vibrating or rotating star was thought to be either a) a white dwarf or b) a neutron star. While it was difficult to discriminate between these various possibilities as long as only pulsars with periods of the order of a second were known, the discovery of pulsars with periods of less than 0.1 s practically eliminated all choices but one. Since the free oscillations of white dwarfs have periods considerably longer than
0.1 s; since white dwarfs cannot rotate at 10 rps or more without being disrupted; since the free oscillations of a neutron star are believed to be rapidly damped through the production of gravitational waves, it became practically certain that pulsars (or at least the «fast» pulsars such as that in the Crab Nebula) were rotating neutron stars.

We can estimate the kinetic energy of rotation $E$ of the pulsar in the Crab by assuming that its mass is of the order of one solar mass ($\sim 2 \times 10^{35}$ g) and by taking the conventional value of 10 km for its radius. With the observed angular velocity of $2\pi \times 30 \approx 190$ s$^{-1}$ we obtain

$$E \approx 1.4 \times 10^{49} \text{ erg}.$$  

From this figure and from the observed rate of increase of the period it follows that the pulsar loses rotational energy at the rate

$$- \frac{dE}{dt} \approx 3.7 \times 10^{38} \text{ erg s}^{-1}.$$  

From the data reported previously we may estimate the total energy of the electromagnetic radiation of all frequencies emitted by the Crab Nebula to be several times $10^{37}$ erg s$^{-1}$. It seems likely that an amount of energy, perhaps of the same order of magnitude, may be spent by the Crab Nebula in the production of cosmic rays. Thus, within the large uncertainties of the present estimates, $-dE/dt$ appears to be remarkably close to the total energy output of the Crab Nebula, which naturally suggests that this energy is supplied the gradual slowing down of the rotating neutron star at the center of the Crab [26, 27]. An additional justification for accepting this suggestion as a working hypothesis in the formulation of our model may be found in the fact that previously it had been necessary to resort to ad hoc assumptions in order to account for the energy storage in the Crab Nebula.

It appears natural to interpret the pulsating signals received from a rotating object as due to a light-house effect [28]. As another working hypothesis, we shall therefore assume that the electromagnetic radiation from a neutron star is confined to one or more narrow beams, which sweep past the observer as they corotate with the star. In the case of the Crab, there would be at least two such beams. The narrow principal peak requires a beam whose angular width in the direction perpendicular to axis of rotation is at most $2\pi/20$ (less if the axis of rotation is not perpendicular to the line of sight.) If this beam were in the shape of a circular cone, the a priori probability of its being detected by an observer on the earth would be 5% or less. Similarly the probability of detecting the beam responsible for the wider pulse would be 10%
or less. We conclude that either the earth is in a peculiarly favourable position for the observation of the signals from the pulsar in the Crab; or the beams responsible for these pulses are fan-shaped rather than circular; or there are more than two beams.

The emission of the radiation into discrete beams implies an azimuthal anisotropy in the structure of the pulsar with respect to its spin axis. The stability of the beams as observed in the visible and X-ray bands is more easily understandable if the anisotropy is due to a magnetization of the pulsar rather than to «hot spots» or other peculiarities in a plasma atmosphere of the pulsar, as had been suggested when only radio observations were available [28]. It should be noted that the collapse of a star with a moderate magnetic field will, indeed, result in a neutron star with exceedingly large magnetization, even if only a minor fraction of the original magnetic flux is conserved. (For a star similar to the Sun, 100% flux conservation would give rise to fields of the order of $10^9$ G at the surface of the neutron star; field strengths up to $10^{13}$ G have been mentioned as a possibility.) It thus appears reasonable to further specify our model by assuming that the neutron star is strongly magnetized, and that the magnetization is not axially symmetric with respect to the spin axis.

We now come to the problem of the processes responsible for the steady component of the radiation (originating from the nebula) and of the pulsating component (originating from the neutron star). With regard to the former, as already noted, we know for sure that the continuous spectrum extending from the radio waves to the ultraviolet is due to a synchrotron effect, and we have good reasons to believe that the same effect is also responsible for the X-ray emission; which means that the nebula contains electrons with energies up to at least $10^{14}$ eV. According to our model, these electrons derive their energy from the kinetic energy of rotation of the neutron star. We may think of a direct process, whereby the electrons are accelerated by the strong time-varying electromagnetic field that exists in immediate neighborhood of the star, and are then injected into the surrounding magnetized plasma cloud. Alternately, we may think of an indirect acceleration mechanism; i.e., we may assume that the rotating neutron star loses energy to the cloud giving rise to disturbances (in the form of waves or shocks), which then, through a Fermi-type stochastic interaction with the electrons in the cloud, supply the energy radiated via the synchrotron process.

An analysis of the stochastic acceleration process (for example on the basis of a model based on the interaction between Alfvén waves and individual electrons [29]) shows that the high efficiency needed to maintain the required electron spectrum can be achieved only under rather extreme circumstances. On the other hand, no quantitative treatment of the direct acceleration
process has yet been developed. In this connection one should keep in mind that the electrons will lose energy by synchrotron radiation even as they are accelerated; and that the synchrotron losses are proportional to the square of the magnetic field and to the square of the energy. Therefore it is not easy to figure out how electrons can emerge from the region of strong magnetic field surrounding the neutron star with the enormous energy they need to radiate X-ray photons in the weak field of the nebula.

Let us consider next the pulsating component of the radiation. One may think of a variety of processes capable of generating pulsations in the long-wavelength band of the spectrum. The fundamental problem, however, is to explain the emission in the X-ray band which, by itself, accounts for at least 99% of the pulsating power, as already noted. In this portion of the spectrum, it appears that the only effective emission process is the interaction of electrons with the magnetic field. This process presupposes the existence around the neutron star of electrons with a suitable energy distribution. In the frame of reference corotating with the star, the spacial distribution of the electrons must be remarkably stable; i.e., the electron cloud must corotate rigidly with the star. Furthermore, the distribution of the electrons in velocity space, and the pattern of magnetic field lines, must be such as to account for the required beam-shaped emission.

Of course, rigid corotation can only occur up to a maximum distance of the spin axis where the rotational velocity becomes equal to the velocity of light [28]. With an angular velocity of \(190 \text{ s}^{-1}\), this distance amounts to \(1.6 \times 10^8 \text{ cm}\). Note that, if the magnetic field resembles that of a dipole, and therefore varies as the inverse cube of the distance, its magnitude at the «light circle» in the equatorial plane is about \(2.5 \times 10^4\) times smaller than at the surface of the neutron star.

Of course, electrons require a much smaller energy to radiate X-ray photons in the strong magnetic field surrounding the neutron star than they do in the weak magnetic field of the cloud. In this connection it is important to keep in mind that the motion of electrons in the plane perpendicular to the magnetic field is actually quantized [30]. In the subrelativistic region the energy levels are equidistant with a separation \(\Delta \varepsilon = h\omega /2\pi\), where \(\omega\) is the cyclotron frequency. With \(\Delta \varepsilon\) measured in eV and the magnetic field \(B\) in gauss, the following relation holds

\[
\Delta \varepsilon = 1.16 \times 10^{-8} B.
\]

If \(\Delta \varepsilon\) is very small compared with the photon energy, quantum effects are negligible and the interaction of the electrons with the magnetic field may be described by the classical theory of magnetic bremsstrahlung. In this
case the average energy of the radiated photons is much smaller than the electron energy. If, however, $\Delta e$ is close to the photon energy then the emission occurs via a process similar to an atomic quantum transition between two bound levels, and the energy of the emitted photons is equal to or a sizeable fraction of the electron energy. Even hard X-ray photons, then, may be produced by subrelativistic electrons.

Quantized emission in the X-ray band requires magnetic fields of the order of $10^{12} \text{ G}$ or more. While these fields are not ruled out, it appears more likely to the author that X-rays are produced in a region of lower magnetic field, in which case relativistic electrons are needed. One must then assume that electrons are first accelerated to relativistic, but not necessarily extremely high, energies by the rotating neutron star. While in the vicinity of the star, they partake of its rotation and generate the pulsating component of the radiation. They then diffuse into the surrounding cloud, where, after perhaps gaining further energy, they give rise to the steady radiation.

6. — To summarize, the model developed here pictures the Crab Nebula as a thin plasma cloud containing a weak magnetic field, with a fast-rotating, strongly-magnetized, neutron star at its center. The magnetization of the star does not have axial symmetry with respect to the spin axis, so that the rotation gives rise to time-varying electromagnetic fields, which, in some way or another, are capable of accelerating electrons. For a while these electrons remain within the corotating magnetosphere of the neutron star, where they give rise to corotating beams of electromagnetic radiation. Subsequently they diffuse into the surrounding cloud, where perhaps they acquire further energy by a Fermi-type stochastic process. Synchrotron emission by these electrons in the weak magnetic field of the cloud gives rise to the steady flux of radiation.

Presumably the kinetic energy of rotation of the neutron star was initially derived from the conversion of some fraction of the gravitational energy released during the stellar collapse following the supernova explosion. From the time of its birth, the Crab Nebula has drawn from the rotating neutron star the energy needed to produce the various kinds of rays which it has been pouring out into space.

Whether or not the general features of this model will survive future observations and future theoretical discussions is still an open question. Here the model is presented as a working hypothesis, that may be useful in suggesting further lines of investigation. From the theoretical point of view, one of the basic problems is clearly a quantitative analysis of the possible mechanisms for the acceleration of the electrons. From the observational point of view, it would be desirable to examine the polarization of the X-ray
emission in order to test the assumption that it originates from a synchrotron process. Furthermore it would be very illuminating to extend the observations of the steady and of the pulsating components of the electromagnetic spectrum to considerably higher photon energies. Finally we may hope that high-resolution X-ray pictures of the Crab, possibly taken at different wavelengths, will furnish important information on the mechanism responsible for the acceleration of electrons and help discover the region of space where this acceleration occurs.

REFERENCES

The $K_L - K_S$ Mass Difference.

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1. Introduction.

The existence of a very small mass difference between the long- and the short-lived components of the neutral $K$ system has been firstly postulated in the theory of particle mixture of Gell-Mann and Pais [1].

The $K_0$ and $\bar{K}_0$ particles, produced in strong and electromagnetic interactions, belong to the isotopic spin doublets ($K^+, K_0$) and ($K^-, \bar{K}_0$) and have definite hypercharges, $+1$ and $-1$, respectively. Furthermore, as a consequence of CPT invariance, particle and antiparticle have identical masses, i.e., $M_{K_s} = M_{\bar{K}_s}$.

However, weak interactions do not conserve hypercharge and they can induce not only decays but also transition between $K_0$ and $\bar{K}_0$. For instance transitions are mediated to the second order by any common decay channel $F$, i.e.

$$K_0 \leftrightarrow F \leftrightarrow \bar{K}_0.$$ 

After an infinitesimal proper time interval $\delta t$, the hypercharge eigenstates $K_0$ and $\bar{K}_0$ have evolved as follows [2]:

$$|K\rangle \rightarrow |K\rangle - i\delta t\{M|K\rangle + B|\bar{K}\rangle\},$$

$$|\bar{K}\rangle \rightarrow |\bar{K}\rangle - i\delta t\{A|K\rangle + \bar{M}|\bar{K}\rangle\}.$$ 

The (complex) terms $A$ and $B$, represent the $K_0 \leftrightarrow \bar{K}_0$ mixing effects and $M$ and $\bar{M}$ are related to the $K_0$ and $\bar{K}_0$ masses decay rates:

$$M = m_{K_s} - \frac{i}{2} \Gamma_{K_s}, \quad M = m_{\bar{K}_s} - \frac{i}{2} \Gamma_{\bar{K}_s}.$$
There exist two definite linear combinations of the $|K\rangle$ and $|\bar{K}\rangle$ states:

$$
\begin{align*}
|L\rangle &= p|K\rangle + q|\bar{K}\rangle, \\
|S\rangle &= r|K\rangle + s|\bar{K}\rangle,
\end{align*}
$$

(2)

which have simple, uncoupled time evolutions:

$$
|L\rangle \rightarrow |L\rangle - i\delta t M_L|L\rangle, \quad |S\rangle \rightarrow |S\rangle - i\delta t M_S|S\rangle
$$

and which obviously describe the long- and short-lived states of simple exponential decay observed experimentally. The quantities $M_L$ and $M_S$ are defined:

$$
M_L = m_L \frac{i}{2} \Gamma_L, \quad M_S = m_S \frac{i}{2} \Gamma_S,
$$

where $\Gamma_L$, $m_L$, $\Gamma_S$, $m_S$ are, respectively, the masses and decay rates of the long- and short-lived states. Relating expressions (1) and (3) with the help of formula (2) one finds easily:

$$
\begin{align*}
M &= (M_L sp - M_S qr)/(sp - qr), \quad \bar{M} = (M_S sp - M_L qr)/(sp - qr), \\
A &= (M_S - M_L) sp/(sp - qr), \quad B = (M_L - M_S) sq/(sp - qr).
\end{align*}
$$

Introducing the condition $M = \bar{M}$ required by CPT invariance and with some phase conventions, the above expressions become considerably simple, since $s = -q$, $r = p$:

$$
\begin{align*}
M &= \bar{M} = \frac{1}{2} (M_S + M_L), \\
A &= \frac{1}{2} \frac{p}{q} (M_L - M_S), \quad B = \frac{1}{2} \frac{q}{p} (M_L - M_S).
\end{align*}
$$

The masses of the long- and short-lived states are therefore different, their difference being proportional to the terms $A$ and $B$. As we have seen, these terms are originated by $K_0 \leftrightarrow \bar{K}_0$ transitions which now give different contributions to the $K_L$ and $K_S$ self-energy diagrams.

A first estimate of the magnitude of $\delta m = m_L - m_S$ is easily obtained from dimensional considerations [3]. If weak interactions to the first order satisfy to the rule $|\Delta S| = 1, 0$ it is possible to connect $K_0$ and $\bar{K}_0$ only with second or higher order diagrams. Consequently $\delta m$ is of order $(G_F \sin^2 \theta)^2$, where $G_F$ is the Fermi constant, and $\theta$ is the Cabibbo angle:

$$
\delta m \simeq \frac{G_F^2}{4\pi} \sin^2 \theta \cdot m^5 \simeq 10^{-5} \text{ eV} \quad \text{(for } m = m_K\text{).}$$
This is of the same order of magnitude of the total $K_S$ transition rate, $\Gamma_S = 1.145 \times 10^{10} \text{ s}^{-1}$, equivalent to $\Gamma_S \hbar = 0.58 \times 10^{-5} \text{ eV}$.

If $K_0 \leftrightarrow \bar{K}_0$ transitions occur directly ($\Delta S = 2$) with strength $f \cdot G_F$, the corresponding mass difference is approximately:

$$\delta m = fG_F m^3 \simeq 1.2 \times 10^3 f \text{ eV} \quad \text{(for } m = m_K)$$

Initially the interest in the $K_L - K_S$ mass difference has been simply whether it was of the same magnitude as $\Gamma_S \hbar$ or much larger. Since the discovery of $CP$ violation in neutral K-decay [4] it has become important to know with precision the mass difference.

An accurate knowledge of $\delta m$ is at the basis of the phenomenological analysis of the nature of $CP$ violation in neutral decay. A more accurate value of $\delta m$ is also demanded in the analysis of experiments measuring the phase of the $CP$ violating amplitude $\eta_{+-} = \lambda(K_L \rightarrow \pi^+\pi^-)/\lambda(K_S \rightarrow \pi^+\pi^-)$. The most direct way to determine the phase of $\eta_{+-}$ is to observe interference between the $K_L \rightarrow \pi^+\pi^-$ and $K_S \rightarrow \pi^+\pi^-$ decay amplitudes close to the production point of $K_0$ state [5]. Since $|\eta_{+-}| \sim 2 \times 10^{-3}$, such interference effects are large only relatively far from the production point where the $K_S$ amplitude has decayed down to the $|\eta_{+-}|$ level: $\exp[i\Gamma_S t/2] \simeq 1$. The error in the measured phase then will come to the greatest extent from the error in $\delta m$ which enters in the argument of the interference term in the form $\delta m i$, with $i \simeq 12/\Gamma_S$. This technique is then limited to the accuracy with which the mass difference is known.

2. **How to measure the mass difference?**

Many different techniques have been employed to determine $\delta m$, since the existence of such a mass difference was suggested. All these experiments show the mass difference as an interference process between a coherent superposition of $K_L$ and $K_S$ states, the phase difference between the two states showing a characteristic precession due to the mass difference $\delta m$. The attractive feature of all these experiments is that the smallness of the $K_L - K_S$ mass difference makes it possible to realize experimental conditions in which a coherent superposition of $K_L$ and $K_S$ states can be observed over long times and over a variety of arrangements. In the present paper we shall consider only the most recent techniques which almost invariably make use of the phenomenon of coherent regeneration. Pais and Piccioni [5] first pointed out the existence of a regeneration process for neutral K mesons but it was M. L. Good [6] who gave a complete description of this process and the application of this technique to the determination of $\delta m$. Whenever a beam
of long-lived $K$ meson traverses a slab of material, the « regenerator » because of difference of strong interaction cross-section for $K_0$ and $\bar{K}_0$ components, the propagation of the incoming state is modified and a small coherent $K_S$ amplitude is regenerated. Thus the time evolution of the $K$ and $\bar{K}$ states instead of (PCT)

$$
\begin{align*}
|K\rangle &\rightarrow |K\rangle - iM\delta t |K\rangle - iB\delta t |\bar{K}\rangle , \\
|\bar{K}\rangle &\rightarrow |\bar{K}\rangle - iM\delta t |\bar{K}\rangle - iA\delta t |K\rangle ,
\end{align*}
$$

we shall have in matter the additional terms due to nuclear absorptions

$$
-\frac{1}{2}N\sigma dz \quad \text{and} \quad -\frac{1}{2}N\bar{\sigma} dz ,
$$

where $N$ is the number of nucleus for unit volume and $\sigma$ and $\bar{\sigma}$ are the total cross-sections for $K_0$ and $\bar{K}_0$. In terms of the forward scattering amplitudes $f$ and $\bar{f}$, from the optical theorem, $\sigma = (4\pi/k) \text{Im}(f)$ and $\bar{\sigma} = (4\pi/k) \text{Im}(\bar{f})$. If the forward amplitude is not purely imaginary, the beam is not only attenuated but also refracted, then instead of $i\text{Im}(f)$ and $i\text{Im}(\bar{f})$ we should write simply $f$ and $\bar{f}$. Then

$$
\begin{align*}
|K\rangle &\rightarrow |K\rangle - i\delta t \left\{ M'_L |K\rangle - \frac{2\pi N}{k} f \frac{\delta z}{\delta t} |K\rangle + B |\bar{K}\rangle \right\} , \\
|\bar{K}\rangle &\rightarrow |\bar{K}\rangle - i\delta t \left\{ M'_s |\bar{K}\rangle - \frac{2\pi N}{k} f \frac{\delta z}{\delta t} |\bar{K}\rangle + A |K\rangle \right\} .
\end{align*}
$$

Taking $p$ times the first equation $\pm q$ times the second we obtain

$$
\begin{align*}
|L\rangle &\rightarrow |L\rangle - i\delta t \left\{ M'_L |L\rangle - \frac{\pi}{k} (f - \bar{f}) N\Gamma_s A_s |S\rangle \right\} , \\
|S\rangle &\rightarrow |S\rangle - i\delta t \left\{ M'_s |S\rangle - \frac{\pi}{k} (f - \bar{f}) N\Gamma_s A_s |L\rangle \right\} ,
\end{align*}
$$

where

$$
\begin{align*}
M'_L &= M_L - \frac{\pi}{k} (f + \bar{f}) N\Gamma_s A_s , \\
M'_s &= M_s - \frac{\pi}{k} (f + \bar{f}) N\Gamma_s A_s ,
\end{align*}
$$

and $\delta z/\delta t = \Gamma_s A_s = k/m_K$, where $\Gamma_s$ is the $K_S$ decay length.

Therefore, if $f \neq \bar{f}$, $|L\rangle$ and $|S\rangle$ do not propagate independently but they are coupled. To the first order in a finite slab of material after a time $\tau$, 


the state becomes:

\[ |L\rangle \rightarrow \exp \left[ iM_L \tau \right]|L\rangle - \frac{\pi}{k} \frac{f-f'}{M_L-M_S} \text{NG} \, A_S \left( \exp \left[ -iM_S \tau \right] - \exp \left[ -iM_L \tau \right] \right) |S\rangle. \]

Thus there is a coherent regeneration of the state \( |S\rangle \). The state at the exit of the plate is of the general form:

\[ |L\rangle + \varrho |S\rangle, \]

where \( \varrho \), the regeneration amplitude, is proportional to

\[ \frac{\pi}{k} \frac{(f-f')}{M_L-M_S} \text{NG} \, A_S \left( 1 - \exp \left[ -(M_S-M_L)\tau \right] \right). \]

3. Single regenerator experiments. The \( K_L - K_S \) interference in the \( \pi^+\pi^- \) decay channel.

It was realized promptly after the discovery of \( CP \) violation that the existence of both \( K_L \to \pi^+\pi^- \) and \( K_S \to \pi^+\pi^- \) decays provides a very powerful method for investigating the \( K_L - K_S \) mass difference. Let us consider the forward going beam after a slab of material. As we have just seen the state after the proper time interval \( t \), measured from the exit face of the regenerator is:

\[ |\text{out}, t\rangle = \exp \left[ -iM_L t \right]|L\rangle + \varrho \exp \left[ -iM_S t \right]|S\rangle. \]

Since both \( |L\rangle \) and \( |S\rangle \) states decay into the state \( \pi^+\pi^- \) with amplitudes \( \langle \pi^+\pi^- | T | L \rangle \) and \( \langle \pi^+\pi^- | T | S \rangle \), the decay amplitude for the state \( |\text{out}, t\rangle \) is:

\[ \langle \pi^+\pi^- | T | \text{out}, t\rangle = \exp \left[ -iM_L t \right]\langle \pi^+\pi^- | T | L \rangle + \varrho \exp \left[ -iM_S t \right]\langle \pi^+\pi^- | T | S \rangle \]

and the decay rate per unit of time is promptly obtained by squaring the above expression:

\[
\frac{dN}{dt}(t) = \Gamma(K_S \to \pi^+\pi^-) \{ |\varrho|^2 \exp \left[ -\Gamma_S t \right] + |\eta_{+-}|^2 \exp \left[ -\Gamma_L t \right] + 2|\eta_{+-}| \exp \left[ -(\Gamma_S + \Gamma_L) t/2 \right] \cos \left( \delta m t - \text{Arg}(\varrho) + \text{Arg}(\eta_{+-}) \right) \},
\]

where \( \Gamma(K_S \to \pi^+\pi^-) \) is the decay rate of the short-lived state into \( \pi^+\pi^- \) and \( \eta_{+-} = \langle \pi^+\pi^- | T | L \rangle / \langle \pi^+\pi^- | T | S \rangle \). The time dependence of the \( \pi^+\pi^- \) decay rate in addition to the two terms due to \( K_S \) and \( K_L \) decaying into \( \pi^+\pi^- \) has an interference term showing the characteristic precession between the \( K_L \) and \( K_S \).
states due to the mass difference. This interference term can be observed over a relatively long time interval. In the case of a thick regenerator $|\varrho| \gg |\eta_{+-}|$ and for early times the term in $|\eta_{+-}|^2$ can be neglected and to an excellent approximation also $I'_s + I'_L \simeq I'_s$:

$$\frac{dN}{dt}(t) \simeq \Gamma(K_S \to \pi^+\pi^-) \cdot |\varrho|^2 \exp[-I'_s t] \left\{ 1 + \frac{2|\eta_{+-}|}{\sqrt{|\varrho|^2 \exp[-I'_s t]}} \cos(\delta m t + \alpha) \right\}.$$  

The statistical accuracy to which the decay rate can be determined at each $t$ is also proportional to $1/\sqrt{|\varrho|^2 \exp[-I'_s t]}$. Therefore the error with which the interference term $|\eta_{+-}| \cos(\delta m + \alpha)$ can be determined is the same at all values of $t$ for which the approximation $|\eta_{+-}|^2 \simeq 0$ is valid. In practice the time interval over which the interference can be observed is determined by the largest regeneration amplitude which can be realized, $|\varrho/\eta_{+-}| \simeq 50$ and by the contributions of systematic errors.

The time-dependent interference pattern has been very extensively studied at CERN for carbon [7] and copper [8, 9] regenerators. A typical inter-

![Fig. 1. - Time-dependent interference between the CP violating amplitude $K_L \to \pi^+\pi^-$ and the $K_S \to \pi^+\pi^-$. The oscillation is showing the characteristic precession between the $K_L$ and $K_S$ phases due to the mass difference. •, ref. [14]; ○, ref. [9].](image-url)
ference term is shown in Fig. 1. The precession between the $K_L$ and $K_S$ states due to the mass difference is beautifully demonstrated over a complete oscillation. From the frequency of the oscillation observed in the three investigations at CERN [7, 8, 9] the mass difference is measured to be:

$$\delta m = (0.543 \pm 0.016) \times 10^{10} \text{s}^{-1}.$$


Another approach to the determination of the $K_L - K_S$ mass difference consists in using two sheets of matter. In the so-called «gap» method [10] the regenerated $\pi^+\pi^-$ intensity after the second one is determined as a function of their separation. In the «gap» method events are collected from the same decay volume for different regenerator geometries whereas the $K_L - K_S$ interference method is based on the observation of the decay rate from different decay regions for a fixed geometry. This technique is less sensitive to variations of the detection efficiency over the decay volume but it requires an excellent monitor and substantial corrections due to interferences with the $CP$ violating decay $K_L \rightarrow \pi^+\pi^-.$

The method can be easily understood expressing the $\pi^+\pi^-$ decay amplitude at the proper time $t$ from the exit face of the second regenerator as the sum of the contributions of the two slabs and of the component arising from $K_L \rightarrow \pi^+\pi^-$ decays. Let $\Delta t$ be the time interval between the exit faces of the two slabs of regeneration amplitudes $\varrho_1$ and $\varrho_2,$ respectively. Then:

$$\langle \pi^+\pi^- | T | t, \Delta t \rangle = [\varrho_1 \exp [-i M_S (t + \Delta t)] + \varrho_2 \exp [-i M_S t] \exp [-i M_L \Delta t]] \cdot \langle \pi^+\pi^- | T | S \rangle + \exp [-i M_L (t + \Delta t)] \langle \pi^+\pi^- | T | L \rangle =$$

$$= \exp [i M_L \Delta t] \{ [\varrho_1 \exp [-i (M_S - M_L) \Delta t] + \varrho_2] \exp [-i M_S t] \cdot \langle \pi^+\pi^- | T | S \rangle + \exp [-i M_L t] \langle \pi^+\pi^- | T | L \rangle \}.$$  

As $\Delta t$ is varied for a constant $t,$ the last two terms combine to give a fixed vector, whereas the first one decreases in magnitude and precesses around this fixed direction. The two pion decay rate can be easily calculated taking the square of the expression above:

$$I(t, \Delta t) = I_1(t + \Delta t) + I_2(t) + 2\sqrt{I_1(t + \Delta t)I_2(t)} \cos (\delta m \Delta t) - I_0,$$

where:

$$I_0 = I(K_S \rightarrow \pi^+\pi^-) \cdot |\eta_{+-}|^2,$$
MINIMUM $\chi^2$ SOLUTION USING THE EXPERIMENTAL VALUES OF $F_{21}$.

$\chi^2 = 1.3$

$\delta = \pm 0.43$

$\text{ARG } f_{21} - \text{ARG } \eta_{+} = \pm 40^\circ$

Fig. 2. – Number of $\pi^+\pi^-$ events as a function of the gap size and the best fit to these data (from ref. [11]).
\[ I_1(t + \Delta t) = \Gamma(K_S \to \pi^+\pi^-)[-|\varrho_1|^2 \exp[-\Gamma_S(t + \Delta t)] + |\eta_{+-}|^2 + 2|\varrho_1\eta_{+-}| \exp[-\Gamma_S(t + \Delta t)/2 \cos(\delta m(t + \Delta t) + \omega)]\],

\[ I_2(t) = \Gamma(K_S \to \pi^+\pi^-)[|\varrho_2|^2 \exp[-\Gamma_S t] + |\eta_{+-}|^2 + 2|\varrho_2\eta_{+-}| \exp[-\Gamma_S t/2 \cos(\delta m t + \omega)]\].

The terms \( I_0, I_1, \) and \( I_2 \) have a very simple physical meaning. They are the decay rates one would observe for no regenerator, and only regenerators 1 and 2, respectively. Therefore \( I_1 \) and \( I_2 \) have a form analogous to the one of the single regenerator experiments.

Determinations of \( \delta m \) with the «gap» method have been reported by the Princeton group [10, 11]. The results are shown in Fig. 2. The two pion decay rate has been observed as a function of the spacing between two regenerators of the same material. The ratio \( \varrho_2/\varrho_1 \) is then known. However the \( \Delta t \)-dependence of the term proportional to \( \cos(\delta m \Delta t) \) and of \( I_1(t + \Delta t) \) are not completely separated out in the fit and the result depends somewhat on the choice of the parameters like \( |\eta_{+-}|, \Gamma_S, \text{Arg}(\eta_{+-}) \). The result of the most recent Princeton experiment [11] is \( \delta m/\Gamma_S = (0.445 \pm 0.038) \).

In order to overcome these difficulties, a slightly different method, called the «zero-cross» method has been used by a CERN group [12]. These authors have determined experimentally both \( I(t, \Delta t) \) and \( I_0, I_1(t + \Delta t), I_2(t) \),

![Fig. 3. «Zero-crossing» point of the interference between two slabs in the experiment of ref. [12]. The two curves correspond to different spacings between the two blocks. a) Geometry 1: distance between blocks is 529.9 mm. b) Geometry 2: distance between blocks is 777.8 mm.](image-url)
owing to their very simple physical meanings. Then the precession due to
the presence of $\Delta t$ can be evidenced directly from the experimentally observed
rates, as follows:

$$\cos(\delta m \Delta t) = \frac{I(t, \Delta t) - I_2(t, \Delta t) - I_1(t) + I_0}{2 \sqrt{I_1(t + \Delta t) I_2(t)}}.$$ 

In order to reduce the possibility of errors coming from incorrect monitoring,
rather than varying the spacing $\Delta l$ between the two slabs, different values
of $\Delta t$ have been explored taking events of different kaon momenta $p_K$, since
$\Delta t = \Delta l \cdot m_K/p_K$. The most sensitive region is around the value for which
$\cos(\delta m \Delta t) \approx 0$. For this reason the method is denominated «zero-cross».

Preliminary results from the CERN group [12] are shown in Fig. 3. The
result is

$$\delta m = (0.542 \pm 0.0060) \times 10^{10} \text{ s}^{-1}.$$ 

5. – Concluding remarks.

The recent experiments clearly demonstrate the effects of a very small
but finite mass difference between the $K_L$ and $K_S$ states; when completed
by the investigations of Piccioni and collaborators which have beautifully
demonstrated that the the long-lived state is heavier [13] they give a precise
determination of the mass difference.

If the results can be combined with experiments which observe the inter-
ference of $K_L$ and $K_S$ in the $\pi^+\pi^-$ decay channel from a state which is pre-
ponderantly $K_0$ ($S = +1$) at the production one finds that [14, 15]

$$\text{Arg}(\eta_{+-}) = (41 \pm 5)^\circ.$$ 

This result is in agreement with a class of theories which give $\text{Arg}(\eta_{+-}) =
= \tan^{-1}(2 \delta m/\Gamma_S) \approx 41^\circ$, and in particular with the «super-weak » for which
such a prediction is exact.

REFERENCES

p. 165. In the present paper we shall follow rather closely the formalism developed
in this reference.
The $K_L-K_S$ mass difference 267


Suggestion for a More Precise Measurement of the $\eta_{+-}$ Phase.

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This volume represent an effort, on the part of Professor Amaldi’s friends and colleagues, to show some token of their affection and respect. In the case of this particular contribution, it is unfortunately a very small token. Quite apart from personal limitations, experimental particle physics moves slowly and requires the collaboration of many, so that it is rarely possible to produce a result for an occasion. I am in this way constrained to offer a mere suggestion for an experiment. It is done with mixed feelings: normally, I would prefer to wait for the result, but for the occasion I would like to make some contribution, even if it must be incomplete. I hope that it will be judged in this way.

It is some years since the discovery of $CP$ violation, but despite substantial effort, it has been observed only in the $K^0$ system. It is possible that we will continue to be restricted to the $K^0$ in the future as well, in our efforts to learn about $CP$ violation. It is then fortunate that several $CP$ violating parameters in $K^0$ decay, and in particular

$$\eta_{+-} = \frac{\langle + - |\tau|K_L\rangle}{\langle + - |\tau|K_S\rangle},$$

the relative amplitudes of long- and short-lived transitions to the charged pion state, can be measured with precision. In this note I point out a variation in the present line of these experiments which should permit an improvement in the measurement of the phase of $\eta_{+-}$. The same technique is in principle also applicable to the phase of the corresponding neutral decay amplitude ratio $\eta_{00}$, but the experimental difficulties in $\pi^0\pi^0$ decay are somewhat greater, so that it will be some time before the method can be expected to be useful also here.
The $\eta_{+-}$ phase can be measured by observing the time dependence of the $\pi^+\pi^-$ decay of a kaon state which is given at time $\tau = 0$: $\Psi(0) = |S\rangle + |L\rangle$. This time distribution has the form:

\begin{equation}
I_{+-}(\tau) = |q|^2 \exp \left[ -G_S \tau \right] + |\eta_{+-}|^2 \exp \left[ -G_L \tau \right] + 2|q\eta_{+-}| \exp \left[ -T\tau \right] \cos (\Delta m \tau - \varphi_{\eta_{+-}}).
\end{equation}

Here $G_S$ and $G_L$ are the short- and long-lived widths, respectively; $T$ is $(G_S + G_L)/2$; $\tau$ is the time in the $K^0$ rest frame; and $\Delta m = m_L - m_S$.

$I_{+-}(\tau)$ is plotted in Fig. 1 for a state with $q = 1$ ($\Psi(0) = |K^0\rangle$). It is experimentally possible to measure this time distribution accurately, except in the short time region (shaded in Fig. 1), which is inaccessible for shielding reasons. Unfortunately, this does not yield a corresponding precision in $q_{\eta}$: Since the $q_{\eta}$ dependent interference term is strongest at $\tau = 10/G_S$, what is measured best is the quantity $10 \Delta m/G_S - q_{\eta_{+-}}$. A precise determination of $q_{\eta_{+-}}$ is only possible if a correspondingly precise value of $\Delta m$ is available. To be completely explicit, if the error in $q_{\eta_{+-}}$ is to be less than some number, say $\Delta q$, the uncertainty in the mass difference must be less than $\Delta (\Delta m/G_S) \approx \Delta q/10$. The measurement of the mass difference with this precision turns out to be the bigger experimental problem.

It is the main contribution of this note to point out that the mass difference can be measured with the same apparatus as the $\pi^+\pi^-$ interference term, and simultaneously.

The suggestion is to measure, simultaneously with the $\pi^+\pi^-$ decay rate, also the charge asymmetry in the lepton decay as a function of $\tau$. This charge asymmetry is also governed by an interference between $K_S$ and $K_L$ amplitudes; the dominant term has in fact the same form as the interface term in $2\pi$ decay.

If

\begin{equation}
x = \frac{<\pi^+e^+\tau|K>}{<\pi^+e^+\tau|K>} = \frac{\Delta S = \Delta Q \text{ violating amplitude}}{\Delta S = \Delta Q \text{ conserving amplitude}}
\end{equation}

and if CPT is assumed, then

\begin{equation}
\delta(\tau) \equiv \frac{N_+ - N_-}{N_+ + N_-} \approx 2(1 - |x|^2) \left[ \exp \left[ -G_S \tau \right] + \exp \left[ -G_L \tau \right] \right] \frac{Re \epsilon + \exp \left[ -T\tau \right] \cos \Delta m \tau}{\left[ 1 + x^2 \exp \left[ -G_S \tau \right] + |1 - x|^2 \exp \left[ -G_L \tau \right] - 4 \Im x \exp \left[ -T\tau \right] \sin \Delta m \right]},
\end{equation}

where $N_+$ and $N_-$ are the decay rates to positive and negative leptons, respec-
Figs. 1. – Two pion intensity and leptonic decay asymmetry as a function of the time in the $K^0$ rest frame.

tively, and $\varepsilon$ is the parameter in the expansion of $K_L$ and $K_\pi$ in terms of $K$ and $\bar{K}$:

$$|S\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|)^2}} [(1+\varepsilon)|K\rangle + (1-\varepsilon)|\bar{K}\rangle].$$

$CPT$ is assumed, and higher order terms in $\varepsilon$ have been omitted.
Expression (2) is also shown in Fig. 1. At long times expression (2) reduces to the already observed small \((\sim 2.7 \times 10^{-3})\) asymmetry of \(K_L\):
\[
\delta(\tau \to \infty) \equiv \delta_L = 2 \text{Re} \varepsilon \frac{1 - |x|^2}{1 + |x|^2}.
\]
At short times the term \(\exp \left[- \bar{I} \tau \right] \cos \Delta m \tau\) dominates and at time \(\tau = 0\),
\[
\delta(\tau = 0) = \frac{1 - |x|^2}{1 + |x|^2} \approx 1.
\]

It may be useful here to point out an important feature of this approach. Both the \(2\pi\) distribution (1) and the asymmetry distribution (2) are slightly modified in an experimental situation, due to several small effects, such as the propagation of the kaon in the target in which it is produced, and the scattering of the kaon on the collimators and \(\gamma\)-ray filters which are commonly introduced. The main effect of this on the measurement of \(\varphi_\eta\) and \(\Delta m\) is to introduce a small phase change \(\varphi_0\) into both expressions (1) and (2), so that the arguments of the cosine function in the interference terms are changed to \(\Delta m \tau - \varphi_{\eta\pi} + \varphi_0\) and \(\Delta m \tau + \varphi_0\), respectively. \(\varphi_0\), in an experiment currently planned, is of the order of \(1 \pm 2\). This is somewhat greater than the error which is anticipated. However, \(\varphi_0\) is common to expressions (1) and (2) and if we think of the charge asymmetry measurement as a determination of the quantity \(\Delta m \tau + \varphi_0\), we can see that in the comparison of the two distributions \(\varphi_\eta\) can be extracted without separate knowledge of \(\Delta m\) or \(\varphi_0\): This, however, is only a heuristic way of understanding the manner in which the errors enter. In practice it will be necessary to analyse the two experiments to find \(\Delta m\) and \(\varphi_0\). However, the error in \(\Delta m \tau + \varphi_0\) for \(\tau \approx 10/\Gamma_\beta\) will be smaller than the error in \(\varphi_0\) or in \(\Delta m \tau\).

It is necessary to discuss here a problem in connection with the measurement of the rest-frame time \(\tau\) in the leptonic decay. Let \(\tau = dm/cp\), where \(d\) is the (measured) distance before decay, and \(m\) and \(p\) are kaon mass and momentum, respectively. The difficulty is that \(p\) is not directly measurable since the neutrino is not observed. It is however possible to proceed as follows. For each event the directions and momenta of the two charged particles are measured. We can then define \(p' = |\vec{p}_{+} + \vec{p}_{-}|\) and \(\tau' = dm/cp'\), and tabulate the experimental asymmetry as a function of \(\tau'\). It is then necessary to fold the transformation \(\tau \leftrightarrow \tau'\) into the expression (2), a process which

\(\text{*}\) Other definitions of \(p'\), which serve equally well or perhaps even slightly better, are possible. This definition serves, however, to illustrate the method.
requires a knowledge of the geometry of the apparatus and the beam momentum distribution. Both of these can be known with sufficient accuracy so that this step need not necessarily increase the error appreciably.

In the remainder of this paper we will discuss the precision which may be obtained in \( q_\eta \) in a particular experiment. In a proposed experiment, the kaons are produced at 75 mrad to a 24 GeV/c proton beam. The detector is assumed to have a sensitive decay region between 2.2 and 11.5 m from the point of kaon production. The main contribution to the result will be from kaons between 6 and 12 GeV/c, exploring the \( \tau \) interval \( 4 < \Gamma_s^{\tau} < 30 \). It is expected that \( 1.5 \times 10^7 \) leptonic decays and \( 5 \times 10^4 \) \( K_L \rightarrow \pi^+\pi^- \) decays per short-lived lifetime can be accumulated in an extended experiment. The theoretical expressions (1) and (2) are modified to account for the fact that both \( K \) and \( \bar{K} \) are produced at the target. This has the effect of diminishing the magnitude of the interference term by the factor

\[
\chi = \frac{I_K - I_{\bar{K}}}{I_K + I_{\bar{K}}}.
\]

Experimentally it is known that \( \chi \) varies from 0.5 at 6 GeV/c to 0.85 at 12 GeV/c for the postulated conditions.

Calculations have been performed which consist of « generating » experimental data according to expressions (1) and (2), using the present experimental values for \( \Delta m, \Gamma_s, \eta, \varphi_\eta, \chi \), etc., and then in turn taking these « data » and inverting the analysis to find the same parameters. In this latter part \( \chi \) and \( \varphi_\eta \) are left as functions of the momentum. The result is that the statistical error for \( \varphi_\eta \) is expected to be \( \sim \frac{3^o}{4} \).

Of course there may be unanticipated problems in the successful exploitation of this suggestion, but the experiment is in process of preparation at CERN, and in a year or two should be completed.

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I wish to thank Drs. K. Kleinknecht and P. Steffen for discussions and the calculations referred to in the last paragraphs of this note.
An Amateur's View (*) of Particle Physics (**).

V. F. WEISSKOPF


Here are some impressions of a non-expert on the present state of particle physics.

1. – The third spectroscopy.

One of the most striking aspects in the recent development is the ever-growing list of excited states of the baryon and meson. Remember the fact that nine years ago the only states known were p, n, N*(1236), Σ, Λ, Ξ, and π, K. Compare this with today's Rosenfeld table. The list is increasing, more levels are found every year and their quantum numbers become better known. It is a slow and painful task indeed; each simple level requiring many man-years of work. It is not made easier by the fact that, often, the widths of the levels are comparable to the level distances. This brings in a slightly disturbing feature: Some baryon levels are established only by phase analysis of meson scattering; they do not appear as a bump over a background in a scattering experiment. Such things happen very rarely in atomic or nuclear spectroscopies.

A relatively recent innovation is the production of certain mesons by electron-positron collisions. This is an unusually clean way of producing a single vector meson—only that type can be produced singly—free of nearby sources of strong interaction; it allows a better determination of the relevant properties.

(*) Amateur: 1) A person who does something for the pleasure of it rather than for money. 2) A person who does something more or less unskilfully. (Webster's New World Dictionary.)

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There are a few immediately obvious features in the level structure of hadrons. In contrast to atomic and nuclear spectra the level distances are comparable to the mass of the object. A striking feature is the existence of practically degenerate isotopic multiplets, another is the restriction of strangeness to negative values \( S > -3 \) for baryons and to \( |S| < 1 \) for mesons; finally there is a connection between the isotopic spin and the strangeness quantum numbers. This has led to the classification of levels by \( SU_3 \) and the discovery of \( SU_3 \) supermultiplets. The newly discovered levels seem to fit reasonably well into this scheme.

For me, the \( SU_3 \) classification is based on the quark model, with three types of quarks, which hadrons are composed of baryons being three quarks and mesons being quark-antiquark pairs. Thus, mesons, but not baryons, can be singly created and destroyed. The trichotomy of quark types is represented by a formalism with three « unitary spins »: isotopic, \( u \)-spin, and \( v \)-spin, each representing the alternatives given by one pair of the three types, just as the ordinary spin represents the alternative of spin-up and spin-down. Isospin and strangeness of hadrons are directly obtained by summing the corresponding quantities of the quarks; this simple rule explains the actual relations and restrictions among these quantum numbers. Indeed, one finds only the angular momentum quantum number to be unrestricted; a reflection of internal orbital excitations of the quarks which gives rise to families of hadrons with equal intrinsic quantum numbers but different angular momenta.

If one assumes that the binding force which keeps the quarks together is independent of the isotopic spin, and weakly dependent on the other spins, the main features of the baryon and meson spectrum can be reproduced. The systematics of quantum numbers, the multiplet structure, some features of transition probabilities between levels, and ratios between decay rates of mesons fall into place.

Of course, quarks have never been observed. Grave problems arise if that model is taken too seriously. However, it serves as a simply describable realization of \( SU_3 \) symmetry. The latter is what remains of the quark model, if one removes the quarks—the grin of the Cheshire cat. Why it works, is still one of the great miracles.

Whether the quark-idea is correct or not—it is improbable that it will turn out to be correct in its present simple form—the experimental evidence of hadron spectra points to an internal structure of the nucleons and mesons. In some ways the situation is reminiscent of atomic and molecular physics before Rutherford and Bohr. We know much of the spectrum of the nucleon, we know something about the force between nucleons; it is a relatively complicated force, attractive at larger distances, repulsive at smaller distances,
spin and symmetry dependent. In this respect it is similar to the chemical force between atoms. Only after the atomic structure was elucidated, one found out that the chemical force is based upon the more fundamental electric force between atomic constituents. It may well turn out that the nuclear force also will be understood as a consequence of more fundamental interactions and processes within the nucleon.

It would be misleading, however, to overstress the analogy between atomic structure and nucleon structure. As it was remarked before, the excitation energies of the nucleon are of the order of its rest mass energy, a circumstance which introduces new and badly understood features. With the present mathematical techniques, we are not able to deal with composite systems where the interaction is so strong that binding energies become comparable to the rest mass. One of the consequences of this situation is the important role of virtual particle pairs; such a system becomes an agglomeration of pairs of particles and antiparticles; the number of constituents, as it were, is always large and indefinite. It is to be hoped that methods can be found to deal with such conditions.

2. — Electron scattering.

The growing availability of high-energy electron beams is noticeable in the increasing number of interesting experiments with electrons. The resulting elastic form factors (electric and magnetic) of the nucleon are not yet understood. Their dependence on the momentum transfer $q$ goes as $q^{-4}$ at high $q$-values. The fact that the form factor decreases smoothly to zero at high $q$'s indicates that the nucleon is an extended system and that there is no hard and small core noticeable at the center of the nucleon. It does not exclude the possibility that the nucleon is made of hard and small constituents. It only shows that the charge and magnetic distribution has no accumulation at the center as it has in the hydrogen atom.

The inelastic electron scattering has turned out to be most interesting. The excitation of higher baryon states by this process is a repetition of the Franck-Hertz experiment, more than 50 years later at a billion times larger energy scale. The form factor of these excitations seems to have a similar $q$-dependence as the elastic one — no wonder, since we expect the excited states to have similar charge and magnetic distributions. An interesting feature appears when one looks at the very large energy transfers of the scattered electrons, way above the known resonances. Then the strong $q$-dependence disappears and the scattering seems to be independent of the momentum transfer, apart from the trivial electric charge effects (Mott scattering).
lack of $q$-dependence indicates a scattering object smaller than the length associated with the momentum transfer. Did we hit here some very small entity or entities within the nucleon, some constituent or perhaps the quark? Whatever the detailed interpretation of these results may be, the absence of a $q$-dependence at high-energy losses certainly indicates the existence of a length, small compared to $10^{-14}$ cm, which should play an important role within the nucleon.

3. – Current algebra.

How can we look into the dynamic situation inside the hadrons? Strong interaction processes seriously distort the hadron under observation. Weak and electromagnetic processes, however, leave it intact and can be directly interpreted as the effect of currents inside the hadron. The most obvious example is the electromagnetic current density $j'^{\mu}_{EM}$ (four vector) whose matrix elements determine, and can be determined by, electromagnetic phenomena. Similarly there exist four more current densities which, in the same way, determine the weak interaction phenomena. Why four? Firstly, the weak interaction as exemplified by lepton-pair emission, has two realizations, non-relativistically speaking, the spins of the leptons may be parallel or antiparallel. This is connected with the fact that any weak process is determined by two currents, $j'^{\nu}_{\mu 0}$ and $j^{A}_{\mu 0}$, the vector and the axial current. In addition, there are two types of weak processes: strangeness conserving and strangeness changing ($|\Delta S| = 0, 1$), which leads to two more currents: $j'^{V}_{\mu 0}$, $j^{A}_{\mu 1}$. Ideally one could measure all matrix elements of the five current densities, if every conceivable electromagnetic or weak process were entirely known. So far, we known very little about them. One most important known fact is the discovery (Gershtein-Zeldowitch, Feynman–Gell-Mann) that $j'^{V}_{\mu 0}$ apart from a constant, is an isotopic brother of $j^{V}_{EM}$, that is, it differs only by the fact that the charge changes in the former. (This difference is a rotation in the isotopic spin space.) Current algebra is a bold generalization of this idea, which assumes that all five of them are brothers, they are supposed to belong to a family of sixteen current densities which I will now describe. Again we make use of the quark model. Whatever the dynamics are, there are currents running in this model; for example, we can define three currents which describe the flow of each one of the three components of the isotopic spin. Since there are two more unitary spins, the $U$-spin and the $V$-spin, one would think that there will be nine currents. However, the three types of spin are not independent—an $I$-flip followed by a $U$- and a $V$-flip brings us back to the original state; therefore, there are, in fact eight
independent current densities: \( j_\mu^\lambda, \lambda = 1, \ldots, 8 \). All of them are vector fields. The electromagnetic current is included and they are all conserved currents.

The second half of the family is obtained by considering something which—non-relativistically—corresponds to the spin density (ordinary angular momentum spin) of the quarks. It represents an axial vector field within the hadron. If we associate the angular momentum spin density of the quark (in this non-relativistic approximation) with its unitary spin components \( (I, U, V-\text{spin}) \), we obtain eight axial current densities \( j_\mu^a, \lambda = 1, \ldots, 8 \). They are not conserved currents since a «spin density» is not a conserved vector field.

The weak currents are supposed to be some linear combinations of the sixteen currents, namely the ones that give rise to the relevant charge and strangeness change.

What follows from this assumption? Evidently the Feynman–Gell–Mann relations between the electric current and the vector part of the weak current is part of this assumption. But there are more; there exist simple relations between those sixteen currents since, in terms of the quark model, they can be represented by simple operators. For example, equal-time commutation relations exist, such as

\[
[j_0^a(x), j_0^b(x^1)] = \delta(x - x^1)f_{\alpha\mu\nu\rho}j_0^\nu(x),
\]

where \( f_{\alpha\mu\nu\rho} \) are constants and the zero index refers to the time components of the current densities. This is equivalent to a sum rule

\[
\sum_b (a|M|b)(b|N|a) - \sum_b (a|N|b)(b|M|a) = \text{const} (a|P|a),
\]

where \( M, N, P \), are operators connected with the currents and \( a \) and \( b \) are quantum states. One example [1] refers to neutrino cross-sections:

\[
\lim_{E_\nu \to \infty} \left[ \frac{d\sigma(\bar{\nu}p)}{dk^2} - \frac{d\sigma(\nu p)}{dk^2} \right] = \frac{G^2}{\pi} (\cos^2 \theta_C + 2 \sin^2 \theta_C),
\]

where \( \sigma(\nu p) \) is the total neutrino cross-section of a proton, \( k \) is the momentum transfer, \( G \) the weak coupling constant, and \( \theta_C \), the Cabibbo angle. We are far from able to test such relations.

Current algebra proper establishes connections between currents, but says nothing about the currents themselves. There are approximate ways to get some limited information about the current distribution within the hadrons. A baryon, for example, can be considered as surrounded by virtual meson fields. The mesons with lower mass contribute more strongly and at larger
distances compared to those with higher mass. Thus particularly the pions, but also the kaons and vector mesons such as the ρ or ω will play a major role in the «meson cloud». Therefore, it is plausible that the vector and axial vector current densities have some relations to those meson fields which have similar geometrical properties. For example, the vector current densities, such as the electromagnetic one, should be related to the ρ, ω, φ fields; a suitable axial vector current density (the one that carries isotopic spin) should be related to the pion field, since the divergence of the axial current density is a pseudo-scalar field, like the pion field. Such assumptions are known under the name of «vector meson dominance» and «pion dominance». The former is equivalent to an assumption that for small momentum transfers—only for those will this restriction to the lowest mesons hold—a light quantum interacts with a baryon in a similar way as a vector meson would, apart from a proportionality constant. That constant is determined by the electromagnetic properties of the vector mesons, and can be deduced from the decay probabilities of these mesons into electron pairs. This assumption has proved to be quite useful for the prediction of photoprocesses with baryons.

The relation between the axial current density and the pion field has many interesting consequences. It is assumed that the divergence of the axial current density is proportional to the pion field. The proportionality constant is given directly by the pion decay into a lepton pair and it represents in some way, the «axial charge» of the pion. This assumption (together with some assumptions about a reasonable behavior of the matrix elements of the pion field) gives rise to a connection between the «axial charge» (the pion-nucleon coupling constant), and the axial weak interaction coupling constant of the nucleon (Goldberger-Treiman relation). The connection comes about in the following way: The weak interaction of the nucleon is caused by the axial current, which is proportional to the pion field which, in turn, is coupled to the nucleon and therefore dependent on the pion-nucleon coupling constant.

The Adler-Weisberger relation is another example which can be derived from this connection between axial weak interaction and pion-nucleon coupling. Here one also uses current algebra which establishes a connection between axial currents and vector currents. One then gets an expression for the ratio of the axial—to the vector coupling of weak interactions, in terms of meson-nucleon cross-sections.

The relation between the axial current and the pion density has an interesting bearing on the question of the conservation of the axial current. The divergence of the axial current density—which should vanish if the current were conserved—was assumed to be proportional to the pion field. The
pion field in the vicinity of a hadron is spread out over relatively large distances of the order of $m^{-1}_\pi$. If this spread is reasonably smooth, one would conclude that the Fourier components for wave numbers much higher than $m_\pi$ should be small. Hence, matrix elements of the divergence of the axial current density will be very small for momentum transfers larger than $m_\pi$. This means that the axial current is conserved in this limit. The Partial Conservation of Axial Currents (PCAC) gives rise to a number of simple relations for the interaction of soft mesons with hadrons. In electrodynamics where the coupling is also mediated by a conserved current, absorption, emission and scattering of long wave length light quanta is given by simple expressions proportional to a power of the electric charge. We find similar approximate expressions involving the axial charge for the corresponding pion processes.

4. - Strong interaction processes.

The theoretical description of the interaction between hadrons is a more complex problem than that of the interaction of leptons or the electromagnetic field with a hadron. The concept of current densities is adapted to the case where the interaction is weak and the interacting field is not much distorted near the hadron. This is the case when one can use perturbation theory and the first approximation of an interaction is the dominant one. Then the interacting field is coupled to those features of the hadron which exhibit the symmetries of the unperturbed field. When hadrons interact among each other, this approximation method is no longer applicable since the interactions are strong. Yet it remains to be seen whether the current concept can be sufficiently generalized and adapted to problems of strong interactions. Perhaps attempts at using a so-called phenomenological Hamiltonian are efforts in that direction.

Most of the work in hadron interactions is based on a different approach. The scattering amplitude $A$ of one hadron scattered by another is a function of the relative energy and the momentum transfer. Since there is no theory for the calculation of this magnitude, one establishes some general rules to which the amplitude $A$ is subjected, and then one tries to extract some theoretical predictions which may follow. The general rules are based on the following four items:

\begin{itemize}
  \item[a)] relativistic invariance;
  \item[b)] causality;
  \item[c)] unitarity;
  \item[d)] analyticity.
\end{itemize}
Relativistic quantum mechanics tells us how to describe unambiguously a group of non-interacting particles with definite masses and spins. Every scattering process begins and ends with such a group. It, therefore, defines the variables on which the scattering amplitude depends. Items b) and c) establish relations which the scattering amplitudes must fulfill, such as dispersion relations and the optical theorem. Item d) contains the assumption that the scattering amplitude is an analytic function of the relevant variables apart from certain singularities which have well-defined physical significance. I suppose that any imaginable and reasonable theory will always give rise to such functions with certain poles, cuts, and definite asymptotic properties.

The importance of item d) comes from two circumstances: First, a pole of the scattering amplitude represents a stationary or metastable state of the composite system of the two scatterers; the relative energy at which the pole occurs is the mass of the stationary state and the residue is connected with the coupling constant of the binding force. Some of these poles may have an overriding influence on the energy dependence of $A$, which sometimes may lead to a simple expression for $A$ involving only a few dominant poles. The second point is the crossing relations: There is a relation between the amplitudes of two reactions which differ by replacing a particle coming in by its antiparticle going out, and also replacing an outgoing particle by its antiparticle coming in. The amplitude of one reaction is an analytical continuation of the other in certain variables. If we know something about one reaction, we can arrive at some conclusions about the other; for example, a strong pole in one channel may have a noticeable effect in the other. Here again the fact that the relevant energies are of the order of the rest masses of the particles involved plays an important role. Therefore a pole in one reaction channel may not be so much farther away from an interesting energy region in the crossed reaction, than it is from a relevant energy region in its own channel. This consideration also shows how crossing relations are relatively unimportant in nuclear or atomic physics, where the relevant energies are small compared to the rest masses.

The most important experimental fact of hadron collisions is the following one: In the case of elastic scattering and of those inelastic scatterings where the quantum numbers of the incident and outgoing particles are the same, the cross-sections seem to reach an energy independent asymptotic values at high energies. For inelastic processes, where quantum numbers are exchanged, the cross-sections vanish as a negative power of the energy. This experimental result is not yet explained by any theory, but it is often used as a basis of deriving other results.

A number of conclusions can be derived from this observed asymptotic behavior, with the help of the items a)-d). One is the Pomeranchuk theorem,
An amateur's view of particle physics

Concerning the quality of particle and antiparticle elastic scattering (e.g., \( \pi^+ + P \) and \( \pi^- + P \) at high energies). Other recent conclusions are the «finite energy sum rules». If one assumes—in some cases there are good reasons—that the difference between the actual and the asymptotic amplitude of a reaction vanishes stronger than a certain power of the energy (stronger than \( s^{-1} \)), the integral of the actual amplitude over the energy from zero up to a finite energy \( E \) is essentially the same as the integral of the asymptotic amplitude over the same energy range. The upper limit \( E \) must be an energy at which the asymptotic form is already applicable. This relation establishes an interesting connection between the low energy behavior governed by resonances, and the high-energy behavior which sometimes can be predicted by the Regge-pole theory discussed later.

One of the important concepts in describing interactions between hadrons is the concept of exchange. In every two-body reaction something is transferred from one partner to the other: It is momentum only in an elastic reaction, it can be any intrinsic quantum number in others. (We call «intrinsic» any quantum number except angular momentum.) It is sometimes useful to describe the reaction by asserting that a suitable particle is exchanged, which is coupled by an interaction with both partners and carries over the quantities. The situation could be described in the following way: A hadron is considered as a conglomerate of hadron pairs whose quantum numbers add up to the intrinsic quantum numbers of the hadron under consideration—a proton is a conglomerate of \( n\pi^+ \), \( K^+ \Lambda \), etc. When hadrons meet they may exchange a member of those pairs. The rules of these exchanges are hard to get at, since particles in the conglomerate do not behave like free particles. One must find means how to deal with this situation. The Regge-pole method is such an attempt: One makes use of the crossing relations and starts from another reaction channel (\( t \)-channel) in which the exchanged particle appears under more «natural» conditions, that is as a free particle. It is the channel where it is created by the fusion of two hadrons (more exactly, by the fusion of one of the incoming hadrons and the antiparticle of an outgoing one). In this channel the exchanged particle appears as a composite system made up by the two hadrons. Such a system may assume a number of states of different angular momentum each of which may be identified with some observed particle (Regge families). Each member of this family has the same intrinsic quantum numbers and could serve as an exchange particle in the reaction. In terms of a continuous \( J \)-variable, a Regge family is represented as a function \( m(J) \), where the actual particle masses are the values of \( m \) at integer (or half integer) \( J \)'s (Regge trajectory).

The existence of these families or trajectories is exploited to express the scattering amplitude in terms of a sum of contributions from special poles.
(Regge poles) which appear if the angular momentum is used as a continuous variable $J$. Each Regge pole represents not only a single state of the composite system, but it encompasses the effects of a whole Regge family upon the scattering amplitude. It is assumed that these Regge-pole contributions have an overriding influence so that they dominate the scattering amplitude not only in the $t$-channel, but also in the actual scattering channel. This may be an appropriate way of dealing with the concept of exchange in strong interactions, which takes into account not only the effect of exchange of one particle, but of each member of a Regge family. Its main results are statements about the scattering amplitude in the limit of small momentum transfer and high-energy. Hence, it is relevant for the asymptotic energy behaviour discussed before.

It must be said that these Regge extrapolations from one channel to another introduce a number of arbitrary functions into the picture such as the «coupling strength» of the Regge trajectories (residues of the Regge poles). In order to satisfy the analyticity of $\mathcal{A}$, these arbitrary functions must fulfill a lot of complicated conditions (evasions and conspiracies), and new «daughter» trajectories must be introduced which may not necessarily give rise to observed particles. In view of all these complications, together with the introduction of Regge-cuts—the same kind of treatment for the exchange of pairs of particles—one may ask whether the Regge-pole method provides deeper insight into what is really going on. I quote Van Hove [2] «The Regge-pole model is not a theory with a high predictive power, but a refined framework to correlate collisions—especially of inelastic type—to exchange processes».

Recently Veneziano has proposed an expression for the scattering amplitude for the case of meson-meson scattering which, perhaps, goes a little further than the Regge-pole model. It is a simple expression representing a specific case of a Regge-pole model, in which all Regge trajectories and the functions describing their coupling strength are well defined. The trajectories are straight lines ($m^2$ is a linear function of $J$) and the fact that only mesons are involved, introduces symmetries which define all the coupling strength functions in an unambiguous way. The simplicity and the internal consistency of this expression are impressive. It seems that some of the conclusions one can draw from this expression are in approximate agreement with the present experimental knowledge on meson-meson scattering. Obviously, the Veneziano expression can only be a first approximation to reality; there are theoretical and experimental reasons for this. The lack of unitarity is of the former kind, the approximate nature of the results is of the latter. It is important to keep in mind that the Veneziano expression is not a theory of meson interaction. It is a guess at a result, in the form of a simple math-
emathical example for a scattering amplitude which fulfils most of the conditions that are imposed. At worst, it is a simple mathematical example of a scattering amplitude fulfilling most fundamental requirements; at best, it may be a first approximation to reality and may have the value which the Balmer formula had in the development of our knowledge of the hydrogen atom.

5. — \textit{CP-violation.}

After the discovery of parity violation in 1957, Pauli exclaimed in a famous letter: «God is left-handed». Actually, it turned out soon, that this is not so. God is right-handed in the antiworld where parity is violated in the opposite sense. However, seven years later we had reasons to feel uneasy; Christenson and Cronin, Fitch, and Turlay found out that world and antiworld are not equivalent even if left is replaced by right. In other words, nature is not \textit{CP} invariant. (\textit{C} transforms world in antiworld, \textit{P} left into right.)

In order to find extremely small differences between a particle and its antiparticle, one must set up beat frequencies. That means, one must be able to create a linear combination of the two with well-defined phases. This can only be done if there is an interaction which can transform one into the other. No baryon-antibaryon pair would do, since all interactions conserve baryon numbers. No charged meson pair would do because all interactions conserve charge. An uncharged meson with strangeness zero is its own antiparticle so that it cannot be a candidate for particle-antiparticle mixing. All that is left are uncharged mesons with a strangeness different from zero. Weak interactions can transform one into the other since they do not conserve strangeness. Hence, the \((K_0, \bar{K}_0)\) pair rose to such great fame.

Thus \(K_0\) and \(\bar{K}_0\) are related by the transformation \textit{CP}: \(CP K_0 = -\bar{K}_0\). As long as the weak interaction which transforms \(K_0\) into \(\bar{K}_0\) is \textit{CP}-invariant, the two states are equivalent and two linear combinations with slightly different energy are formed, a symmetric and an antisymmetric one:

\[
K_1 = K_0 - \bar{K}_0, \quad K_2 = K_0 + \bar{K}_0.
\]

Since the mixing of \(K_0\) and \(\bar{K}_0\) is done by a second-order weak process the splitting is extremely small; it is of the order of \(10^{-5}\) eV. It is instructive to think of the analogous situation with two identical coupled pendulae.
(This analogy was used by F. Crawford in a colloquium talk.) Such coupling also produces a symmetric and an antisymmetric proper vibration with slightly different frequencies. Usually the antisymmetric one has more friction than the symmetric one so that, after some time, the latter one only survives. The same is true in the \((K_0, \bar{K}_0)\) system. Friction corresponds to weak decays. We direct our attention mainly to decay into pions, either into two, or into three pions, of the total charge zero. Two pions are even under \(CP\)-transformation, three pions are predominately odd if they emerge from a small source. Hence, the symmetric combination \(K_2\) (which is odd under \(CP\)) can only decay into three pions, the antisymmetric \(K_1\) decays mostly into two. Since two pions have more phase space, the \(K_1\) mode has more «friction» and the \(K_2\) mode survives longer.

If the weak interactions were not \(CP\)-invariant, \(K_0\) and \(\bar{K}_0\) would no longer be equivalent in this interaction. This would have two consequences: 

\(a\) the eigenstates, which we call \(K_S\) and \(K_L\) (short-lived and long-lived), would no longer be either symmetric or antisymmetric. They would be some other linear combination: \(K_S \neq K_1\), \(K_L \neq K_2\). The same would happen in the case of two coupled pendulae, if the two pendulae were not exactly equal.

\(b\) The number of decay pions would no longer be an indication of the symmetry of the decaying state; \(K_2\) would also be allowed to emit two pions. As the result of both consequences, one would observe that the long-lived eigenstate sometimes emits two pions and not always three. This is what Christenson et al., have found to our surprise and bewilderment, and they have shown that \(K_0\) and \(\bar{K}_0\) are not equivalent; world and antiworld are distinguishable. This simplest situation is realized if only consequence \(a\) of \(CP\)-violation occurs. This would be the case with a so-called «super-weak» interaction, which violates \(CP\) but is very much weaker than the ordinary weak interaction. Hence, it would practically not influence the ordinary weak decay into two or three pions, so that this alternative can still be used as a distinction between symmetry and antisymmetry. But, because of the extremely small split between the two modes—second-order weak interaction—a very small \(CP\)-violating term (of the size of the second-order of weak interaction), would destroy the pure symmetry or antisymmetry of the modes. The long-lived mode would not be purely symmetric but would have a small antisymmetric admixture, which shows up in the form of a two pion decay. There is an indication that consequence \(b\) may in fact not be present: The \((\pi^+\pi^-/\pi^0\pi^0)\)-ratio seems to be the same in the short-lived and in the long-lived, two-pion decay. This fact does not exclude consequence \(b\) but if it were not so, one would be sure that \(b\) is present.

There is new independent evidence regarding the two modes of the \((K_0, \bar{K}_0)\) system, which came from the comparison of the following two
decays of the long-lived mode:

\[ K_L \rightarrow \pi^+ + e^- + \bar{\nu}, \quad K_L \rightarrow \pi^- + e^+ + \nu. \]

If \( K_L \) were exactly symmetric in \( K_0 \) and \( \bar{K}_0 \), the two decays would be exactly equally strong. Actually, a group at Stanford and at Columbia [3] have found a difference of the order of one percent, a direct manifestation of an inequality of world and antiworld in the \( K_L \)-mode.

6. – General situation.

It is questionable, whether our present understanding of high-energy phenomena is commensurate to the intellectual effort directed at their interpretation. We are able to describe the phenomena in terms of particle fields, scattering amplitudes, current densities, etc., by using a language which implicitly assume that there exists a valid quantum theory of particles with interactions—strong, electromagnetic, and weak—which explains everything in a consistent way along the accepted principles of quantum field theory. The experiments do not reveal any inconsistency with these principles. On the other hand, no such theory has yet been formulated. It is impossible to decide at this stage, whether this lack of success is caused by the mathematical difficulties of a field theory with strong interaction, or by the fact that we have not yet found the conceptual framework necessary to understand the situation.

The present theoretical activities are attempts to get something from almost nothing; attempts to conclude as much as possible from a few general restrictions of the formalism, such as relativity, causality, unitarity, and analyticity. It is astonishing and exhilarating how much, in fact, one can conclude from so little input. It is a veritable «bootstrap» operation. But does it lead to a deeper understanding of what is going on within the hadrons? Does it give us any insight into what lies behind that wealth of experimental material on resonances and reaction rates? It is true that the present formulation of weak interaction phenomena is impressive in its eloquent simplicity. Ideas such as \( SU_3 \) and current algebra were very exciting when they were introduced; they supplied an appropriate terminology for the description of facts, they showed how certain phenomena are connected with others, but it does not appear that they wave brought us much nearer to an understanding of the subnuclear world. The existence of three types of seemingly unconnected interactions is still an unsolved problem, although the close relation between electromagnetic and weak vector currents may be the first
hint at some deeper connection. We also have no understanding yet of the nature of the electric charge with its double manifestation as electron and muon.

High-energy physics today is an experimental science. We are exploring unknown modes of behaviour of matter under completely novel conditions. The field has all the excitement of new discoveries in a virgin land, full of hidden treasures, the hoped-for fundamental insights into the structure of matter. It will take some time before we can produce a rational map of that new land.

REFERENCES

Some Questions Concerning Adiabatic Transformations.

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1. -- Introduction.

The old questions connected with the so-called «adiabatic theorem» [1] still play an important role in many questions of quantum field theory such as renormalization, definition of the vacuum state, etc. It may therefore be of some interest to simplify or clarify the proof of certain results, even though they are generally regarded as well known. We refer in particular to the following: A very common situation arises when a coupling constant is switched on exponentially, \( g(t) = g \exp [\alpha t] \); the adiabatic limit is then obtained, of course, for \( \alpha \to 0^+ \). For finite \( \alpha \), the state \( \psi_g \) attained by the system at, say, the time \( t = 0 \), satisfies a well-known differential equation with respect to the coupling constant \( g \). This equation was obtained by Gell-Mann and Low [2] by means of a perturbation expansion in the interaction representation. We note in Sect. 2 that a much simpler and more transparent derivation can be given, without resorting to a perturbation expansion. Our main topic, however, is the limit \( \alpha \to 0 \). As is well known, the state vector \( \psi_g \) contains a phase factor \( \exp [-i\theta(g, \alpha)] \) which becomes singular in the limit \( \alpha \to 0 \), in the sense that \( \theta \sim 1/\alpha \). Physically, this infinite phase must be the integral over time, from \( t = -\infty \) to \( t = 0 \), of \( \Delta E(g)/\hbar \), where \( \Delta E(g) \) is the level shift produced by the perturbation; when \( \alpha \) is finite, this shift operates effectively for a time of the order of \( 1/\alpha \), hence the above-mentioned result. Formally, the existence of the singularity can be easily checked [2], if one examines the behaviour, in the limit \( \alpha \to 0 \), of the successive terms in the perturbation expansion; one finds that the term of order \( g^n \) contains singular terms in \( \alpha \), of order \( \alpha^{-1}, \alpha^{-2}, \ldots, \alpha^{-n} \). It is intuitively plausible, but not obvious mathematically that these terms arise from

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the expansion of the exponential phase factor. If this is true, it becomes possible to eliminate the singularity at \( z = 0 \) by renormalization of \( \psi_g \), for example [2], by the definition

\[
\chi_g = \psi_g/(\varphi, \psi_g),
\]

where \( \varphi \) is the initial value (at \( t = -\infty \)) of the state vector. Here \((..., ...)\) indicates as usual the scalar product of two vectors. It is, of course, a fundamental result that \( \chi_g \) should have a finite limit for \( z = 0 \), because it is this limit, which is then shown to be an eigenstate of the perturbed Hamiltonian, as stated by the adiabatic theorem.

Now Gell-Mann and Low content themselves with asserting that all the above statements can be verified «up to any desired order in \( g \rangle \), i.e., presumably as far as one's patience and willingness to be convinced allow. The direct calculations actually become quite laborious very soon. The question, however, is so fundamental and simple that it should be settled completely for an arbitrary order \( n \). We show in the following that this is indeed possible by utilizing the differential equations in \( g \) satisfied by the state vectors \( \psi_g \) and \( \chi_g \), equations already given by the above-mentioned authors. The connection between the adiabatic solution and the Rayleigh-Schrödinger perturbation series is thus completely established.

2. – The differential equations.

The problem to be dealt with is the following [2]: The Hamiltonian of interest is of the form

\[
\mathcal{H} = \mathcal{H}(g) \equiv \mathcal{H}_0 + g\mathcal{H}_1.
\]

One asks for a solution of the time-dependent Schrödinger equation

\[
i\frac{\partial \psi}{\partial t} = \{\mathcal{H}_0 + g \exp [\lambda t] \mathcal{H}_1\} \psi,
\]

namely, the solution which for \( t \to -\infty \) behaves asymptotically according to

\[
\psi \sim \varphi \exp [-i\epsilon_0 t],
\]

where \( \varphi \) is a normalized eigenstate of the unperturbed Hamiltonian corresponding to a nondegenerate eigenvalue \( \epsilon_0 \):

\[
(\mathcal{H}_0 - \epsilon_0)\varphi = 0.
\]

In eq. (3), \( \lambda \) is a real constant \( > 0 \).
Some questions concerning adiabatic transformations

We now denote by \( \psi_g \) the value of this solution for \( t = 0 \). More explicitly, let \( \psi(t, g, \alpha) \) be the unique solution of eqs. (3) and (4), then

\[
(6) \quad \psi_g = \psi(0, g, \alpha).
\]

We derive a differential equation with respect to \( g \), satisfied by \( \psi_g \), as follows. Consider the function:

\[
(7) \quad \psi_1(t) = \exp [ie_0 \tau] \psi(t + \tau, g, \alpha)
\]

which, apart from a phase factor, is obtained from the solution \( \psi \) by a time-displacement \( \tau \). Clearly \( \psi_1 \) satisfies the asymptotic condition (4), and also the Schrödinger equation (3), for a different value of \( g \): \( g_1 = g \exp [\alpha \tau] \). Therefore, \( \psi_1(t) = \psi(t, g \exp [\alpha \tau], \alpha) \) or:

\[
(8) \quad \psi(t + \tau, g, \alpha) = \exp [-ie_0 \tau] \psi(t, g \exp [\alpha \tau], \alpha).
\]

Differentiating with respect to \( \tau \) and setting \( \tau = 0 \), we can express the time derivative in eq. (3) in terms of \( \alpha g (\partial \psi/\partial g) - ie_0 \psi \). At \( t = 0 \), eq. (3) becomes

\[
(9) \quad i\alpha g \frac{\partial \psi_g}{\partial g} = \{ \mathcal{H}(g) - e_0 \} \psi_g,
\]

which is Gell-Mann and Low's eq. (A.5).

This derivation does not depend on a perturbation expansion and it can be easily generalized. For example, assume

\[
(10) \quad \mathcal{H}(g) = \mathcal{H}_0 + g \mathcal{H}_1 + g^2 \mathcal{H}_2
\]

(or even a polynomial in \( g \) of order \( n \)) and then consider the time-dependent Schrödinger equation in which \( \mathcal{H}(g) \) is replaced by \( \mathcal{H}(g \exp [\alpha t]) \). Then everything goes through as before, and eq. (9) is obtained. In the following, however, we shall restrict ourselves to the simple case, eq. (2). The generalization (10) is not without interest, however, since Lagrangians and Hamiltonians with quadratic terms in the coupling constants are quite common.

Finally we notice with Gell-Mann and Low the following equations of which the second one is a differential equation for the renormalized state vector \( \chi_g \):

\[
(11) \quad i\alpha g \frac{\partial}{\partial g} (q, \psi_g) = (q, [\mathcal{H}, \psi_g] - e_0) \psi_g = g(q, \mathcal{H}_1 \psi_g),
\]

\[
(12) \quad i\alpha g \frac{\partial \chi_g}{\partial g} = \{ \mathcal{H}(g) - e_0 \} \chi_g - \chi_g i\alpha g \frac{\partial}{\partial g} \ln(q, \psi_g).
\]
Both equations, of course, follow from eq. (9). In eq. (12), furthermore, the last term can be modified as follows

\[ i x g \frac{\partial}{\partial g} \ln(q, \psi_g) = g(q, \mathcal{H}_1 \chi_g), \]

so that finally

\[ i x g \frac{\partial \chi_g}{\partial g} = (\mathcal{H}(g) - E(g, x)) \chi_g, \]

\[ E(g, x) = \varepsilon_0 + g(q, \mathcal{H}_1 \chi_g). \]

As we shall see, in these two last equations, one can go to the limit \( x = 0 \) without any trouble, and \( E(g, 0) \) is then the perturbed eigenvalue [of the operator \( \mathcal{H}(g) \)].

3. – The adiabatic limit.

It is customary to obtain the perturbation expansion of \( \psi_g \)

\[ \psi_g = q + \sum_{n=1}^{\infty} \frac{g^n}{\varepsilon_0 - \mathcal{H}_0 + i x n} \cdot \mathcal{H}_1 \frac{1}{\varepsilon_0 - \mathcal{H}_0 + i x(n-1)} \mathcal{H}_1 \cdots \frac{1}{\varepsilon_0 - \mathcal{H}_0 + i x} \mathcal{H}_1 q, \]

from the time-dependent equation (2). From this, in principle, one can obtain the expansion of \( \chi_g \), eq. (1), but, as we have said, this is rather cumbersome. Notice, however, that (16) can be obtained directly from eq. (9). It should be almost equally simple to obtain an expansion for \( \chi_g \) directly from eqs. (14) and (15). For \( g = 0 \), \( \chi_g = \psi_g = \psi \); assume therefore

\[ \chi_g = \sum_{n=0}^{\infty} g^n q_n(x), \]

where \( q_0 = \psi \). Notice also that, from (1): \( (q, \chi_g) = 1 \), so that we must have

\[ (q, q_n) = \delta_{n0}; \quad (n = 0, 1, 2, ...). \]

Substituting (17) into (15) we obtain

\[ E(g, x) = \varepsilon_0 + \sum_{n=1}^{\infty} g^n \varepsilon_n(x), \]

where

\[ \varepsilon_n(x) = (q, \mathcal{H}_1 q_{n-1}(x)). \]
In particular, \( \varphi_1(x) = (\varphi, \mathcal{H}_1 \varphi) \) is the first-order perturbation of the energy level, and is independent of \( x \). Next substitute (17) and (19) into (14) and obtain the recurrence relation

\[
(\epsilon_0 - \mathcal{H}_0 + ixn)\varphi_n(x) = \mathcal{H}_1 \varphi_{n-1}(x) - (\varphi, \mathcal{H}_1 \varphi_{n-1}(x)) \varphi - \sum_{m=1}^{n-1} \epsilon_{n-m}(x) \varphi_m(x); \quad (n = 1, 2, \ldots).
\]

Thus the \( n \)th term in the expansion of \( \varphi_n \) is at first sight much more complicated than the corresponding term in (16). It has, nevertheless, much simpler properties in the limit \( x = 0 \). Notice in fact that the solution of an equation \((\epsilon_0 - \mathcal{H}_0 + ixn)\varphi_n = \omega_n\) or \( \varphi_n = (\epsilon_0 - \mathcal{H}_0 + ixn)^{-1} \omega_n \) becomes singular as \( x \to 0 \) only if \( (\varphi, \omega_n) \neq 0 \). But the right-hand side of (21) may be rewritten

\[
\omega_n = \Lambda \mathcal{H}_1 \varphi_{n-1} - \sum_{m=1}^{n-1} \epsilon_{n-m} \varphi_m,
\]

where \( \Lambda \) is a projection operator (projection on the subspace orthogonal to \( \varphi \)). Together with (18) this implies that \( \omega_n \) is orthogonal to \( \varphi \), and \( \varphi_n = \lim_{x \to 0} \varphi_n(x) \) exists, and may be obtained by solving the recurrence equations

\[
(\epsilon_0 - \mathcal{H}_0)\varphi_n = \Lambda \mathcal{H}_1 \varphi_{n-1} - \sum_{m=1}^{n-1} \epsilon_{n-m} \varphi_m,
\]

\[
\epsilon_n = (\varphi, \mathcal{H}_1 \varphi_{n-1}).
\]

These are, however, precisely the equations of the Rayleigh-Schrödinger expansion of the perturbed wave function and perturbed eigenvalue \( \epsilon_0 + \Delta E(g) \). The cancellation of the singular phase factor is therefore demonstrated to arbitrary order.

4. Remarks and extensions.

It is clearly possible to extend the above calculations in various directions. For example the conclusions of the last Section could be easily extended to the case of eq. (10). Also, one could expand \( \varphi_n(x) \) and \( \epsilon_n(x) \) in powers of \( x \) and examine non-adiabatic effects; for example, terms up to order \( x \) in \( \epsilon_n(x) \) are easily connected to the normalization factor in the denominator in eq. (1), since one can prove, by means of eq. (13), that

\[
\ln (\varphi, \varphi_g) = -i \int_{q}^{g} \Delta E(g') \frac{dg'}{g} - i \sum_{n=1}^{\infty} g^n \frac{g_n^{(1)}}{n} + \ldots,
\]
where
\[ \varepsilon_n^{(1)} = (d\varepsilon_n/dz)_{z=0} . \]

We want to deal briefly with a different extension. The methods of perturbation theory, adiabatic switching on, and renormalization, have been applied not only to the Schrödinger equation but also directly to the field equations in the Heisenberg representation, as, for example, in the work of Källén [3] and of Yang and Feldman [4]. The most important difference from the previous case is that the equations are nonlinear. Nevertheless a method similar to that of Sect. 2 leads to some interesting equations. Consider as an example the scalar field equation
\[ (\Box + m^2)\phi(x) = -\lambda \exp[\alpha t]\phi^n(x) . \]

Here \( x = (x, t) \) and the coupling constant \( \lambda \) is «switched on» by the exponential factor. One seeks a solution which satisfies the boundary condition
\[ \phi(x) \sim \phi_0(x) \quad \text{when} \quad t \to -\infty , \]

where \( \phi_0 \) is a solution of the homogeneous equation \((\Box + m^2)\phi_0 = 0\). For simplicity, we limit ourselves to the nonquantized version of the theory [5]. We notice that instead of the boundary condition (26) we may consider more generally
\[ \phi(x) \sim \phi_0(x + \xi) \quad \text{when} \quad t \to -\infty , \]

where \( \xi = (0, \tau) \) is a displacement in the time direction. In fact \( \phi_0(x + \xi) \) is also a solution of the homogeneous equation (and in the quantized case, it satisfies the same commutation law as \( \phi_0(x) \)). Now if we call \( \Phi(x; \lambda, \xi) \) the solution of (25) and (26'), it is easy to prove by reasoning as before, that \( \Phi \) satisfies the functional equation:
\[ \Phi(x; \lambda, \xi) = \Phi(x + \xi; \lambda \exp[-\alpha \tau], 0) , \]

in other words \( \Phi \) depends on the three variables \( t, \lambda \) and \( \tau \) only as a function of two variables: \( t + \tau \) and \( \lambda \exp[-\alpha \tau] \). Hence \( \Phi(x; \lambda, \xi) \) satisfies identically the differential equation
\[ -\frac{\partial \Phi}{\partial t} + \alpha \lambda \frac{\partial \Phi}{\partial \lambda} + \frac{\partial \Phi}{\partial \tau} = 0 . \]

By differentiation one derives from this
\[ \frac{\partial^2 \Phi}{\partial t^2} = \frac{\partial^2 \Phi}{\partial \tau^2} + 2\alpha \lambda \frac{\partial^2 \Phi}{\partial \lambda \partial \tau} + \alpha^2 \lambda^2 \frac{\partial^3 \Phi}{\partial \lambda^2} + \alpha^2 \lambda \frac{\partial \Phi}{\partial \lambda} . \]
We now write the differential equation (25) at time \( t = 0 \), replacing the second derivative with respect to time by means of (29). The boundary condition (26) is replaced by
\[
\Phi = \varphi_0(x, \tau) \quad \text{when} \quad \lambda = 0.
\]

The equation obtained may play a similar role as eq. (9). The solution can be expanded in powers of \( \lambda \), and the coefficients of \( \lambda^n (n = 1, 2, \ldots) \) obey a set of recurrent equations. We do not wish to deal with this any further here, but we shall content ourselves with the remark that the method seems to be of some interest also in the more complicated case of the quantized theory.

REFERENCES

Range and Straggling of Muons.

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The range of muons is calculated including radiation, pair production, and nuclear effects. It is shown that the large individual energy losses characteristic of these processes reduce the average range by a factor of ln 2 from that which one would get on the basis of simply integrating the average energy loss. The fractional straggling first increases as radiation and pair production effects become important and then decreases as the energy is further increased.

1. — Introduction.

The study of the passage of fast-moving particles through matter has been important since the early days of nuclear physics [1]. Many of the experimental techniques of detection and measurement of particles depend on such specific properties of penetration as the total range or as the specific energy loss. Protons and pions at energies below a few hundred million electron volts have a well-defined average range, but the effects of nuclear collisions obscure this definite range at higher energies: The track of the incident proton becomes completely lost in the accompanying nuclear shower of secondary particles at energies higher than a billion electron volts. Individual electrons never show a well-defined range: At low energy, multiple scattering causes them to diffuse through matter; and at high energies a shower conceals the initial electron.

Muons are more satisfactory particles to consider from this point of view because at low energies multiple scattering is not too serious, and at high energies the effect of nuclear collisions is small. On the other hand, at high energies, bremsstrahlung and direct pair production do occur to increase the energy loss above that due to ionization. The nature of the large
individual energy losses due to radiation markedly increase the fluctuations of individual track lengths.

The present study is concerned with a quantitative evaluation of the range and straggling of muons at very high energies. Briefly stated, the range is smaller by a factor \( \ln 2 \) from what one would calculate by neglecting fluctuations; on the other hand, the fluctuations on the ranges of individual tracks are smaller than might be expected intuitively.

2. – Range calculations.

First let us calculate the range assuming only bremsstrahlung losses because this can be done more or less rigorously, then we will include pair production, nuclear absorption, and ionization. The calculation will parallel that made by the author for electrons [2]. Bethe and Heitler [3] give the energy loss of a particle that has traversed a thickness of matter \( t \). They approximate the radiation spectrum by

\[
\sigma(k)\, dk = \frac{dk\, dt}{E\ln\left[E/(E-k)\right]},
\]

where \( \sigma(k)\, dk \) is the probability of the muon energy \( E \) radiating a photon of energy \( k \) in passing through a distance \( dt \) measured in units of shower length, \( i.e., \) for this application in muon radiation lengths divided by \( \ln 2 \). Then they find that the probability of the particle of initial energy \( E_0 \) having an energy \( E \) after traversing a finite distance \( t \) is

\[
w(y, t) = (t-1, y)!/(t-1)!
\]

in terms of the incomplete gamma-function \( (t-1, y)! \), where \( y = \ln (E_0/E) \). From this one can find [4] the result we seek, namely

\[
w(y, t)\, dt = e^{-y}y\, dt!/t!.
\]

For large values of \( y \) and \( t \), the above equation can be approximated by the Gaussian form

\[
w(y, t)\, dt = (2\pi y)^{-\frac{1}{2}} \exp \left[-(t-y)^2/2y\right] \, dt.
\]

From this we see directly that the mean range \( r \) is

\[
r = y_{\text{max}},
\]
where $y_{\text{max}}$ is the value of $y$ at the peak of the Gaussian, and that the root-mean-square straggling of the range $s$ is given by

$$s = y_{\text{max}}^4 = r^4. \tag{6}$$

2'1. Ionization loss. — If only radiation contributed to the energy loss, the range and the straggling would be infinite, however, as the energy degrades, ionization losses become important and allow us to evaluate $y_{\text{max}}$. Expressing eq. (5) in terms of energy and then differentiating the mean range with respect to the initial energy gives the average radiation loss on travelling a distance $dt$,

$$-\frac{dE}{dt} = E. \tag{7}$$

That this is neither obvious nor trivial is clear if we remember that we are using shower units of length which introduce the factor $\ln 2$.

Now let us express the energy in units equal to $\ln 2$ times the critical energy $\beta$, i.e., the energy lost by a muon to ionization in going a distance of one muon radiation length. Here we are making the rough approximation that the ionization loss is independent of energy: we will examine the validity of this later. We add the ionization loss to (7) to get

$$-\frac{dE}{dt} = E + 1 \tag{8}$$

and integrating this over the energy gives the mean range,

$$r = \log (E_0 + 1), \tag{9}$$

which in units of radiation lengths becomes

$$r = \ln 2 \ln [(E_0/\beta \ln 2) + 1]. \tag{10}$$

Now let us turn to straggling which is manifest in the distribution described by eq. (3). Equation (6) shows us rather surprisingly that on the basis of radiation loss alone the fractional straggling $s/r$ varies as $1/r^4$, i.e., becomes smaller as the energy increases. Actually the straggling at low energies will be less than given by (6) because the energy loss due to ionization has been neglected. The energy loss due to ionization for a muon traveling a distance $r$ is just equal to $r$ in the peculiar energy units of the above theory, hence a fraction of the range $r/E_0$ can be ascribed to ionization loss. The straggling of this fraction will be less than 1%; hence for any energy
for which this calculation can apply, we can neglect it completely. The remaining fraction of energy that is lost to radiation processes \((1 - r/E_0)\) will vary roughly as given by eq. (4), which will only be valid when the fraction \((1 - r/E_0)\) is large. As a rough interpolation formula, suggested by Monte Carlo calculations [2], we can write

\[ s = \left(1 - \frac{r}{E_0}\right)^{\frac{1}{3}} \]

and remember that the Gaussian distribution has a cut-off at a range equal to \(E\) in shower units or \(E/\beta\) in radiation lengths.

2'2. – *Pair production and other energy losses.* – Up until now we have neglected direct pair production as well as nuclear interactions. In fact, the loss of energy due to the direct production of electron pairs is compa-

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**Fig. 1.** – The range and straggling in various materials.
rable (within about 10%) to the loss due to bremsstrahlung [5]. The energy
loss due to nuclear interactions amounts at most to a few percent of
the total loss, being relatively larger for light elements and at high energies.
On the basis of a Weissacker-Williams consideration of direct pair pro-
duction, we can expect the atomic shielding factor to be almost identical to that
for bremsstrahlung. Thus, except for the lowest energy losses where the
mass of the created pair becomes significant, we can expect the two pro-
cesses to be roughly the same.

In order to include these effects, I suggest that we define a muon inter-
action length in place of the muon radiation length that we have been using
thus far so that the above theory will be valid. Until a more exact calcu-
lation of the muon interaction length is made, I suggest that we use simply
one half of the muon radiation length. On this basis, the range and strag-
gling in various materials has been computed and is plotted in Fig. 1. The
range in uranium is significantly smaller than that given in ref. [5] where the
effects of fluctuations were neglected.

REFERENCES

[4] Using

\[ \int_0^y x^t e^{-x} dx = t! - e^{-y} \sum_{\mu=1}^t t!/\mu! y^\mu \]

which can be derived by successive partial integrations (for integral t), one obtains
eq. (3) by taking the difference between t and (t − 1). This procedure was shown to
me by S. Pasternack: originally, I simply guessed the result [3].
The Basic $SU_3$ Mixing: $\omega_2 \leftrightarrow \omega_1$.

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1. – Introduction.

The purpose of the present paper is to review a basic problem of $SU_3$ symmetry, the ($\omega$-$\phi$) mixing, and to describe the first direct measurement of the so-called ($\omega$-$\phi$) mixing angle.

1'1. The origin. – The ($\omega$-$\phi$) mixing has played a basic role in the understanding of hadron spectroscopy and therefore in the discovery of $SU_3$ [1, 2]. In fact, without guessing the $SU_3$ symmetry-breaking it would have been impossible to establish its existence. The regularity of $SU_3$ symmetry-breaking follows the observed regularity of the other symmetry breakings (such as: isospin by electromagnetic interactions, and $C, P$ by weak interactions), and represents an aspect of particle physics that is even more spectacular than the existence of $SU_3$.

As is well known, $SU_3$ symmetry is not an exact symmetry law. When medium-strong interactions are switched on (their strength in supposed to be about one-tenth of the strong interaction strength), $SU_3$ symmetry is broken. By assuming that the symmetry-breaking term is a coherent superposition of an $SU_3$ singlet and the eighth component of an $SU_3$ octet, the celebrated unitary symmetry mass formula, first derived by Gell-Mann [1] for a unitary octet and then generalized by Okubo [3] to any unitary multiplet, is obtained. This Gell-Mann–Okubo mass formula (linear for fermions, quadratic for mesons (*)) was found to be in good agreement with all observed

(*) A simple argument to justify the rule of linear expressions for fermions and of quadratic expressions for mesons is due to Feynman, and is based on the fact that the mass term in the Dirac and in the Klein-Gordon equations is linear and quadratic, respectively.
mass values for the pseudoscalar meson octet, the spin $\frac{3}{2}^+$ baryon octet and the spin $\frac{1}{2}^+$ baryon decuplet (*). Surprisingly enough, the Gell-Mann–Okubo mass formula failed to explain the observed vector meson masses. More precisely, the situation concerning the vector meson octet was the following: from the known mass values of the $K^*$ and of the $\rho$ mesons, $m_{K^*} = 888$ MeV, $m_\rho = 750$ MeV. the isoscalar vector meson was expected to be at $m_{\pi=0} = 930$ MeV. But the $\omega$-meson mass was known to be at $m_\omega = 780$ MeV. It was indeed very disturbing to find that the regularity of the Gell-Mann–Okubo breaking failed so badly to explain the observed spectrum of the vector mesons. This was the situation in 1962 when Sakurai [5], on the basis of the discovery of the $I = 0$ vector meson at 1020 MeV (the $\phi$-meson) by the Brookhaven-Syracuse Group [6], put forward the proposal that the cause of the failure of the Gell-Mann–Okubo mass formula was the fact that the $\omega$ and $\phi$ mesons were two particles with identical quantum numbers, as far as spin parity, isospin, and $G$-parity are concerned. The only difference (if any) between $\omega$ and $\phi$ would be their $SU_3$ attribute. But $SU_3$ is only an approximate symmetry. Therefore it is reasonable to suppose that the 780 MeV $\omega$-meson and the 1020 MeV $\phi$-meson are coherent superpositions of two pure $SU_3$ states: a pure $SU_3$ singlet $\omega_1$, and the eighth component of a pure $SU_3$ octet $\omega_8$. On this basis Sakurai [7] was able to construct a simple dynamical model of basic $SU_3$ symmetry-breaking that could account for the success of the Gell-Mann–Okubo mass formula.

It should be clear at this point that it is more appropriate to speak in terms of $(\omega_8-\omega_1)$ mixing, rather than $(\omega-\phi)$ mixing (**).

The crucial point was then to check if this mixing really exists in nature. For a long time the only experimental information on the $(\omega-\phi)$ mixing was obtained from the observed mass spectrum of the various vector mesons. But this is more a way of adjusting the Gell-Mann–Okubo mass formula, than a measurement of the $(\omega_8-\omega_1)$ mixing.

12. Attempts to measure the $(\omega_8-\omega_1)$ mixing. – A way of measuring the $(\omega_8-\omega_1)$ mixing was suggested by Sakurai [7], who pointed out that if the $\phi$

(*) To be more precise, at that time the tenth member of the $J^P = \frac{3}{2}^+$ decuplet, the $\Omega^-$ baryon singlet, was still unobserved, but only theoretically postulated [4]. Nevertheless, the famous equal-spacing rule, i.e., the proportionality of the mass $M$ to the hypercharge $Y$, $M = M_0 (1 + aY)$, which is a straightforward consequence of the Gell-Mann-Okubo mass formula, was very well satisfied for all the other observed members of the baryon decuplet $N^+_\frac{8}{3}(1235)$, $Y^+_\frac{1}{2}(1380)$, and $\Xi^+_\frac{1}{2}(1530)$.

(**) An analogous situation occurs in weak interactions where the $(K^0-\bar{K}^0)$ mixing produces the physically observed $K^0_\lambda$ and $\bar{K}^0_\lambda$ states. Notice that $K^0$ and $\bar{K}^0$, like $\omega_8$ and $\omega_1$, remain unobserved as particle states.
width $\Gamma_{\phi \to \text{all}}$ is calculated from the known $\phi$ width and compared with the partial width $\Gamma_{\phi \to K\bar{K}}$, the mixing angle $\theta$ is obtained through the relation

$$\frac{\Gamma_{\phi \to K\bar{K}}}{\Gamma_{\phi \to \text{all}}} = \cos^2 \theta.$$ 

This is because in the decay $\phi \to K\bar{K}$ the $\omega_1$ cannot contribute. In fact $\omega_1$ can only be coupled to a symmetric bilinear expression involving two pseudoscalar $K$’s, which consequently cannot be in a state with $J = 1$.

The value of $\theta$ deduced in this way has been controversial because of the different results obtained for the branching ratios of the $\phi$ decays ($\phi \to 3\pi$ and $\phi \to K\bar{K}$) in various laboratories [8]: these results led to values of $\theta$ with one standard deviation limits ranging from $0^\circ$ to $55^\circ$. Other ways of determining $\theta$, as for instance those suggested by Glashow [9], have also not led to any positive result. The first significant «indirect» value for the ($\omega_8$-$\omega_1$) mixing angle was obtained by Massam and Zichichi [10] via a world analysis of the nucleon electromagnetic form factors.

13. The direct way. – All these suggestions were put forward and the corresponding attempts were made because the direct clean way of checking the ($\omega_8$-$\omega_1$) mixing hypothesis seemed to be quite remote from the experimental area. As we shall see later, this «clean» and «direct» way is either via the study of the electromagnetic decay modes of the $\omega$ and $\phi$ mesons, i.e.,

$$\begin{align*}
\omega & \to \gamma \to e^+e^-; \\
\phi & \to \gamma \to e^+e^-,
\end{align*}$$

or via their production through ($e^+e^-$) colliding beam machines:

$$\begin{align*}
e^+e^- & \to \gamma \to \omega; \\
e^+e^- & \to \gamma \to \phi.
\end{align*}$$

The major difficulty connected with reactions (1) is that the $\omega$ and $\phi$ mesons decay via strong interactions, and their electromagnetic channels are expected to be depressed by a factor of the order of $\alpha^2$ (where $\alpha$ is the fine structure constant): This means that the branching ratios

$$\frac{\Gamma_{\omega \to e^+e^-}}{\Gamma_{\omega \to \text{all}}}$$

and

$$\frac{\Gamma_{\phi \to e^+e^-}}{\Gamma_{\phi \to \text{all}}}.$$
are expected to be of the order of $10^{-4}$. Moreover, there were also difficulties associated with the production processes of the $\omega$ and $\phi$.

The $\omega$ production could go with a reasonable cross-section via the reaction

\begin{equation}
\pi^- + p \rightarrow \omega + n,
\end{equation}

but, as the $\omega$-meson mass is enveloped in the large width of the $\rho$ mass, a large background of $\phi$'s was expected to be present; thus it was necessary to choose, if possible, those experimental conditions where the $\rho$ contribution is depressed in favor of the $\omega$ production.

The $\phi$ production was observed with a reasonable cross-section in reactions where strange particles were present, such as $K^- p \rightarrow \Lambda^0 \phi$, but was expected to be depressed by a large factor (due to the $\lambda$-quark spin conservation), and in fact remained unobserved for a long time, in simple reactions such as [11]

\begin{equation}
\pi^- + p \rightarrow \phi + n.
\end{equation}

Finally, in order to observe $\omega$ and $\phi$ decays into $(e^+e^-)$, it was obviously necessary to devise a large acceptance experimental apparatus, able to select and measure angles of emission and energies of the final products of reactions (5) and (6), which are neutrons and $(e^+e^-)$ pairs. The feasibility of such an experimental program was shown in an unpublished paper by Dalpiaz et al. [12].

The difficulty connected with reactions (2) lay in the fact that $(e^+e^-)$ storage rings had to be built. On the other hand, no serious problems of particle identification or of strong background could be foreseen in the study of these processes, the feasibility of the experiment being confined to the problems connected with the construction of moderate-energy $(e^+e^-)$ storage rings [13].

As we shall see later, the first measurement of the $(\omega_8-\omega_1)$ mixing was obtained via the study of reactions (1).

As mentioned before, the $(\omega_8-\omega_1)$ mixing was at first described by Sakurai [5] using a unique mixing angle $\theta$, but by now there are four $(\omega_8-\omega_1)$ mixing angles quoted in the literature [5, 6, 8, 14-18]; the original $\theta$, then $\theta_Y$ and $\theta_N$, and finally the generalized mixing angle $\theta_G$.

2. – The four mixing angles.

2.1. – Schematic derivation. – We shall now try to review the origin of all these mixing angles. The starting point is: two particles with identical quantum numbers ($J^{P,Q}$, $I$, $Y$) such as $\omega_8$ and $\omega_1$ will convert into each other

\begin{equation}
\omega_8 \leftrightarrow \omega_1,
\end{equation}
because process (7) does not violate any conservation law but that of \(SU_3\) symmetry, which is broken by the moderately strong interactions. As it is impossible to switch off these interactions, process (7) will go.

When two particle states can convert into each other, as in process (7), the inverse propagator that describes the mixed system can be shown to have the familiar form

\[
D = AK^2 + BM^2,
\]

where \(A\) and \(B\) are \(2 \times 2\) matrices (if we want to describe mixing between two particles only), \(K\) is the quadrupole momentum, and \(M\) is the mass of the two states. Let \(D_0\), \(A_0\), and \(B_0\) be the quantities defined above before the mixing starts. Without mixing the two matrices, \(A_0\) and \(B_0\) are diagonal, i.e.,

\[
A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B_0 = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix},
\]

and the inverse propagator

\[
D_0 = A_0K^2 + B_0M^2
\]

gives the two propagators of the two unmixed states, each having momentum \(K\) and masses \(\beta_1 M^2\) and \(\beta_2 M^2\) (\(\beta_1\) and \(\beta_2\) are just numerical coefficients). The effect of mixing can be of two types. These two ways of treating the mixing between two particles have been discussed first by Coleman and Schnitzer [15] (CS) and later by Kroll, Lee and Zumino [16] (KLZ), who particularly emphasized the need of having two mixing angles.

2.1.1. Mass mixing. Here it is supposed that the effect of mixing (process (7)) is that of destroying the diagonality of the matrix \(B_0\), which becomes \(B = B_0 + \delta B\), without disturbing the matrix \(A_0\).

The problem is to diagonalize \(B\) and hence \(D\) without destroying the diagonality of \(A_0\). It is well known that in order to achieve this, the matrix that is needed can be an orthonormal matrix. As the elements of a \(2 \times 2\) matrix are four, and the orthonormality conditions are three, all the mixing can be described using a single parameter: the mixing angle \(\theta\), which is the angle first introduced by Sakurai [5]. The left side of Fig. 1 shows a synthesis of the above chain of arguments.
(\omega_3 - \omega_1) MIXING

\[ D = AK^2 + BM^2 \]

Without mixing: \( A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \); \( B_0 = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \)

**Effect of mixing**

i) Mass-mixing

\[ \downarrow \]

Destroy diagonality of

\( B_0 \)

\[ \downarrow \]

*Problem*: diagonalize

\( B = B_0 + \delta B \)

Without destroying diagonality of

\[ A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ \downarrow \]

Matrix needed:

\[ \begin{pmatrix} a & \beta \\ \gamma & \delta \end{pmatrix} \]

with orthonormality conditions:

\[ a^2 + \beta^2 = 1 \]
\[ \gamma^2 + \delta^2 = 1 \]
\[ a\gamma + \beta\delta = 0 \]

4 Parameters - 3 Conditions

\[ \downarrow \]

Only one parameter

\[ \theta \]

ii) Current mixing

\[ \downarrow \]

Destroy diagonality of

\( A_0 \)

\[ \downarrow \]

*Problem*: diagonalize

\( A = A_0 + \delta A \)

Without destroying diagonality of

\[ B_0 = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \]

\[ \downarrow \]

Matrix needed:

\[ \begin{pmatrix} a & \beta \\ \gamma & \delta \end{pmatrix} \]

no orthonormality conditions

4 parameters:

\[ \{ \text{2 coupling constants: } \varepsilon_R, \varepsilon_S \} \]
\[ \{ \text{2 angles: } \theta_R, \theta_S \} \]

Correlated

\[ \downarrow \]

\[ \frac{m_{\omega}}{m_{\beta}} \tan \theta_R = \frac{m_{\beta}}{m_{\omega}} \tan \theta_S = \tan \theta_\alpha \]

Fig. 1. - \( (\omega_3 - \omega_1) \) mixing: schematic comparison between the «mass-mixing» type and the «current-mixing» type models.
2'1.2. Current mixing. Here it is assumed that the effect of mixing is to destroy the diagonality of $A_0$, leaving $B_0$ diagonal. The problem is now to diagonalize $A = A_0 + \delta A$, without destroying the diagonality of $B_0$. Notice that $B_0$ is diagonal but (unlike $A_0$) not unit matrix. In order to diagonalize $A$ without destroying the diagonality of the nonunit $B_0$, a $2 \times 2$ matrix, without orthonormality conditions, is required. The mixing must therefore be described using four parameters, which can be expressed in terms of two coupling constants $g_Y$ and $g_N$, and of two mixing angles $\theta_Y$ and $\theta_N$ (here we use the same notation as KLZ). However, because of $T$ invariance, $A$ and $B_0$ are symmetric matrices; this gives one condition for the four free parameters. This condition can be used in order to establish a relation between the two mixing angles $\theta_Y$ and $\theta_N$, i.e.,

$$\frac{\tan \theta_Y}{\tan \theta_N} = \frac{m_\phi^2}{m_\omega^2},$$

first derived by KLZ. This relation can be rewritten as

$$m_\omega \tan \theta_Y = m_\phi \tan \theta_N = \tan \theta_\phi,$$

thus allowing the mixing to be expressed in terms of the «generalized» mixing angle $\theta_\phi$ [18]. The right-hand side of Fig. 1 illustrates the above chain of arguments.

2'2. Why are there all these complications? – After the introduction of the ($\omega_8$–$\omega_1$) mixing hypothesis by Sakurai [5], CS [15] emphasized that the Sakurai-type of mixing, called by them «particle mixing» and by KLZ [16] «mass-mixing», was not adequate enough to describe mixing between «vector particles». This is because vector particles are believed to be associated with conserved quantities, and «mass mixing» is incompatible with this requirement, as can be easily shown with the following example. Suppose that the inverse propagator $D$ describes the isoscalar form factor of the nucleon (*) and that we choose the «mass mixing» model. After mixing, $D_0$ becomes $D$,

$$D_0 = A_0 K^2 + B_0 M^2^{\text{mixing}} \Rightarrow D = A_0 K^2 + (B_0 + \delta B)M^2.$$  

At $K^2 = 0$, $D_0 \neq D$. But the value of $D_0$ and $D$ at $K^2 = 0$ is related to the nuclear isoscalar electric charge (i.e., electric charge of the proton divided

(*) Pole dominance is of course assumed.
by two). The effect of « mass mixing » is to change the value of the nuclear isoscalar electric charge and this is unacceptable.

If we choose « current mixing » we have

\[ D_0 = A_0 K^2 + B_0 M^2 \text{ mixing} \rightarrow D = (A_0 + \delta A) K^2 + B_0 M^2, \]

and at \( K^2 = 0 \) it is \( D_0 = D \). This is the reason why « current mixing » is believed to be more adequate for the description of the mixing between vector particles.

It is interesting to notice [15] that if the force mixing the particles is truly weak, « mass mixing » and « current mixing » are indistinguishable (as in the case of the \( (K^0, \bar{K}^0) \) mixing which produces the physically observed states \( K^0_0 \) and \( K^0_\pm \); in fact here the transition \( K^0 \rightarrow \pi^{\mp} \bar{K}^0 \) is a second-order weak interaction (*)). Notice that in the above models of mixing it has always been assumed that the mixing alters only the propagators and not the vertex functions. Notice also that both « current mixing » and « mass mixing » are compatible with the transversality conditions for the source of the vector mesons [16], i.e., the currents to which they are coupled are conserved currents.

2'3. The crucial point. – The conclusion of all the above arguments is that the physically observed states \( \omega \) and \( \phi \) are mixtures with certain percentages (\( \% \)) of two pure \( SU_3 \) states, \( \omega_8 \) and \( \omega_1 \):

\[ \omega \equiv (\% \omega_8 + (\% \omega_1, \]

\[ \phi \equiv (\% \omega_8 + (\% \omega_1 \]

The problem is how to measure these percentages.

Suppose we have a selective interaction, i.e., an interaction which is coupled to \( \omega_8 \) and not to \( \omega_1 \). If we can find such an interaction, we can then see how much \( \omega_8 \) there is in the physical states \( \omega \) and \( \phi \).

There is a good candidate for this selective interaction: the electromagnetic interaction. In fact, remember that all known particles obey the famous Gell-Mann–Nishijima relation

\[ (11) \]

\[ Q = I_3 + \frac{Y}{2} + \text{nothing}. \]

(*) A simple way of seeing why in this case there is no difference between mass mixing and current mixing is to notice that \( CPT \) implies \( m_{K^0} = m_{\bar{K}^0} \), and therefore \( B_0 \) turns out to be a diagonal and unit matrix.
The basic $SU_3$ mixing: $\omega_8 \approx \omega_1$

It could be argued that even if the electric charge $Q$ of all elementary particles has no contribution from quantum numbers which are not $I_3$ and $Y$, the electromagnetic current $J_\mu$ can still contain a singlet $SU_3$ term: $J_\mu = J^{(8)}_\mu + J^{(1)}_\mu$. In fact, if the fourth component of $J^{(1)}_\mu$ has vanishing volume integral, then:

\begin{equation}
Q = \int J^{(8)}_\mu(x, t) d^3x + \int J^{(1)}_\mu(x, t) d^3x,
\end{equation}

and the Gell-Mann–Nishijima relation (11) remains unaltered. Here the difficulty with the electromagnetic current becomes clear. In fact the octet part of the electromagnetic current is a $U$-spin singlet ($U$ transformations leave the electric charge invariant). All $SU_3$ predictions based on $U$-spin conservation alone cannot distinguish between the octet part and the singlet part of the electromagnetic current. In order to measure the octet part $J^{(8)}_\mu$ and the possible existence of a singlet part $J^{(1)}_\mu$ in the electromagnetic current, it is necessary to devise an experiment where these two parts can be directly observed; the cleanest known example is the measurement of the $(e^+e^-)$ decay rates of $\rho$, $\omega$, $\phi$. In fact, if we believe in the one-photon approximation, these decays go via the following Feynman diagrams:

The octet part of the electromagnetic current $J^{(8)}_\mu$ couples to the isospin ($i.e.$, the $\rho$ meson) and to the hypercharge $Y$ ($i.e.$, the $\omega_8$ part of the $\omega$ and $\phi$ mesons). The singlet part of the electromagnetic current $J^{(1)}_\mu$ couples to the $\omega_1$ part of the $\omega$ and $\phi$ mesons. Therefore if we measure the decay rates of $\rho$, $\omega$, and $\phi$ into $(e^+e^-)$, we do study the coupling of the photon to the isospin, the hypercharge, and the $SU_3$ singlet.
To recapitulate, we have said that we wanted a selective interaction, coupled only to $\omega_3$, in order to check the $(\omega_3, \omega_1)$ mixing hypothesis; but what we have found is a «good candidate» for the required selective interaction.

2'4. Conclusion. – The study of $\omega \rightarrow e^+e^-$ and $\phi \rightarrow e^+e^-$ will allow the $(\omega_3, \omega_1)$ mixing hypothesis (in the limit where the photon is not coupled to $\omega_1$) to be checked; the comparison of the decay widths $\Gamma_{\omega \rightarrow e^+e^-}$, $\Gamma_{\phi \rightarrow e^+e^-}$ with $\Gamma_{\rho \rightarrow e^+e^-}$, will allow checking of the consistency of the selectivity hypothesis we made on the electromagnetic current, i.e., the $SU_3$ nature of the electromagnetic current.

3. Vector-meson photon interaction.

3'1. The first derivation. – In order to study the processes

$$\begin{align*}
\rho & \rightarrow e^+e^- \\
\omega & \rightarrow e^+e^- \\
\phi & \rightarrow e^+e^-
\end{align*}$$

(13)

it is necessary to know how to describe the coupling of a vector meson with the photon; in fact, as mentioned above, these processes, in the one-photon approximation, are represented by the following Feynman diagram, where $V$ stands for the three vector mesons $\rho$, $\omega$, $\phi$.

![Feynman diagram](image)

Gell-Mann and Zachariasen [19] were the first to treat the problem of vector-meson photon interaction and to find out the effective vector-meson-photon coupling constant on the basis of vector-meson dominance. Their argument is as follows. Consider the $\pi$ electromagnetic form factor (EMFF), $F_\pi(q^2)$. If we assume $\rho$-meson dominance, i.e., that the isovector photon is always coupled to the $\rho$-meson, then the pion EMFF will be given by the following expression:

$$F_\pi(q^2) = \frac{g_\gamma \gamma \cdot g_\rho \gamma \pi}{m_\rho^2 + q^2},$$

(14)
which can be easily derived by inspecting the corresponding Feynman diagram for the elastic \((e-\pi)\) scattering, where \(g_{\rho\gamma}\) is the effective \((\rho-\gamma)\) coupling constant and \(g_{\rho\pi\pi}\) is the \((\rho-\pi)\) coupling constants and \((m_{\rho}^2 + q^2)^{-1}\) is the \(\rho\) propagator. At \(q^2 = 0\), the pion EMFF is by definition equal to 1 (in units of the electron charge):

\[
F_{\pi}(0) = 1 = \frac{g_{\rho\gamma} \cdot g_{\rho\pi\pi}}{m_{\rho}^2},
\]

therefore:

\[
(15) \quad g_{\rho\gamma} = \frac{m_{\rho}^2}{g_{\rho\pi\pi}}.
\]

If we introduce the universality condition for the \(\rho\)-meson hadron coupling, \(g_{\rho\pi\pi} = f_{\rho} \cdot I_{\pi}^\pi\), \(g_{\rho\gamma}\) becomes:

\[
(16) \quad g_{\rho\gamma} = \frac{m_{\rho}^2}{f_{\rho}},
\]

where \(f_{\rho}\) is the coupling of the \(\rho\)-meson to the isospin current. In fact universality of the \(\rho\) coupling to the hadrons means that the coupling of the \(\rho\)-meson with its source density, the isospin current, is universal (at \(q^2 = 0\)), i.e., the ratio of amplitudes for any hadronic state \(A\) going into \((A + \rho)\) and any hadronic state \(B\) going into \((B + \rho)\) is just proportional to the ratio of the appropriate \(i\)th components of the isospin of \(A\) and \(B\):

\[
\frac{A \leftrightarrow A + \rho}{B \leftrightarrow B + \rho} = \frac{I_i^A}{I_i^B}.
\]

3.2. The question of gauge invariance and pole dominance. – The simple relation (16) was obtained by Gell-Mann and Zachariasen [19] by treating the coupling between the vector mesons \(V_{\mu}\) and the electromagnetic field \(A_{\mu}\) in the simple way

\[
(17) \quad e \cdot V_{\mu} \cdot A_{\mu},
\]
where \( e \) is the electromagnetic coupling constant. This interaction produces pole dominance but manifestly violates gauge invariance.

Feldman and Mathews [20] remarked that in order to have a gauge invariant electromagnetic interaction between the vector mesons and the electromagnetic field, it is necessary to work with interaction terms of the type

\[
G_{\mu \nu} \cdot F_{\mu \nu},
\]

where \( G_{\mu \nu} \) is analogous to

\[
F_{\mu \nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu},
\]

for the vector mesons, i.e.

\[
G_{\mu \nu} = \frac{\partial V_\nu}{\partial x_\mu} - \frac{\partial V_\mu}{\partial x_\nu}.
\]

The interaction (18) has the advantage of being obviously gauge invariant. It is in fact constructed using the fields instead of the potentials, but gives no pole dominance. In fact (*)

\[
G_{\mu \nu} \cdot F_{\mu \nu} = - \frac{\epsilon_\mu \cdot J_\mu}{q^2 + m_p^2} \cdot q^2,
\]

and this expression vanishes for \( q^2 = 0 \) (real photons) (**).

At this point it seemed there was no way out: one either had to

i) choose \( e \cdot V_\mu \cdot A_\mu \), then get pole dominance but lose gauge invariance;

ii) or choose \( G_{\mu \nu} \cdot F_{\mu \nu} \), then keep gauge invariance but lose pole dominance in the sense that the interaction of vector mesons with real photons vanishes.

3'3. The Kroll-Lee-Zumino theory. – The solution to this trouble was found by Kroll, Lee, and Zumino [16], who were able to reconcile pole dominance and gauge invariance in the description of vector-meson photon interaction. For simplicity we will again consider only the \( \phi \)-meson photon term. Their argument goes as follows: Add to formula (19) another term

\[
K \cdot \epsilon_\mu \cdot J_\mu,
\]

(*) For simplicity we include only the \( \phi \)-meson photon term. The \( \omega \) and \( \psi \) terms are analogous but are longer because of their mixing.

(**) The interested reader can find these points discussed further in: The Nature of the Photon, lectures given by A. Zichichi at the Enrico Fermi International School of Physics, Varenna, September 1969.
where $K$ is a constant to be chosen later. Thus the total interaction term is

$$
K(e_{\mu} \cdot J_{\mu}^\rho) + \frac{e_{\mu} \cdot J_{\mu}^\rho}{q^2 + m_{\rho}^2} q^2 = \frac{e_{\mu} \cdot J_{\mu}^\rho}{q^2 + m_{\rho}^2} [K(q^2 + m_{\rho}^2) + q^2].
$$

In the KLZ theory, $K$ turns out to be: $K = -1$, as a consequence of the physical requirement that the expression (20) should vanish for $q^2 \to \infty$. The interaction term (20) then becomes:

$$
\frac{e_{\mu} \cdot J_{\mu}^\rho}{q^2 + m_{\rho}^2} m_{\rho}^2.
$$

Pole dominance is re-established in a gauge invariant theory of vector-meson photon interaction. It should, in fact, be emphasized that the «pole dominance» result (21) is achieved by KLZ not via the introduction of a term of the form $g_{\mu} \cdot A_{\mu}$ in the Lagrangian, but by establishing a special relation between the «direct» source term $J_{\mu}^\rho \cdot A_{\mu}$ and the gauge invariant $\rho$-meson photon interaction $G_{\mu \nu} F_{\mu \nu}$. For clarity we draw the Feynman diagrams corresponding to these terms:

\[
\begin{array}{ccc}
\text{direct photon-hadron coupling} & = & A_{\mu} \cdot J_{\mu}^\rho \\
\text{(only isovector hadronic current)} & = & J_{\mu}^\rho \\
\hline
\text{gauge invariant} & = & G_{\mu \nu} \cdot F_{\mu \nu} \\
\text{$\rho$-meson photon interaction} & = & G_{\mu \nu} \cdot F_{\mu \nu}
\end{array}
\]

An interesting point to remark is that as far as the effective coupling between vector mesons and real photons is concerned, the KLZ result coincides with that of Gell-Mann and Zachariasen:

$$
g_{\rho \gamma} = e \frac{m_{\rho}^2}{f_{\rho}},
$$

and analogously for $\omega$ and $\phi$.

In the language of KLZ, the interaction of the electromagnetic field with $\rho$, $\omega$, $\phi$ takes place through the coupling of the electromagnetic field with the isospin current $J_{\mu}^\rho$ and the hypercharge current $J_{\mu}^8$, whose explicit expres-
sions are (*)

\[ J^{3}_{\mu} = \left( \frac{m^{2}_{e}}{f_{\rho}} \right) \varrho_{\mu}^{3}, \]

\[ J^{8}_{\mu} = \frac{1}{f_{Y}} \left( \cos \theta_{Y} \cdot m^{2}_{\phi} \cdot \phi_{\mu} - \sin \theta_{Y} \cdot m^{2}_{\omega} \cdot \omega_{\mu} \right), \]

where \( \varrho_{\mu}, \omega_{\mu}, \phi_{\mu} \) are the \( \varrho, \omega, \phi \) field operators, and \( \theta_{Y} \) is the hypercharge mixing angle already mentioned. Notice that the electromagnetic field couples only to \( J^{3}_{\mu} \) and \( J^{8}_{\mu} \) (the exact validity of this statement has already been discussed in Sect. 2.3), and from formulas (22) and (23) the effective \( (\varrho, \omega, \phi) - \gamma \) coupling constants are derived to be:

\[
\begin{align*}
\varrho_{\gamma} & = e \cdot \frac{m^{2}_{e}}{f_{\rho}}, \\
\omega_{\gamma} & = e \cdot \frac{m^{2}_{\omega}}{f_{Y}} \sin \theta_{Y}, \\
\phi_{\gamma} & = e \cdot \frac{m^{2}_{\phi}}{f_{Y}} \cos \theta_{Y}.
\end{align*}
\]

(15a)

The predictions for the \((e^{+}e^{-})\) decay rates of the vector mesons are, according to KLZ [16] (and to previous estimate [19, 21, 22]):

\[ \Gamma_{\varrho \rightarrow e^{+}e^{-}} = \frac{\alpha^{2}}{3} \left( \frac{4\pi}{f_{\rho}^{2}} \right) m_{\varrho}, \]

\[ \Gamma_{\omega \rightarrow e^{+}e^{-}} = \frac{\alpha^{2}}{3} \left( \frac{4\pi}{f_{Y}^{2}} \right) m_{\omega} \sin^{2} \theta_{Y}, \]

\[ \Gamma_{\phi \rightarrow e^{+}e^{-}} = \frac{\alpha^{2}}{3} \left( \frac{4\pi}{f_{Y}^{2}} \right) m_{\phi} \cos^{2} \theta_{Y}, \]

(24)

(25)

(26)

(for the notation we follow KLZ).

In these expressions \( SU_{3} \) symmetry has not been used. If \( SU_{3} \) is valid, \( f_{Y} = \sqrt{3} f_{\rho} \) and \( \theta_{Y} = 0 \). If we assume the «naive» \( SU_{3} \) symmetry-breaking, then \( \theta_{Y} = 35^{\circ} \) and we obtain the well-known relation between the partial decay widths of the various vector mesons:

\[ \Gamma_{\varrho \rightarrow e^{+}e^{-}}: \Gamma_{\omega \rightarrow e^{+}e^{-}}: \Gamma_{\phi \rightarrow e^{+}e^{-}} = 9:1:2w, \]

(27)

(*) Notice that \( J^{3}_{\mu} \) and \( J^{8}_{\mu} \) are the third and eighth components of \( J^{(8)}_{\mu} \). However, the \( SU_{3} \) relation between \( f_{\varrho} \) and \( f_{Y} \) will be fixed later.
where \( w \) is an unknown factor resulting from the fact that the mass of the \( \phi \) is different from that of the \( \rho \) and \( \omega \).

3.4. The generalized Weinberg spectral function sum rules. – For the purpose of checking the \((\omega_\pi - \omega_\lambda)\) mixing hypothesis, it is sufficient merely to show that \( \omega \to e^+e^- \) and \( \phi \to e^+e^- \) both exist (*). Further developments of the mixing theory can be checked if we compare the partial decay rates (25) and (26), which give us

\[
\tan \theta_y = \frac{\sqrt{m_\phi \cdot \Gamma_{\omega \to e^+e^-}}}{\sqrt{m_\omega \cdot \Gamma_{\phi \to e^+e^-}}},
\]

and in terms of the generalized mixing angle \( \theta_\phi \) (see eq. (10)):

\[
(28) \quad \tan \theta_\phi = \frac{\sqrt{m_\omega \cdot \Gamma_{\omega \to e^+e^-}}}{\sqrt{m_\phi \cdot \Gamma_{\phi \to e^+e^-}}} = \frac{m_\omega}{m_\phi} \tan \theta_y.
\]

As mentioned before, in order to have a self-consistency check on the \( SU_3 \) nature of the photon, it is necessary to compare \( \Gamma_{\omega \to e^+e^-} \) and \( \Gamma_{\phi \to e^+e^-} \) with \( \Gamma_{\rho \to e^+e^-} \), using for example the relation (27) (where it should be emphasized that the factor \( \langle w \rangle \) remains unknown).

A more stringent relation can be established between \( \Gamma_{\rho \to e^+e^-}, \Gamma_{\omega \to e^+e^-} \) and \( \Gamma_{\phi \to e^+e^-} \) if we believe in the First Generalized Weinberg Spectral Function Sum Rule (FGWSR). Great interest in the Weinberg Sum Rules [23] was sparked from

\[
m_{A_1} = \sqrt{2}m_\rho,
\]

obtained from the first and second WSR, assuming pole dominance and the KSFR relation [24]. The two Weinberg Sum Rules related objects carrying the same isospin. The generalization produced sum rules relating objects of different isospin. According to Das, Mathur, and Okubo [17], and to Oakes and Sakurai [18] (we shall refer to them as DMS and OS, respectively) the generalization of the first WSR establishes the following relation among the \((e^+e^-)\) vector meson decay rates: (**)

\[
(29) \quad \frac{1}{2} m_\rho \cdot \Gamma_{\rho \to e^+e^-} = m_\omega \cdot \Gamma_{\omega \to e^+e^-} + m_\phi \cdot \Gamma_{\phi \to e^+e^-}.
\]

(*). Assuming the selectivity of the electromagnetic interaction (see Sect. 2.3 and 2.4).

(**) This relation can easily be derived using eqs. (31), (15a), (24), (25), and (26), and remembering that the isovector coupling of the photons is \( \sqrt{3} \) times stronger than the isoscalar coupling.
Furthermore, the «current mixing» result [16] between the two mixing angles $\theta_Y$ and $\theta_N$ (see relation (10)) is also derived from the 1st GWSR [17, 18]

\[
\tan \theta_\phi = \frac{\sqrt{m_\omega \cdot \Gamma_{\omega \to e^+ e^-}}}{\sqrt{m_\phi \cdot \Gamma_{\phi \to e^+ e^-}}} = \frac{m_\omega}{m_\phi} \tan \theta_Y = \frac{m_\omega}{m_\phi} \tan \theta_N.
\]

This seemed to imply [18] that in the vector-meson dominance approximation, the «current mixing» model of KLZ and CS is the only theory of $(\omega_8-\omega_1)$ mixing which is compatible with the 1st GWSR. As shown later by Majumdar [25], Weinberg’s first Sum Rule and the vector dominance hypothesis do not exclude any of the $(\omega_8-\omega_1)$ mixing models, i.e., either «mass mixing» or «current mixing». What happens is that a particular model of the $(\omega_8-\omega_1)$ mixing demands a particular form of the spectral function sum rule. For example, in order to have relation (30), it is necessary to assume the Schwinger term between $J_{\mu}^p$ and $J_{\nu}^{(1)}$ to be zero [25], besides assuming the 1st GWSR and the vector dominance hypothesis.

Relation (29) is more stringent than (27); however, it gives no predictions for $\theta_Y$. In order to predict $\theta_Y$, a precise model for $SU_3$ symmetry-breaking is needed. Various models have been presented in the literature:

i) the quark model of Van Royen and Weisskopf [26];

ii) two models of the «mass mixing» type by KLZ [16];

iii) three models of the «current mixing» type by KLZ [16], DMO [17], and OS [18], respectively.

It turns out that the two models of KLZ and OS are in fact identical [27]. We shall just mention one point of the DMO and KLZ+OS models in connection with the Generalized Weinberg Sum Rules.

In fact, as pointed out by Das-Mathur-Okubo [17] and by Sakurai [28], the 2nd GWSR must be abandoned as long as vector meson dominance approximation is considered valid. This is because, if we assume that the spectral functions are dominated by the known vector mesons $\rho$, $\omega$ and $\phi$, we have

\[
\text{from the 1st GWSR:} \quad \frac{g^2_{\rho \gamma}}{m_\rho^2} = \frac{g^2_{\omega \gamma}}{m_\omega^2} + \frac{g^2_{\phi \gamma}}{m_\phi^2},
\]

(31)

\[
\text{from the 2nd GWSR:} \quad g^2_{\rho \gamma} = g^2_{\omega \gamma} + g^2_{\phi \gamma}.
\]

(23)

For the consistency of eqs. (31) and (32) it is necessary that $m_\rho^2 = m_\omega^2 = m_\phi^2$, which does not agree with observation. Das, Mathur and Okubo [17] and Oakes and Sakurai [18], proposed to change the 2nd GWSR à la Gell-
The basic $SU_3$ mixing: $\omega_8 \leftrightarrow \omega_1$

Mann and Okubo, i.e.:

\[
\text{DMO} \int dm^2 \{ \rho_8(m^2) + 3\rho_8(m^2) - 4\rho_4(m^2) \}
\]

2nd GWSR: \( \int \{ \rho_8(m^2) - \rho_8(m^2) \} dm^2 \) changed into

\[
\text{OS} \int \frac{dm^2}{m^4} \{ \rho_8(m^2) + 3\rho_8(m^2) - 4\rho_4(m^2) \},
\]

where \( \rho_i \) are the spectral functions and the \( i \)'s refer to the $SU_3$ component in the octet. It is interesting to notice that the DMO proposal [17] clearly implies that the Weinberg spectral function integral satisfies an octet-breaking formula, while the OS proposal [18] corresponds to the fact that it is the inverse propagator matrix for the current that satisfies an octet-breaking formula (*). These two proposals give different values for \( \theta_Y \). All the above-mentioned theoretical prediction will be reported in Fig. 15, Sect. 5, where they are compared with experimental data.

4. – The first experimental measurement of the \((\omega_8^8-\omega_4^8)\) mixing.

4'1. Introduction. – As mentioned in Sect. 1'3, there were two ways of attempting a direct check of the \((\omega_8-\omega_4)\) mixing hypothesis: i) either by using strong interactions for the production of the vector mesons \( \omega \) and \( \phi \), and subsequently detecting their rare decay modes into $e^+e^-$:

\[
\begin{align*}
\pi^- + p & \rightarrow \omega + n \\
& \rightarrow e^+e^-, \\
\pi^- + p & \rightarrow \phi + n \\
& \rightarrow e^+e^-
\end{align*}
\]

(33)

or ii) by using the electromagnetic production processes of \( \omega \) and \( \phi \) from \((e^+e^-)\) collisions, and detecting the \( \omega \) and \( \phi \) via their strong decay modes:

\[
\begin{align*}
e^+ + e^- & \rightarrow \omega \rightarrow \text{strong decay modes,} \\
e^+ + e^- & \rightarrow \phi \rightarrow \text{strong decay modes,}
\end{align*}
\]

(34)

(*) As emphasized by DMO [17], the 2nd GWSR is obtained assuming for the spectral functions \( \rho_i \)'s, superconvergent conditions much stronger than those needed to obtain the 1st GWSR. This is why one expects the 1st GWSR to be much better than the 2nd GWSR, and therefore one tries to improve the last one.
As reported by Ting at the Vienna Conference [29], the first successful experiment on the direct determination of the \(\theta_{12} - \theta_{13}\) mixing angle was done at CERN by the Bologna-CERN Collaboration [30, 31] using reactions (33).

It is obvious from the examination of the final states in reactions (33) that in order to perform the experiment it is necessary to have a large « neutron » detector and a large « electron » detector so as to be able to measure with good acceptance all particles present in the final states of the above-mentioned reactions.

4'2. The experimental set-up. – A schematic diagram of the experimental set-up is shown in Fig. 2. It consists of the following:

i) A system of «beam-defining counters» ČUSŘ: Č is a gas Čerenkov counter to anticoincide the electrons present in the primary beam; \(U\) is an important plastic scintillator counter used in the timing of the neutron; \(S\) is a very thin (0.05 cm) plastic scintillator counter in order to reduce as much as possible the interactions outside the \(H_2\) target; \(R\) is an anticoincidence counter to remove beam halo.

ii) A 40 cm long, 5 cm diameter \(H_2\) target. A veto counter, not shown in Fig. 2, is placed behind the target in order to anticoincide noninteracting pions.

iii) Two electron detectors called « top » and « bottom ». In front of them there are coincidence counters and thin-plate spark chambers, which, for the sake of clarity, have all been omitted in Fig. 2.

iv) Two neutron detectors, called « left » and « right », with anticoincidente counters \(\bar{G}_L\) and \(\bar{G}_R\) in front of them to reject charged particles.
impinging in the «neutron counters». These two identical neutron detectors had a sensitive surface and volume equal to 2.16 m² and 0.78 m³, respectively.

A neutron detector is made of 12 elements of plastic scintillator, each having dimensions (100 x 18 x 18) cm³. Each element is viewed by two XP-1040 photomultipliers placed on its two small faces (see Fig. 3). The

large volume of scintillator, in the particular geometrical arrangement chosen, allows a mean detection efficiency of about 26% in the range (40 ÷ 560)MeV neutron kinetic energy, for a laboratory solid angle of 0.14 sr at 4 m radial distance from the centre of the H₂ target. An interesting feature of this instrument is the accuracy achieved in locating incident particles; this accuracy is ±1.4 cm for charged particles, and ±2.5 cm for neutrons. The accuracies achieved for the time-of-flight measurement are ±0.35 ns for charged particles and ±0.7 ns for neutrons. It is interesting to note that the relative timing of all photomultipliers in the neutron counters could be equalized to ±0.1 ns. An example of this time-equalization is shown in Fig. 4, where \( t_1 \) is the time difference between the \((U)\) signal and the signal from any photomultiplier at one side of the neutron counter, \( t_2 \) corresponds to signals from the other side of the neutron counter, and \( \theta \) is the difference between the two, obtained electronically (*).

(*) For more details on this instrument we refer the reader to Bollini et al. [32].
Fig. 4. – Relative timing of the 12 elements of the neutron detector «Right». The abscissa indicates the identification number of an element, the ordinate the relative timing.

Fig. 5. – Spatial resolution of a neutron counter, as measured with a muon beam. Each of the peaks in this spectrum corresponds to a given position of the beam-defining telescope along the neutron counter.
Typical data on position resolution and linearity of the neutron counters are shown in Figs. 5 and 6, respectively. In Fig. 5 the curves are labeled with the distance from the edge of the counter, and the spatial resolution for all positions in the counter is ±1.4 cm for charged particles. In Fig. 6 the ordinate is the distance from one edge of the counter, and the abscissa is the channel number in which the peak corresponding to a certain position (as shown in Fig. 5) falls. The counter is seen to be linear.

Notice that there is a total of 24 elements. For all of them the above calibrations were repeated periodically in order to check the correct performance of the apparatus. For example, the neutron counter stability over a week is shown in Fig. 7a, where the time variation for \( \theta_n \), \( t_1 \), and \( t_2 \) signals is plotted for each element of the neutron counter. The time stability of the neutron detector is remarkably good. Figure 7b) shows the high-voltage variation over a period of one week for the «neutron-right».

The neutron detectors measure the times-of-flight \( t_n \) and angles of emission \( \theta_n \) of neutrons in reactions (33), thus allowing a determination of the missing masses in reactions (33), \textit{i.e.}, of the mass of the produced meson. The mass resolution obtained with the above space and time resolutions depends on the kinematical region in the plane \((t_n, \theta_n)\) (see Fig. 8). It is ±4 MeV in the \( \gamma \)-mass region, ±10 MeV in the \( \omega \)-mass region, and ±15 MeV in the \( \phi \)-mass region.
Fig. 7. – a) Neutron counter stability over a week: the time variation for $\theta$, $t_1$, and $t_2$ signals is plotted for each counter. b) Typical maximum variation of the high-voltage supplies over a period of one week, for all the photomultipliers of one neutron detector. Notice that in fact a variation of 1 V produces a 15 ps shift in the time definition.
The basic $SU_3$ mixing: $\omega_8 \leftrightarrow \omega_1$

Fig. 8. – The neutron time of flight $t_n$ over a 4 m path is plotted vs. the neutron emission angle $\theta_n$ in the laboratory system. The ordinate on the right refers to the neutron kinetic energy. The kinematic curves are labeled by the corresponding neutron missing masses. The dashed lines indicate constant values of $\cos \theta_n$.

The electron detectors are shown in detail in Fig. 9. Each electron detector consists of nine elements, each one being made of a piece of lead followed by a two-gap spark chamber and a plastic scintillation counter. The first layer of lead is two radiation lengths thick; the other layers are one radiation length thick. The over-all detector thickness is half a meter. Before the first lead layer there are two thin-plate spark chambers to allow precise kinematical reconstruction of the events. Thus a long $H_2$ target could be used, when looking for rare events, without losing accuracy in the missing-mass measurement by the neutrons. The two detectors may be rotated independently about a horizontal and vertical axis through the $H_2$ target.
Fig. 9. – Side view of the electron detectors. The $M_p$ and $M_B$ are scintillation counters, $K_p$ and $K_B$ are thin-plate spark chambers. Each electron detector consists of nine layers of lead, spark chamber, and scintillator sandwiched together.

Figure 10 shows a calibration of one of the two electron detectors. The response of the telescope is plotted as a function of the energy of the beam. We see that the instrument is linear. The three sets of points in the upper curve correspond to measurements made at different times (given by the run number), and to two different positions of the beam in the detector. Near the extremes of the detector, the pulse-height decreases and the calibrations are parametrized according to the maximum depth of the detector available to the shower. The two lower curves are two of these edge-effect calibrations. During the calibrations the detectors were rotated to many positions, and calibrated as a function of depth and energy in order to allow the calculation of the total efficiency for any event configuration. For fixed depths we see that the fluctuations are small; in any case the system was frequently calibrated in order to be sure that pions were not wrongly identified as electrons.

Figure 11a shows a complete efficiency calibration at fixed energy. The purpose of this figure is to show that the electron detectors «top» and «bottom» had very similar characteristics. The open circles refer to the bottom detector and the full circles to the top detector. The electron energy for these two sets of points in 1050 MeV. Figure 11b shows a family of curves corresponding to 170 calibration points taken at energies from 1.05 GeV down to 0.45 GeV.
Table I summarizes the efficiencies of the electron detectors. From as low as 400 MeV up to 1100 MeV, one can reject pions with a power of $\sim 3 \times 10^{-4}$. For each particle and each momentum there are three numbers: the electronic efficiency, the picture analysis efficiency, and their product, the over-all efficiency. As mentioned above, the electron detectors consist of counters and spark chambers, so there is an electronic rejection in the trigger; then, once the pictures are taken, there is a further rejection in the picture analysis. The latter is very important because it allows the elimination of charge exchange of pions, which is the greatest source of trouble when you want to distinguish
a pion from an electron. From 400 MeV to 1100 MeV, the power of the telescope against pions is practically the same, and the efficiency for electron detection is very good—between 70 and 80%.

**Table I. Efficiency in the electron detector «bottom»**.

<table>
<thead>
<tr>
<th>Momentum (MeV/c)</th>
<th>Particle</th>
<th>Electronic efficiency (%)</th>
<th>Picture analysis efficiency (%)</th>
<th>Over-all efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>e</td>
<td>(77.5±2.2)</td>
<td>(89.0 ± 2.2)</td>
<td>(69.0±2.6)</td>
</tr>
<tr>
<td></td>
<td>π</td>
<td>(6.3±0.2)</td>
<td>(0.43±0.2)</td>
<td>(2.7±1.6)×10⁻⁴</td>
</tr>
<tr>
<td>1100</td>
<td>e</td>
<td>(94.0±1.5)</td>
<td>(88.0 ± 2.0)</td>
<td>(83.0±2.3)</td>
</tr>
<tr>
<td></td>
<td>π</td>
<td>(17.6±0.6)</td>
<td>(0.16±0.16)</td>
<td>(2.8±2.8)×10⁻⁴</td>
</tr>
</tbody>
</table>

To summarize, each of the two telescopes has ~3×10⁻⁴ rejection against pions, giving a product ~10⁻⁷, which is the rejection factor for charged ππ pairs and any other sort of charged multipion events. It is this rejection power that allows the study of rare events such as (e⁺e⁻) decays of strongly interacting particles.

4.3. **Some relevant details.** — Table II summarizes the most relevant parameters of the experiment for ω and φ decays into (e⁺e⁻). The ω-mass region and the φ-mass region have been investigated using the same experimental
set-up at different angles of acceptance for the neutron and electron detectors, but changing the primary beam momentum in order to maximize the number of observable events, i.e. (production cross-section) × (acceptance). For more details we refer the reader to the original papers (refs. [30] and [31]).

**Table II.** Relevant parameters of the \( \omega \) and \( \phi \) decay experiments (*).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \omega ) (ref. [30])</th>
<th>( \phi ) (ref. [31])</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_\pi )</td>
<td>1.67 GeV/c</td>
<td>1.93 GeV/c</td>
</tr>
<tr>
<td>( \theta_{\text{lab}}^0 )</td>
<td>( 31^\circ \pm 45^\circ )</td>
<td>( 19^\circ \pm 33^\circ )</td>
</tr>
<tr>
<td>( \theta_{\text{c.m.}}^0 )</td>
<td>( 165^\circ \pm 94^\circ )</td>
<td>( 160^\circ \pm 70^\circ )</td>
</tr>
<tr>
<td>( T_n )</td>
<td>( (42 \pm 430) ) MeV</td>
<td>( (95 \pm 560) ) MeV</td>
</tr>
<tr>
<td>( t_n )</td>
<td>( (46 \pm 18) ) ns</td>
<td>( (32 \pm 17) ) ns</td>
</tr>
<tr>
<td>( q^2 )</td>
<td>( (0.08 \pm 0.8) ) (GeV/c)*2</td>
<td>( (0.18 \pm 1.1) ) (GeV/c)*2</td>
</tr>
<tr>
<td>( \Delta M )</td>
<td>( \pm 10 ) MeV</td>
<td>( \pm 15 ) MeV</td>
</tr>
<tr>
<td>( \theta_{\text{lab}}^{\omega,\text{em}} )</td>
<td>( 6^\circ \pm 32^\circ )</td>
<td>( 5^\circ \pm 25^\circ )</td>
</tr>
<tr>
<td>( \theta_B )</td>
<td>( 32^\circ )</td>
<td>( 36^\circ )</td>
</tr>
<tr>
<td>( T_1 = B_1 ) threshold</td>
<td></td>
<td>( 1.7 \times (dE/dx)_{\text{min}} )</td>
</tr>
<tr>
<td>( \sum T = \sum B ) threshold</td>
<td></td>
<td>150 MeV</td>
</tr>
<tr>
<td>( \sum \sum ) threshold</td>
<td></td>
<td>700 MeV</td>
</tr>
</tbody>
</table>

(*\( p_\pi \) is the primary pion momentum. It has been chosen at the maximum of the production cross-section. \( \theta_{\text{lab}}^0 \) is the angular range covered by the neutron detectors in the laboratory system. \( \theta_{\text{c.m.}}^0 \) is the corresponding value in the centre-of-mass system. \( T_n \) is the range of neutron kinetic energies accepted in the above angular range. It follows the corresponding range of neutron time-of-flight \( t_n \). \( q^2 \) is the range of four-momentum transfer. \( \Delta M \) is the mass uncertainty. \( \theta_{\text{lab}}^{\omega,\text{em}} \) is the angular range of vector-meson production. \( \theta_B \) is the angular position of the electron detectors in the vertical plane containing the beam. \( T_1 \) and \( B_1 \) are the thresholds of the first counters in the electron detectors, i.e. after two radiation lengths in lead. \( \sum T \) and \( \sum B \) are the thresholds of the two electron detectors «top» and «bottom». These thresholds were fixed at a very low value of 150 MeV in order to have high efficiency in the detection of electromagnetic showers. \( \sum \sum = \sum T + \sum B \) is the total electromagnetic energy released in «top» plus «bottom». We trigger every time that the total energy is greater than 700 MeV; again this choice of low threshold is taken in order to have good detection efficiency for electromagnetic showers originated either by electrons or photons.)

Another important point worth mentioning is the way in which \( \gamma \gamma \) events are rejected. In the description of the electron detectors, it was pointed out that the rejection power against charged \( \pi\pi \) pairs was \( \sim 10^{-7} \). But in \((\pi^- p)\) interactions, two or more \( \pi^0 \)s can be produced; \( \pi^0 \)s decay into \( \gamma \)s, which then materialize in the target or in the plastic scintillator before the thin-plate spark chambers, thus producing electron-positron pairs which can simulate a genuine e± from vector meson decay. It is possible to recognize
most of these $\gamma$-produced «fake $e^\pm$», because they are really «electron-positron pairs» whose opening becomes sufficiently large by multiple scattering in the trasversal of the material which is in front of the kinematic spark chambers. The distribution of the distance between two tracks of a pair is shown in Fig. 12. The wide part of the spectrum is that expected from multiple scattering. The peak at zero is clearly due to a genuine single $e^\pm$ and not ($e^+e^-$) pairs simulating single tracks. In fact, from the measured distribution (shown in Fig. 12) the number of «fake $e^\pm$» present in the
The basic SU₃ mixing: \( \omega_8 \approx \omega_1 \)

peak is expected to be \( \sim 2 \). If we now plot the mass distribution of the events in the peak of Fig. 12, we obtain the distribution shown in Fig. 13.

Notice that this result represents the first successful attempt to resolve the \( \omega \)-peak from the \( \rho \). As mentioned in the introduction, the experimental

![Graph showing mass distribution](image)

**Fig. 13.** – For \( \omega \rightarrow e^+e^- \), mass distribution of \( e^+e^- \) pairs in the \( \omega \)-region obtained with the same type of analysis as in the \( \phi \)-case. The shape of the \( \rho \)-distribution is determined by its natural width, the known production distribution and density matrix, and the experimental acceptance and resolution. The dashed curve is the result of a maximum likelihood fit to the experimental data.
conditions were chosen in such a way as to minimize the amount of observable ρ's. In fact, the broken curve is the expected ρ-mass distribution calculated from the known production and decay angular distribution combined with the experimental acceptance.

Repeating the same type of analysis for the φ-case gave the mass distribution shown in Fig. 14. In the φ-mass region there is a total of ten events

\[
\phi \to e^+ + e^-
\]

Fig. 14. – For \( \phi \to e^+ e^- \), the mass distribution for those events with zero opening distance in Fig. 12.
minus one background events. To have a small background was an essential feature of the experiment, the limitation in the number of observed $\phi \rightarrow e^+e^-$ being due to the available machine time. Notice the difference between the distribution shown in Fig. 14 and that of the previous one shown in Fig. 13. The background below the $\phi$-peak is flat because there is no $\rho$-like object in the $\phi$-mass region. In conclusion, a total of nine events of unambiguously identified $\phi \rightarrow e^+e^-$ decays were observed.

4.4. Results. – Table III summarize the experimental results obtained on the $(e^+e^-)$ decay of $\omega$ and $\phi$ mesons. Let us start with the $\omega$ column. The first entry is the direct experimental result obtained. Below there is the $\omega$-production cross-section which is well known; these two numbers then give the branching ratio, which together with the total width of the $\omega$, taken from the Rosenfeld tables [33], gives the partial width in the bottom entry.

| Table III. – Experimental results of $\omega$ and $\phi$ decay. |
|-------------------------------------------------|------------------|-----------------|
| $\omega \rightarrow e^+e^-$ (ref. [30])          | $\phi \rightarrow e^+e^-$ (ref. [31]) |
| $\sigma(\pi^-p \rightarrow nV) \rightarrow e^+e^-$ | $(67 \pm 25) \times 10^{-33}$ cm$^2$ | $(18.4 \pm 6.9) \times 10^{-33}$ cm$^2$ |
| $\sigma(\pi^-p \rightarrow nV) \rightarrow$ all   | $(1.67 \pm 0.07) \times 10^{-27}$ cm$^2$ | $(30 \pm 6) \times 10^{-30}$ cm$^2$ |
| $\Gamma(V \rightarrow e^+e^-)/\Gamma(V \rightarrow$ all) | $(4.0 \pm 1.5) \times 10^{-5}$ | $(6.1 \pm 2.6) \times 10^{-5}$ |
| $\Gamma(V \rightarrow$ all) (*)                  | $(12.2 \pm 1.3)$ MeV | $(3.4 \pm 0.8)$ MeV |
| $\Gamma(V \rightarrow e^+e^-)$                   | $(0.49 \pm 1.19)$ keV | $(2.1 \pm 0.9)$ keV |

(*) Date taken from the Rosenfeld tables.

In the case of the $\phi$, the production cross-section is much lower and it is not so well known as that of the $\omega$. In fact, a point in the $\phi$-production cross-section was measured by the Bologna-CERN Collaboration [10], because when the experiment was started, the $\phi$ production had not been observed in pion-nucleon interactions [34]. The energy at which the $\phi$-production cross-section was measured is slightly higher than that at which the decay experiment was performed. In fact, a maximum value in the cross-section had still to be found, when a bubble chamber group [35] published a paper in which the maximum seemed to be 150 MeV lower; so the experiment was performed at the lower beam momentum. Notice that the measured value of the $\phi$-production cross-section [10] is in very good agreement with
the bubble chamber data [35]. Again the total width is taken from the Rosenfeld tables [33] to derive the partial width.

The value of the generalized mixing angle $\theta_G$ was thus determined to be:

$$\tan \theta_G = \frac{\sqrt{m_\omega \cdot \Gamma_{\omega \rightarrow e^+e^-}}}{\sqrt{m_\phi \cdot \Gamma_{\phi \rightarrow e^+e^-}}} \rightarrow \theta_G = 23^\circ \pm 7^\circ - 5^\circ.$$ 

This result is in excellent agreement with the «current mixing» theory of Kroll et al. [16] and of Oakes and Sakurai [18] (the slight difference between the KLZ and OS predictions for $\theta_G$ is due to the use of slightly different mass values, the two models of $SU_3$ symmetry breaking being identical [27]).

In connection with previous remarks, the effect of $(\omega-\rho)$ interference has also been estimated [36], the result being a variation of $\pm 3^\circ$ for complete constructive or destructive interference respectively. It should be noticed, however, that in the OPE model the $(\omega-\rho)$ interference is exactly zero.

Following the theoretical considerations previously reviewed, the results obtained by the Bologna-CERN collaboration led to the following conclusions [30, 31]:

i) the general idea of $(\omega_0-\omega_1)$ mixing is confirmed;

ii) the First Generalized Weinberg Spectral Function Sum Rule (saturated using only $\rho$, $\omega$, $\phi$) is valid within 30% over all experimental uncertainty;

iii) there is no evidence for the coupling of the electromagnetic field to an $SU_3$ singlet;

iv) the old $A$ quantum number [37] is not a good quantum number;

v) the fact that $(e^+e^-)$ decays of $\omega$ and $\phi$ are observed with the measured rates is a direct evidence that the $J^{PC}$ quantum numbers of the $\omega$ and $\phi$ are indeed $1^{--}$. 

5. — Present status and conclusions.

The results obtained using strong production reactions (33) were followed by other measurements of Ting and collaborators at DESY [38] and later by the Orsay group [39].

Ting studied the photoproduction of $\rho$'s and of $\phi$'s, thus obtaining the partial widths $\Gamma_{\rho \rightarrow e^+e^-}$ and $\Gamma_{\phi \rightarrow e^+e^-}$. The DESY result with its uncertainties for $\theta_G$ is shown in Fig. 15, where also the Orsay data are plotted. The Orsay group made use of the $(e^+e^-)$ storage ring ring facility in order to determine the
The basic $SU_3$ mixing: $\omega_8 \leftrightarrow \omega_1$

partial widths $\Gamma_{\rho \rightarrow \pi^+\pi^-}$, $\Gamma_{\omega \rightarrow \pi^+\pi^-}$, and $\Gamma_{\phi \rightarrow \pi^+\pi^-}$. As mentioned previously, the production reactions are given in eqs. (35)-(37),

\begin{align*}
(35) & \quad e^+e^- \rightarrow \rho \rightarrow \pi^+\pi^- \\
(36) & \quad e^+e^- \rightarrow \omega \rightarrow \pi^+\pi^-\eta \\
(37) & \quad e^+e^- \rightarrow \phi \rightarrow KK \rightarrow \pi^+\pi^-\pi^0
\end{align*}

where the vector mesons are produced «electromagnetically» and their strong decays are observed. The identification of the $\rho$, $\omega$, $\phi$ is done using the information coming from the total $(e^+e^-)$ energy, while the identification of the final states in the various reactions is performed via the use of geometrical constraints on the decay products in the various reactions (35)-(37).

All experimental data available so far are reported in Fig. 15, where all the theoretical predictions are also shown. The diagram is constructed so as to reproduce in a graphically clear way the first GWSR, as derived from DMO [17] and OS [18], i.e., relation (29). Notice that the quark model prediction [26] numerically satisfies the first GWSR. This should not be so strange, as the results of the quark model can be derived from the following set of assumptions [40]: i) PCAC; ii) First and Second GWSR; iii) pole dominance in the First and Second GWSR.

It should be emphasized that the two predictions for $\theta_\alpha$, i.e., that of DMO [17] and that of KLZ [16] and OS [18], would coincide to first order $SU_3$ symmetry-breaking. These models are all of the «current mixing» type; they differ only by second-order $SU_3$ symmetry-breaking effects. Also the two «mass mixing» models of KLZ [16] differ only in second-order $SU_3$ symmetry-breaking.

It would be misleading to try combining the experimental data of Fig. 15 in the hope of giving an answer to this extremely interesting question which refers to second-order $SU_3$ symmetry-breaking effects. There are, in fact, no other experiments where second-order $SU_3$ symmetry-breaking effects can so neatly be measured.
Fig. 15. – Theoretical predictions and experimental measurements of the ($\omega_2$-$\omega_1$) mixing.

But we have extended the discussion too far. Let us not forget that a keypoint in the great $SU_3$ castle has withstood the experimental proof: the reason why the unitary-symmetry mass formula of Gell-Mann and Okubo does not hold true for the vector meson multiplet is really the ($\omega_2$-$\omega_1$) mixing mechanism. However, the field is now open for checking second-order effects in $SU_3$ symmetry-breaking.

REFERENCES